# ADAPTIVE MULTIUSER DETECTION ALGORITHM BASED ON SUBSPACE TRACKING

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## 1. Introduction

MultiUser Detection (MUD) [1] is the key technology against interference in CDMA communication system. In real CDMA communication system, correlation of signals exists, which is named Multiple Access Interference (MAI). MAI produced by several users is small, but with the increase of users or the power of signal, MAI becomes the central problem in CDMA communication system. According to classical direct-sequence spread-spectrum theory, traditional detector filters signal of each user according to its signature waveform, which results in bad performance against MAI. Based on traditional detection technique, the multiuser detection detects desire user by utilizing all information causing MAI fully. Thus MUD has fine performance against MAI, solves the near-far effect, and reduces the requirement for precision of controlling power. So spectral resources can be utilized more effectively, which improves the communication capacity notably.

Over the past more than ten years, various detection techniques have been proposed. The main multiuser detector conclude optimum detector [2], linear decorrelating detector [3], MMSE linear multiuser detector [4], multistage interference cancellation [5], decision-feedback detector [6], and detector based on neural network, etc. Reference [7] has proposed an adaptive multiuser detecor based on subspace tracking, which adopts PASTd algorithm to track the subspace. We found that with a random initialization, the convergence is fairly slow. The reason is that approximate estimation of subspace cumulate errors. Reference [7] calculates an initial matrix using SVD, then tracks the subspace with PASTd, which increase the complexity. The paper introduces the SP-1 (Subspace Projection) [8] tracking subspace, which reduces calculation complexity of the algorithm. And performance is equivalent to that of PASTd using initial matrix. The signal subspace estimation is achieved by SP-1 followed by a new demodulating vector which modifies the MMSE detector.

## 2. Signal Model

Consider a synchronous DS-CDMA communication system in additive white Gaussian noise, which is shared by K simultaneous users. At the receiver, chip-matched filtering followed by chip rate sampling yields a N-vector of chip-matched filer output samples within a symbol interval T, which is

$$\mathbf{r} = \sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \sigma \mathbf{n} \tag{1}$$

where $\mathbf{s}_k = (1/N)[\beta_0^k \ \beta_1^k \cdots \beta_{N-1}^k]^T$ is the normalized signature waveform vector of the $kth$ user , $N$ is the processing gain, $(\beta_0^k, \beta_1^k, \cdots, \beta_{N-1}^k)$ is a signature sequence of $\pm 1's$ assigned to the $kth$ user. $\mathbf{n}$ is a white Gaussian noise vector with mean 0 and covariance matrix $\mathbf{I}_N$ ($\mathbf{I}_N$ denotes the $N \times N$ indetity matrix).

For convenience and without loss of generality, we assume that the signature waveforms $\{\mathbf{s}_k\}_{k=1}^K$ of the $K$ users are linearly independent. Denote $\mathbf{S} \triangleq [\mathbf{s}_1 \ \mathbf{s}_2 \cdots \mathbf{s}_K]$ and $\mathbf{A} \triangleq diag(A_1^2, \cdots, A_K^2)$ . The autocorrelation matrix of the received signal $\mathbf{r}$ is then given by

$$\mathbf{C} \triangleq E\{\mathbf{r}\mathbf{r}^T\} = \sum_{k=1}^{K} A_k^2 \mathbf{s}_k \mathbf{s}_k^T + \sigma^2 \mathbf{I}_N = \mathbf{S}\mathbf{A}\mathbf{S}^T + \sigma^2 \mathbf{I}_N \tag{2}$$

By performing an eigen-decomposition of the matrix $\mathbf{C}$, we get

$$\mathbf{C} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T = [\mathbf{U}_s \ \mathbf{U}_n] \begin{bmatrix} \Lambda_s & \\ & \Lambda_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^T \\ \mathbf{U}_n^T \end{bmatrix} \tag{3}$$

where $\mathbf{U} = [\mathbf{U}_s \ \mathbf{U}_n], \mathbf{\Lambda} = diag(\mathbf{\Lambda}_s, \mathbf{\Lambda}_n); \mathbf{\Lambda}_s = diag(\lambda_1, \cdots, \lambda_K)$ contains the $K$ largest eigenvalues of



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$\mathbf{C}$ in descending order and $\mathbf{U}_s=[\mathbf{u}_1\cdots\mathbf{u}_K]$ contains the corresponding orthonormal eigenvectors; $\mathbf{\Lambda}_n=\sigma^2\mathbf{I}_{N-K}$ and $\mathbf{U}_n=[\mathbf{u}_{K+1}\cdots\mathbf{u}_N]$ contains the $N-K$ orthonormal eigenvectors that correspond to the eigenvalue $\sigma^2$. The range space of $\mathbf{U}_s$ is called the signal subspace and its orthogonal complement, the noise subspace, is spanned by $\mathbf{U}_n$.

Assume that user 1 is the desired user and its signature waveform is $\mathbf{s}_1$. A MMSE linear multiuser detector for demodulating the $kth$ user's data in (1) is in the form of a correlator followed by a hard limiter

$$\hat{b}_1=\mathrm{sgn}(\mathbf{m}_1^T\mathbf{r}) \tag{4}$$

where MMSE detector $\mathbf{m}_1\in\Re^N$. Minimizes the MSE, defined as

$$MSE(\mathbf{m}_1)\triangleq E\{(A_1b_1-\mathbf{m}_1^T\mathbf{r})^2\}$$

subject to $\mathbf{m}_1^T\mathbf{s}_1=1$. And we get an linear MMSE detector , in terms of the signal subspace parameters $(\mathbf{U}_s,\mathbf{\Lambda}_s,and\ \sigma)$

$$\mathbf{m}_1=\frac{1}{[\mathbf{s}_1^T\mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^T\mathbf{s}_1]}\mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^T\mathbf{s}_1 \tag{5}$$

3. Subspace Estimation using SP-1 and a new demodulating vector

Classical subspace tracking is EigenValue Decomposition (EVD) and Singular Value Decomposition (SVD). Although their performances are better, the computational complexity $\left(\mathrm{O}(N^3)\right)$ is high. Computational complexity of PASTd is $\left(\mathrm{O}(NK)\right)$, and total computational complexity is $(\mathrm{O}(N^3)+\mathrm{O}(NK^2))$ for solving an initial matrix. The section introduces a subspace tracking method, i.e. SP-1 [8] and its computational complexity is $\mathrm{O}(NK^2)$. Solve a demodulating vector using the estimated subspace.

Assume $Q_t=[q_t^1\ q_t^2\ \cdots q_t^K\ ]$, and $q_t^i$ is the $31\times1$ eigenvector estimate associated with $ith$ largest eigenvalue of $\mathbf{C}_t$. $\tilde{Q}_t=[Q_{t-1}\ \mathbf{r}]$, and $\beta(0\le\beta\le1)$ is the forgetting factor.

Initial Values:

$$Q_0=\begin{bmatrix} I_d \\ 0_{(N-d)\times d}\end{bmatrix},\ U_0=I_{31}\times Q_0,\ g_0=I_{31},\ C_0=I_{31}$$

For $t=1,2\cdots,N$

$$C_t=\beta C_{t-1}+\mathbf{r}\mathbf{r}^T,\ g_t=g_{t-1}\mathbf{r}$$

$$\tilde{Q}_t(:,1:K)=Q_{t-1},\ \tilde{Q}_t(:,K+1)=r,\ U_t=\beta[\tilde{U}_{t-1}\ \ g_t]+\mathbf{r}\mathbf{r}^T\tilde{Q}_t$$

$$A=\tilde{Q}_t^TU_t,\ B=\tilde{Q}_t^T\tilde{Q}_t$$

solve $(A-\Lambda_tB)W_t=0_{N\times(K+1)}$

$$Q_t=\tilde{Q}_t\cdot W_t(:,1:K),\ \tilde{U}_t=U_t\cdot W_t(:,1:K),\ g_t=\mathbf{r}\mathbf{r}^T$$

end

The computational complexity of SP-1 is $\mathrm{O}(NK^2)$, and a little higher than PASTd. But it does not need initial matrix. In real CDMA communication system, computational complexity of SP-1 is lower than PASTd for $N\gg K$. Table 1 shows the compare of computational complexity of the two algorithms.

Table 1 Comparison of computational complexity of PASTd and SP-1

| Algorithm | Computational complexity |
|---|---|
| PASTd | $\mathrm{O}(N^3)+\mathrm{O}(NK)$ |
| SP-1 | $\mathrm{O}(NK^2)$ |

The initial matrix of PASTd is obtained by applying an SVD to the first 50 data vectors, which will result in a delay to the communication of users. SP-1 will not encounter this.

If $Q_0$ has orthogonal columns, all subsequent $Q_t$ will have orthogonal columns. Let $\tilde{\mathbf{U}}_s=Q_t$, so $range(\tilde{\mathbf{U}}_s)=range(\mathbf{S})=range(\mathbf{U}_s)$.

The received signal $\mathbf{r}$ is projected onto the subspace $\tilde{U}_s$ to get a $K$-vector [9]. Its autocorrelation matrix is

$$Y=E\{(\tilde{\mathbf{U}}_s^H\mathbf{r})\cdot(\tilde{\mathbf{U}}_s^H\mathbf{r})^H\}=\tilde{\mathbf{U}}_s^HE\{\mathbf{r}\mathbf{r}^H\}\tilde{\mathbf{U}}_s=\tilde{\mathbf{U}}_s^HC\tilde{\mathbf{U}}_s \tag{6}$$

According to (5) the new linear MMSE detector of user1 is given by

$$\mathbf{m}_{new} = [\mathbf{s}_1^T \tilde{\mathbf{U}}_s \mathbf{Y}^{-1} \tilde{\mathbf{U}}_s^H \mathbf{s}_1]^{-1} \tilde{\mathbf{U}}_s \mathbf{Y}^{-1} \tilde{\mathbf{U}}_s^H \mathbf{s}_1 \quad (7)$$

Since $range(\tilde{\mathbf{U}}_s) = range(\mathbf{U}_s)$, it can be readily proved that the new detector $\mathbf{m}_{new}$ is equivalent to the MMSE detector presented in [7].

4. Simulation Results

A synchronous CDMA system with processing gain $N = 31$ and six active users $(K = 6)$ is assumed. The user 1 is specified as the desired user. There are four 10-dB MAI's and one 20-dB MAI in the channel, all relative to the desired user's signal. The signature sequence of desired user is an $m-$sequence, while the signature sequences of the MAI's are randomly generated. The forgetting factor $\beta = 0.995$. The performance measure is the out signal-to-noise-and-interference ratios (SINR) [10] which is defined as

$$SINR_{av}[i] = \frac{\sum_{l=1}^{M} (\mathbf{m}_l^H [i] \mathbf{s}_1)^2}{\sum_{l=1}^{M} [\mathbf{m}_l^H [i] (\mathbf{r}_l [i] - b_{1,l} [i] \mathbf{s}_1)]^2} \quad (8)$$

where $M = 100$ is the number of algorithm runs, and $l$ indicates that the associated variable depends on the particular run.

Simulation 1: Compare the performance of SVD, SP-1 and PASTd using initial matrix. The initial matrix of PASTd is obtained by applying an SVD to the first 50 data vectors. Figure 1 illustrates SINR of the three algorithms versus the number of samples where the SNR of user 1 is 20dB. From the simulation result, it is clear that convergence and steady-state performance of SVD are best; those of SP-1 are good, and equivalent to those of PASTd using initial matrix.

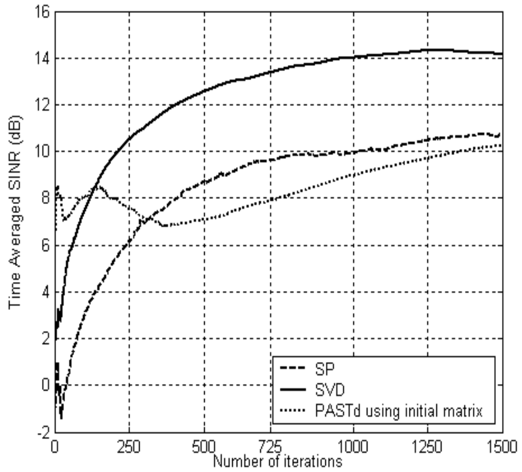


Fig.1 Performance comparison of SVD, SP-1, PASTd

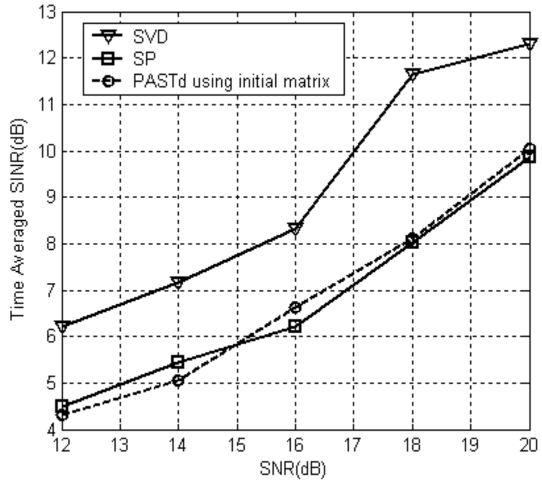


Fig.2 Performance comparison of SVD, SP-1, PASTd versus SNR

Simulation 2: Compare the steady-state performance versus the SNRs of user 1. Simulation result is plotted in Fig. 2, which shows SVD MUD is the steadiest; performance of SP-1 and PASTd using initial matrix are almost the same. But SP-1 decreases computational complexity and is more suited for adaptive environment.

Simulation 3: Compare the BERs of user 1. The SNR of user 1 is 20dB. We can see form Fig. 3 that tracking of SVD MUD is fast and BER performance is best. BER performance of SP-1 is acceptable and equivalent to that of PASTd MUD using initial matrix.

5. Conclusions

In this paper, a new adaptive multiuser detector based on subspace tracking is proposed. The signal subspace estimation is achieved by SP-1 followed by a demodulating vector which modifies the MMSE detector. Simulation results show that performance of the multiuser detection based on SP-1 subspace tracking is good. SP-1 does not need to compute initial matrix, which results in lower computational complexity. And SP-1 has better convergence and steady-state performance. So, the algorithm proposed in this paper is practical.

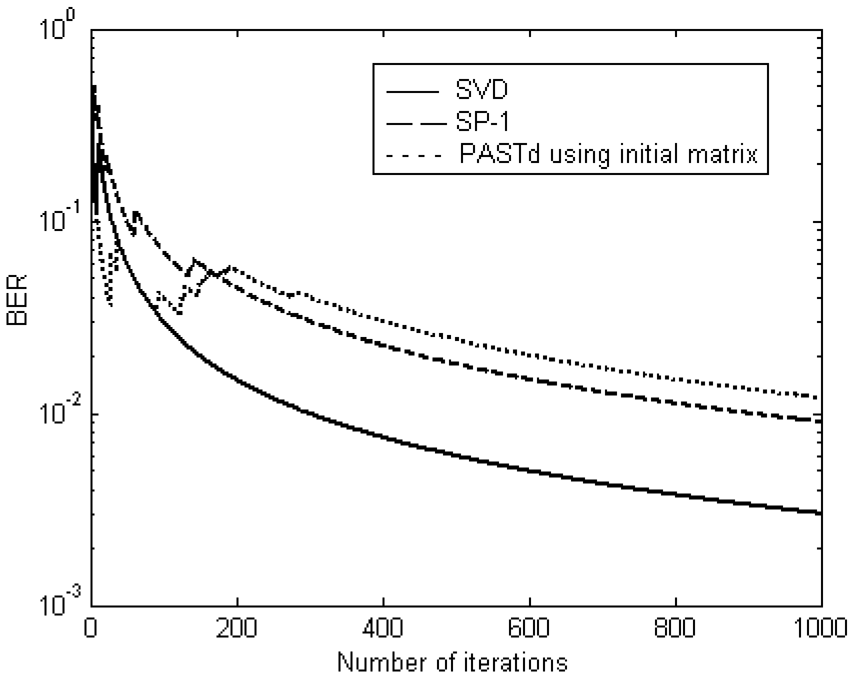


Fig. 3 BERs of SVD, SP-1, PASTd

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