

THE RADIATION OF AN ELECTRIC AND MAGNETIC FIELD
SOURCES IN CHAOTICALLY MOVING MEDIUM

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1. The great interest to the problems of electromagnetic waves radiation and propagation in turbulent media is caused by the importance of these problems for plasma physics, atmosphere and ocean physics. In overwhelming majority of papers, devoted to the investigation of radiation in turbulent media, it is supposed, that medium parameters (for example permittivity) fluctuate in space and time. Moreover, the chaotical motion of turbulent medium is not taken into account. At the same time, chaotical motion of medium leads to the additional radiation, which sometimes is assential (for example the radiation of magnetic field sources). Further, we shall consider some radiation effects in chaotically moving turbulent medium.

On the basis of Maxwell equations and Minkovskii material ratio [1] it is possible to obtain the generalized Pointing equation for slowly moving media (derivatives of velocity with respect to coordinate and time are small, permeability is equal to unit)

$$\begin{aligned} & c(4\pi)^{-1} \operatorname{div}[\vec{E}\vec{H}] + \vec{j}\vec{E} + \partial/\partial t \{ (1/8\pi)(\varepsilon E^2 + H^2) - (\kappa/4\pi c) [\vec{V}[\vec{E}\vec{H}]] \} - \\ & - (\kappa/4\pi c) \left(\frac{\partial \vec{V}}{\partial t} [\vec{E}\vec{H}] \right) = 0 \end{aligned} \quad (1)$$

This equation has such interpretation: in a chaotically moving medium the change of electromagnetic field energy density (which is described by the third term) takes place not only at the expense of radiation or field work upon a current (the first and the second terms respectively), but also at the expense of energy density changing. The latter is caused by the liberation or absorption in the medium by the "external force", which forces the medium to move nonuniformly (the fourth term). So, in nonuniformly moving medium the new channel of energy transfare is open over which the energy can flow in or flow out from the system. The irregular moving medium is the same as an nonstationary medium, its permittivity changes in time.

The expression for electric and magnetic field sources radiation in inhomogeneous and slowly nonuniformly moving medium was derived by perturbation method in the paper [2]. The velocity was a function of coordinates and time. It was shown, that the spectral and angular energy distribution of radiation with λ polarization was described by the ratio

$$W_{\vec{k}, \lambda} d^3k = \frac{\kappa^2 \omega^2 (2\pi)^4}{4c^2 \varepsilon(\omega)} \left| \int d\vec{k}_1 d\omega_1 (\vec{V}(\vec{k} - \vec{k}_1, \omega - \omega_1) ([\vec{H}^q(\vec{k}_1, \omega_1) \vec{e}^\lambda]) + \right.$$

$$+\varepsilon^{1/2} \left[\left[\vec{n} \vec{e}^\lambda \right] \vec{E}^q(\vec{k}_1, \omega_1) \right] \Big|^2 d^3k \quad (2)$$

where $\vec{E}, \vec{H}, \vec{D}$ and \vec{B} are the electric and magnetic fields, electric and magnetic inductions respectively, $\vec{V}=\vec{V}(\vec{r}, t)$ is medium velocity ($V \ll C$), \vec{j} is current density, ε -permittivity in the rest medium frame of reference, $\kappa=\varepsilon-1$, \vec{k} and ω are the wave vector and the frequency of radiated wave, $\vec{n}=\vec{k}/k$, \vec{e}^λ -unit polarization vector ($\lambda=1,2$), \vec{E}^q and \vec{H}^q are electric and magnetic fields caused by external charges and currents.

Let's generalize this theory for the nonuniformly moving media. The velocity of turbulent medium is a random function of coordinate and time. The random velocity field is supposed to be stationary, homogeneous and isotropic [3]

$$\langle \vec{V}_1(\vec{k}_1, \omega_1) \vec{V}_m^*(\vec{k}_2, \omega_2) \rangle = B_{1m}(k_1, \omega_1) \delta(\vec{k}_1 - \vec{k}_2) \delta(\omega_1 - \omega_2) \quad (3)$$

where the brackets mean the statistical average. If the medium is incompressible ($\text{div} \vec{V}=0$) the spectrum of correlation tensor has the form

$$B_{1m}(k, \omega) = F(|\vec{k}|, \omega) (\delta_{1m} - k_1 k_m / k^2)$$

2. At first consider the radiation of magnetic field sources. By averaging (2) we obtain

$$\begin{aligned} \langle W_{\vec{k}, \lambda} \rangle d^3k = & \frac{\kappa^2 \omega^2 (2\pi)^4}{4c^2 \varepsilon(\omega)} \int d\vec{k}_1 d\omega_1 F(|\vec{k}_1|, \omega_1) \{ |[\vec{H}^q(\vec{k}-\vec{k}_1, \omega-\omega_1) \vec{e}^\lambda]|^2 - \\ & - k_1^{-2} |(\vec{k}[\vec{H}^q(\vec{k}-\vec{k}_1, \omega-\omega_1) \vec{e}^\lambda])|^2 \} d^3k \quad (4) \end{aligned}$$

This formula generalizes the expression for magnetic field source energy radiation in a nonuniformly moving incompressible turbulent medium at arbitrary correlation function F . In the case of constant magnetic field \vec{H}_0 , Fourier-component of which is

$$\vec{H}^q(\vec{k}_1, \omega_1) = \vec{H}_0 \delta(\vec{k}_1) \delta(\omega_1), \quad (5)$$

substitution from (5) into (4) leads to the appearing of delta function square and, correspondingly, to the divergence of energy radiation. The latter evidently is connected first of all with infinite duration of the homogeneous in time process and also with infinite dimensions of radiated area. Therefore we must talk about the mean radiation power from unit volume of a turbulent medium. Integration over \vec{k}_1 and ω_1 , and summation over two independent polarization for spectral and angular distributions of the mean radiation power gives

$$P_{\omega, \vartheta} d\omega d\vartheta = \frac{\kappa^2 \pi \varepsilon^{1/2} H_0^2}{c^5} F\left(\frac{\omega}{c}, \varepsilon^{1/2}, \omega\right) \cos^2 \vartheta \sin \vartheta d\vartheta d\omega \quad (6)$$

where ϑ is an angle between \vec{k}_0 and the direction of radiation. By integrating (6) over ϑ we obtain the power spectrum

$$P_\omega = \frac{2\pi^2 \pi \varepsilon^{1/2} H_0^2}{3c^5} \omega^4 F\left(\frac{\omega}{c} \varepsilon^{1/2}, \omega\right) d\omega \quad (7)$$

For example, the spectral component of the correlation tensor of solenoidal velocity fields ($\text{div } \vec{v}=0$)

$$B_{lm}(\vec{\rho}, \tau) = \langle v_l(\vec{r}, t) v_m(\vec{r} + \vec{\rho}, t + \tau) \rangle = \langle v^2 \rangle [(1 - \rho^2/l^2) \delta_{lm} + \rho_l \rho_m / l^2] \exp(-\rho^2/l^2) \Gamma(\tau)$$

at $\Gamma(\tau) = \exp(-\tau^2/T^2)$ is proportional to

$$F(k, \omega) = (\pi^2/4) \langle v^2 \rangle T l^5 k^2 \exp\left(-\frac{k^2 l^2}{4} - \frac{\omega^2 T^2}{4}\right) \quad (8)$$

By inserting (8) into (7) the total power radiation from unite volume of turbulent medium, covered under the constant homogeneous external magnetic field is given by

$$P = 20\pi^{7/2} \pi^2 \varepsilon^{3/2} H_0^2 T \langle v^2 \rangle l^5 / c^7 T_0^7 \quad (9)$$

where l and T are the characteristic spectral and temporal scales of turbulent medium velocity fluctuations, $T_0 = T [1 + \varepsilon^{1/2} l / cT]^2$.

The mean angular and spectral radiate energy distribution of external sources creating the electric field \vec{E}^q in the nonuniformly moving turbulent medium may be written

$$W_{\vec{k}, \lambda} d^3k = \frac{\pi^2 \omega^2 (2\pi)^4}{4c^2} \left| \int d\vec{k}_1 d\omega_1 F(|\vec{k}_1|, \omega_1) k_1^{-2} [\vec{k}_1 [\vec{E}^q(\vec{k} - \vec{k}_1, \omega - \omega_1) \vec{e}^{3-\lambda}]]^2 \right| d^3k \quad (10)$$

The radiation of nonstationary immovable medium under the action of applied constant field was investigated in [4].

3. Let's consider the radiation of fixed q charge source in chaotically moving medium. Fourier-component of an electric field \vec{E}^q is

$$\vec{E}^q = -i \frac{q}{2\pi^2} \frac{\vec{k} \delta(\omega_1)}{\varepsilon(0) k^2} \quad (11)$$

$\varepsilon(0)$ is statical permittivity. Inserting (11) into (10) and summing up the polarization, after integrating over the polar and azimuthal angles, at arbitrary correlation function F we obtain

$$P_{\vec{k}} d^3k = \frac{\kappa^2 \omega^2 q^2}{c^2 \epsilon^2(0)} \int_0^{\infty} dk_1 F(k_1, \omega) \left\{ 1 + k_1^2 k^{-2} + \frac{1}{4} k_1^3 k^{-3} (1 - k^2 k_1^{-2})^2 \right. \\ \left. \ln \left(\frac{k_1 k^{-1} - 1}{k_1 k^{-1} + 1} \right)^2 \right\} d^3k \quad (12)$$

The solution of the same problem on radiation of a fixed point charge in the chaotically immovable medium with spatial-temporal inhomogeneties of permittivity was investigated in [5]. By making use of (8), the mean radiate power spectrum at $k_1 l \gg 1$ becomes

$$P_{\omega} d\omega = \frac{16\pi^{5/2}}{3} \frac{\kappa^2 \epsilon^{3/2}(\omega) q^2}{c^5 \epsilon^2(0)} \langle v^2 \rangle l^2 T \omega^4 \left[1 - \frac{1}{16} \omega^2 c^{-2} l^2 \epsilon(\omega) \right] \\ \exp(-\omega^2 T^2 / 4) d\omega \quad (13)$$

At quite big pulsation of medium velocity fluctuation, when $\omega T \gg 1$, the energy radiation is exponentially small.

References

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