Rapidly converging solutions on the Yasuura Method for Solving Three-Dimensional Scattering Problems

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1.Introduction

The Yasuura method [1,2] is one of the numerical methods to analyze electromagnetic scattering problems and has been used as an efficient technique to analyze scattering problems by bodies of revolution [3]. It is , however, not efficient to analyze scattering problems by 3-D scatterers of arbitrary shape, since we have to solve big and dense matrix. Therefore it is necessary to reduce computational tasks on the Yasuura method in order to solve 3-D scattering problems effectively.

To reduce numerical tasks, we introduce arrays of multipoles in addition to the conventional ones and expand the scattered field into N terms linear combination of multipoles. After optimizing the locations of multipoles for the scatterer shape and the direction of incidence, we numerically show the rapidly converging solutions in comparison with the ones given by the Yasuura method [3]. Throughout this paper, the time factor $\exp(j\omega t)$ is assumed and suppressed.

2.Formulation of problem

We formulate the scattering problem by a 3-D scatterer as shown in Fig.1. A plane wave whose direction of incidence is denoted by (θ_i, ϕ_i) impinges on the scatterer where an angle between the plane of incidence and the incident electric field vector $E^i(P)$ is defined by α (See Fig.1). The surface of the scatterer and its outer region are denoted by S and V, respectively. The point P is inside V and Q on S is given by $r(\theta, \phi)$.

Here we use multipole expansion method for solving 3-D scattering problems. We introduce arrays of multipoles in addition to conventional ones. Referring to [1,4,5], we represent an approximate scattered field by N terms linear combinations of multipoles such that

$$E_N^s(P)=\sum_{\kappa=1}^{K}\sum_{n=1}^{N}\Big[\sum_{m=0}^{n}\{a_{emn}^{\kappa}(N)\sum_{u=1}^{U}m_{emn}(P-R_{\kappa,u})+b_{emn}^{\kappa}(N)\sum_{u=1}^{U}n_{emn}(P-R_{\kappa,u})\}$$

$$+\sum_{m=1}^{n}\{a_{omn}^{\kappa}(N)\sum_{u=1}^{U}m_{omn}(P-R_{\kappa,u})+b_{omn}^{\kappa}(N)\sum_{u=1}^{U}n_{omn}(P-R_{\kappa,u})\}\Big] \qquad (1)$$

where the capital N of the unknown coefficients $a_{\chi mn}^{\kappa}(N)$ and $b_{\chi mn}^{\kappa}(N)$ (χ =e,o) denotes the dependence on the truncation size N, $m_{\chi mn}$ and $n_{\chi mn}$ are conventional multipoles which are defined by spherical vector wave functions [6], and $\sum_{u=1}^{U}m_{\chi mn}(P-R_{\kappa,u})$ and $\sum_{u=1}^{U}n_{\chi mn}(P-R_{\kappa,u})$ mean arrays of multipoles for $U \geqq 2$. The array of multipoles is made by U conventional multipoles located on an open or a closed loop line whose intervals are selected as close as possible. In Eq(1), $R_{\kappa,u}$ (κ =1,2, ··· ,K, u=1,2, ··· ,U) are the location points of the conventional multipoles. When we consider a scattering from a peanut-like scatterer as shown in Fig.2(a), we use several conventional multipoles resulting from a spherical portion of scatterer surface in addition to conventional multipoles located at the origin as shown in Fig3(a).

On the other hand, when we consider a scattering from a doughnut-like scatterer as shown Fig.2-(b), we need several arrays of multipoles resulting from a cylindrical portion of scatterer surface together with the conventional ones at the origin. For axial incidence we use a circular closed-array of multipoles and for oblique incidence we use two semicircular open-arrays of multipoles by considering the asymmetry property of the scattered field as shown in Fig3(b),(c). Since we have introduced arrays of multipoles and locate two kinds of multipoles depending on the scatterer shape, we can obtain rapidly converging solutions compared to that of the conventional Yasuura method [3] and the other multipole expansion methods [4,5]. For 3-D scatterers of arbitrary shape, we can also accelerate convergence rates of solutions in the same way. However, the more complicated the scatterer becomes, the much more locations of multipoles we need.

Now we show how to determine the unknown coefficient $\{ a^{s}_{\chi mn}(N)\ ,\ b^{s}_{\chi mn}(N) \}$ in Eq.(1). We minimize the squared norm about the boundary condition.

$$\Omega(N)=\int_S |\ \nu(Q)\times\ (E^{s}_{N}(Q)+E^{i}(Q))\ |^2 dS \Big/ \int_S |\ \nu(Q)\times E^{i}(Q)\ |^2 dS \qquad (2)$$

Then we obtain a following simultaneous linear equation: $A\,a = f$ (3)

where A is a M × M(M=2KN(N+2)) Hermitian matrix whose elements are defined by inner products of multipoles, f is a column vector whose elements are defined by inner products of multipoles and incident wave, and a is a column vector whose elements are $\{ a^{s}_{\chi mn}(N)\ ,\ b^{s}_{\chi mn}(N) \}$ [3]. By optimizing locations of the multipoles depending on the scatterer shape and the direction of incidence, we obtain rapidly converging solutions.

The original algorithm Eq.(3), however, is not computer-aided one, since every element of A, f needs numerical calculation of surface integrals. To construct a computer-aided algorithm, we discretize the continuous norm using the standard technique employed in the numerical method [7]. Then we have a computer-aided algorithm where the discretized norm monotonically decreases as the truncation size N increases with the total numbers of sampling points which are several times as large as N [8] . In an actual calculation, from the numerical point of view we use QR-decomposition technique [9].

3. Checking the accuracy and the convergence rate of solution

Here we numerically check the accuracy of solutions and demonstrate the significant improvement of convergence rate of solution on the Yasuura method. We examine scattering from 3-D perfectly conducting indented scatterer whose surface is described by

$$r(\theta\ ,\ \phi)=a(1+\gamma\ \sin\theta\ \cos\phi)(1+\delta\ \cos2\theta), \qquad |\gamma|,|\delta|<1 \qquad (4)$$

where a is a positive constant and Eq.(4) represents a peanut-like and a doughnut-like scatterer for positive and negative δ , respectively. First we show how to locate multipoles. For the peanut-like scatterer, we locate conventional multipoles at $(d_p,0,0)$, $(d_p,\pi,0)$, and $(0,0,0)$. On the other hand, for the doughnut-like scatterer we use a circular closed-array of multipoles located at $(d_{dc}\ ,\ \pi/2,\ \phi_u)$ in an axial incidence(θ_i =0 or π) and in an oblique incidence two semicircular open-arrays of multipoles located at $(d_{do}\ ,\ \pi/2,\ \phi_u)$ and $(d_{do}\ ,\ \pi/2,\ \phi_v)$ in addition to the conventional ones at the origin. The upper limit U in Eq.(1) is set to 20 in this paper. Next we adjust the locations of multipoles. In these exapmles we have only one parameter with respect to the locations of multipoles for each scatterer. Therefore we can determine the optimum locations of multipoles numerically. We can see from Fig.4 that there are optimum locations of multipoles. For the peanut-like scatterer the actual locations of conventional multipoles are (0.55a,0,0), (0.55a, π ,0) and (0,0,0), and for the doughnut-like scatterer the locations of two semicircular open-arrays of multipoles and conventional ones are $(0.6a,\ \pi/2,\ \phi_u)$, $(0.6a,\ \pi/2,\ \phi_v)$, and (0,0,0), respectively when $\delta = \pm 0.3$. By adjusting the locations of multipoles, we obtain rapidly converging solutions (See Fig.5). The doughnut-like scatterer has slower convergence rate of solution than the peanut-like scatterer. It may be caused by the difference of scatterer size. Simultaneously we check convergence properties of scattering cross sections $\sigma(\theta_i\ ,\ \phi_i\ ;\ \theta_o\ ,\ \phi_o)$ which are normalized by πa^2 . Here $(\theta_o\ ,\ \phi_o)$ is an observation direction. Numerical solutions by the Yasuura method presented here rapidly converge, while those given by the conventional one [3] shows very slow convergence as shown in Fig.7. As a result we obtain at least 3 significant figures for the scattering cross sections as shown in the Table.

5. Conclusion

We have proposed a rapid algorithm on the Yasuura method to calculate scattering from a 3-D perfectly conducting scatterer of arbitrary shape by introducing arrays of multipoles in addition to the conventional ones and numerically showed rapidly converging solutions.

Acknowledgement

This research is supported in part by Scientific Research Grant-In-Aid from the Ministry of Education, Science and Culture(0665043), Japan.

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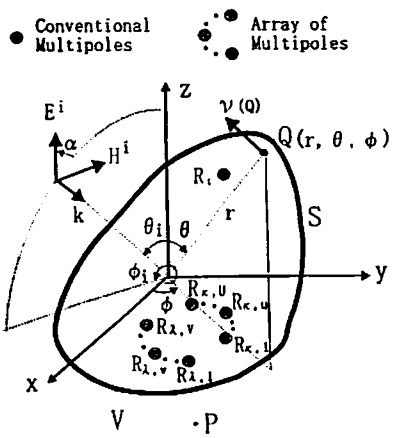

Fig.1 Coordinate system and geometry of three-dimensional scatterer.

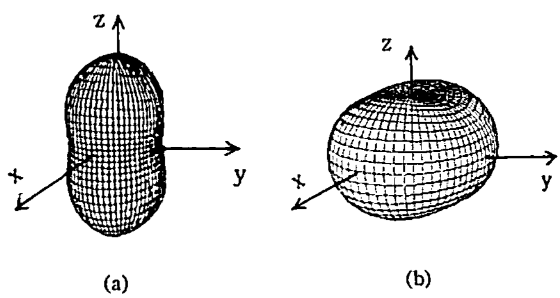

Fig.2 Scatterers shape.
(a) Peanut-like scatterer
(b) Doughnut-like scatterer

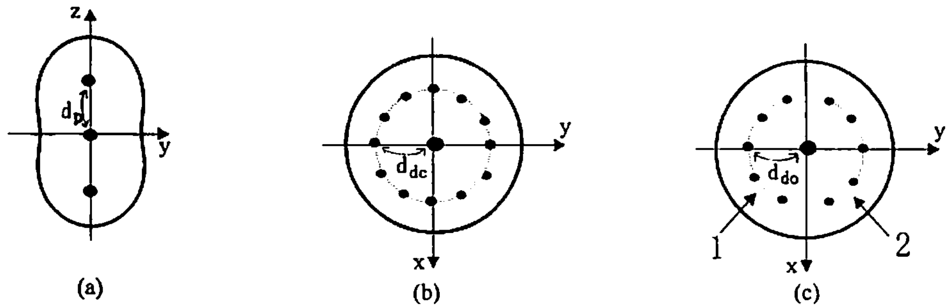

Fig.3 Arrangement of multipoles. Solid contour lines represent cross sectional areas at x=0(a), z=0(b) and (c). For the peanut-like scatterer we locate three conventional multipoles (a). For the doughnut-like scatterer, in addition to the conventional multipoles, we use a circular closed-array of multipoles for the axial incidence (b) and two semicircular open-arrays of multipoles for the oblique incidence.

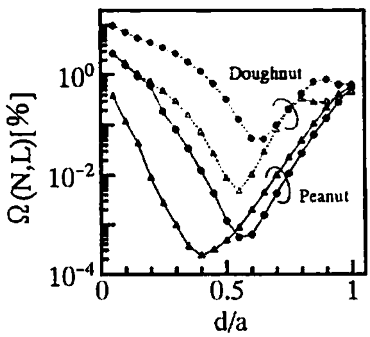

Fig.4 Discretized norms vs. locations of multipoles.
(ka=10, N=10, γ =0.05, $\theta_i = \pi/6$, ϕ_i =0, α =0)
▲: $\delta = \pm 0.2$, ●: $\delta = \pm 0.3$

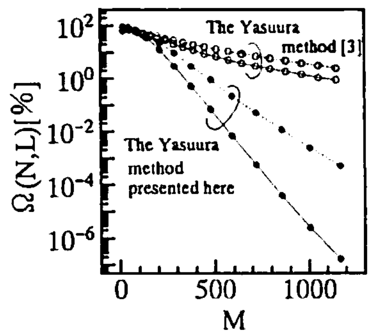

Fig.5 Discretized norms vs. matrix size.
(ka=10, γ =0.05, $\delta = \pm 0.3$, $\theta_i = \pi/6$, ϕ_i =0, α =0)
— :Peanut, ····:Doughnut

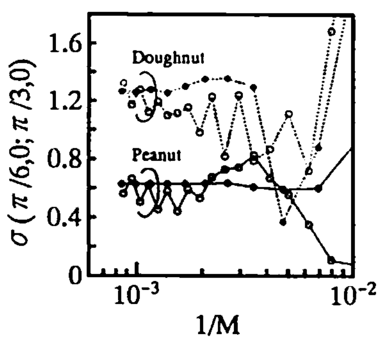

Fig.6 Scattering cross sections vs. matrix size.
(ka=10, γ =0.05, $\delta = \pm 0.3$, α =0)
● :The Yasuura method presented here
o :The Yasuura method [3]

Table. Convergence properties of discretized norms and scattering cross sections.
(ka=10, γ =0.05, δ =±0.3, α =0°)

	Ω(N,L)[%]		σ(π/6,0;π/3,0)	
M	Peanut	Doughnut	Peanut	Doughnut
.378	5.4×10^{-1}	2.9	0.6370	1.357
480	7.0×10^{-2}	9.4×10^{-1}	0.6342	1.355
594	6.9×10^{-3}	2.2×10^{-1}	0.6310	1.301
720	5.6×10^{-4}	5.2×10^{-2}	0.6317	1.255
858	3.9×10^{-5}	1.1×10^{-2}	0.6316	1.278
1008	2.6×10^{-6}	2.5×10^{-3}	0.6317	1.261
1170	1.7×10^{-7}	5.1×10^{-4}	0.6317	1.268