A SPARSE MATRIX/CANONICAL GRID METHOD FOR SOLVING LARGE-SCALE ELECTROMAGNETIC BOUNDARY VALUE PROBLEMS

C. H. Chan*†, L. Tsang† and C.-M. Lin†
*Department of Electronic Engineering, City University of Hong Kong
83 Tat Chee Avenue, Kowloon, Hong Kong
†Department of Electrical Engineering, University of Washington
Box 352500, Seattle, Washington 98195-2500, USA

1. Introduction

With the advent of modern computers, rigorous numerical solutions to many electromagnetic boundary value problems that were once considered intractable are now feasible. One of the most commonly used numerical techniques in computational electromagnetics is the Method of Moments (MOM) which results in a matrix solution that requires an $O(N^3)$ operation and a memory requirement of $O(N^2)$ with N being the number of unknowns. To alleviate these difficulties for large-scale problems in which N is large, fast Fourier transform (FFT) iterative methods such as the conjugate gradient method (CG-FFT) have been developed. In these methods, a uniform discretization is employed to exploit the translational invariant property of the resulting matrix leading to a solution that requires O(N) memory and O(N log N) operations. Therefore, a certain class of large-scale electromagnetic boundary value problems, e.g., scattering of an electrically large flat plate, becomes solvable. However, in many practical situations, conventional implementations of MOM do not possess the translational invariant property that permits uniform discretization, and therefore, an efficient iterative method such as the CG-FFT cannot be applied. Alternative implementation of the MOM is required.

We will present a sparse matrix/canonical grid method which has been successfully applied to a wide variety of large-scale electromagnetic boundary value problems such as one-dimensional (two-dimensional scattering problem) and two-dimensional random rough surfaces (three-dimensional scattering problem), scattering from a dense medium consisting of thousands of cylinders and densely-packed high-frequency microstrip interconnects. This method is also applicable to frequency selective surfaces in which each matrix element requires the summation of hundreds of thousands of Floquet's harmonics. The key idea is to decompose the matrix into a strong near-interaction part and a weak far-interaction part. Based on this decomposition, the matrix-vector multiplication operation in a typical iterative procedure will be divided into a direct sparse matrix-vector multiplication corresponding to the strong interactions and an indirect dense matrix-vector multiplication corresponding to the weak interactions that can be computed efficiently via FFTs. As the sparse matrix only accounts for a small fraction of the whole matrix, both CPU time and computer memory requirements can be reduced significantly. Three different approaches can be used to convert Green's function kernel for the far interactions into the translational invariant-form so that FFTs can be applied, viz, the Taylor series expansion, addition theorem and interpolations. In this paper, we will show how these schemes are employed for various large-scale electromagnetic boundary-value problems.

2. Sparse Matrix/Canonical Grid Method

We started the near- and far-interaction decomposition of the impedance matrix when we analyzed 1-D large-scale random rough surface problems. As the unknowns on the surface profile can be numbered sequentially, the strong near-interaction portion of the impedance matrix, $[Z^s]$ resembles a banded matrix. The updated solution $\{x^{n+1}\}$ in the iterative process is obtained by solving the banded matrix with the excitation vector $\{b\}$ corrected by the product of the far-interaction portion of the matrix $[Z^w]$ and the previous solution $\{x^n\}$, i.e., $[Z^s]\{x^{n+1}\} = \{b\} - [Z^w]\{x^n\}$. We termed the method the banded-matrix iterative approach [1]. It should be noted that the factorization of the banded matrix is done once and the subsequent banded matrix solution only requires backsubstitutions. It was found that when we extended the method into 2-D random rough surfaces, the banded nature of the near-interaction was destroyed. The banded matrix in the 1-D case is now replaced by a sparse matrix in the 2-D case. The convergence of the iterative solution improved when we subtracted $[Z^f]$, which corresponds to a flat surface, from the original 2-D far-interaction portion of the matrix. The modified equation becomes $[Z^s]\{X^{n+1}\} + [Z^f]\{X^{n+1}\} = \{b\} - [Z^w - Z^f]\{X^n\}$. For the 2-D rough surface, a uniform discretization is adopted along the two surface lengths L_x and L_y. In contrast, a non-uniform discretization is chosen along the height of the surface. With the uniform discretization on the x-y plane, the product $[Z^f]\{X^{n+1}\}$ can be obtained by FFT's due to the translational invariant properties of the kernel. We termed this method the sparse-matrix flat-surface iterative approach [2]. This flat-surface term corresponds to the first term of a Taylor series expansion of the 2-D kernel about $z = 0$. Keeping the higher-order expansion terms, the matrix-vector operation of $[Z^w]\{X^n\}$ can be performed by FFT's, this is a version of sparse-matrix/canonical grid method with the grid being $z=0$ [3]. The largest problems we have analyzed have 130,000 unknowns. A similar idea using Taylor series expansion was also reported by Bleszynski *et al*.[6].

An alternate method for converting the far-interaction kernel into a translational invariant one is to make use of the addition theorem [7]. We demonstrated how this can be done for 2-D scattering problems of thousands of infinite cylinders randomly located within a unit square. First, we divide the unit square into a uniformly spaced canonical grid. We translate the scattering centers from the centers of all the cylinders to their nearest grid points by the use of the addition theorem. The interactions among all the grid points will be translationally invariant for which FFTs can be used for speedy evaluations. The scattered field is then translated from the grid point to the centers of the nearest cylinders using the addition theorem for the second time. The whole process can be written as a premultiplication of the scattering amplitudes by a block diagonal matrix followed by convolutions using FFTs. A post-multiplication with a similar block diagonal matrix completes the efficient evaluation of the matrix-vector multiplication for the far interactions. A similar approach using the addtion theorem was later reported by Chew *et al.* for 3-D scattering problems [8].

There are no significant differences in terms of complexity between the Taylor series expansion and the addition theorem for the same order of accuracy. While it is more convenient to use the addition theorem to obtain higher-order expansion terms, the Taylor series expansion is easier to implement. For the analysis of densely-packed high-frequency interconnects presented in this paper, we choose to use the Taylor series expansion. For planar structures parallel to the x-y plane, the Taylor series expansion only involves derivatives w.r.t. x and y while the addition theorem requires the use of spherical Bessel functions.

The third approach to obtain the Green's function kernel for the far interactions is through the interpolations of the interactions among the grid points of a uniform canonical grid. This method was first employed by Phillips and White [9] to analyze quasi-static interconnects and packages. In these problems, the Green's function varies smoothly for the far interactions. In full-wave analyses. the retarded potential oscillates due to the rapidly changing phase term and therefore, interpolation would not work properly without using a very fine grid or extending the range of near interactions, both of which will increase the memory requirement and CPU time drastically. On the other hand, this interpolation scheme can be adopted to analyze periodic structures with an irregular triangular discretization efficient [10]. Here, the near interactions are computed in the spatial domain while the far interactions are computed in the Fourier domain using FFT's with the use of interpolations of the Green's functions on the canonical grid. The summation of Floquet modes using FFT's is identical to the techniques developed for a uniform discretization [11].

3. Numerical Results

We will present numerical results for various large-scale electromagnetic boundary-value problems that illustrate the use of each of the three schemes mentioned for speedy evaluations of the far interactions by FFT's. Representative results will be presented for scattering from 2-D random rough surfaces and dense media comprised of infinite cylinders, high-frequency interconnects, and periodic structures such as frequency selective surfaces. To demonstrate the accuracy of the method, we include a comparison of the Monte Carlo simulation with a laboratory controlled experiment for a 32x32 square wavelength 2-D random rough surface with an rms height of 1 wavelength and correlation length of 2 wavelengths. The bistatic scattering coefficients for both HH and VH are the ensemble average of 600 surface realizations. Very good agreement with experiemntal data is obtained.

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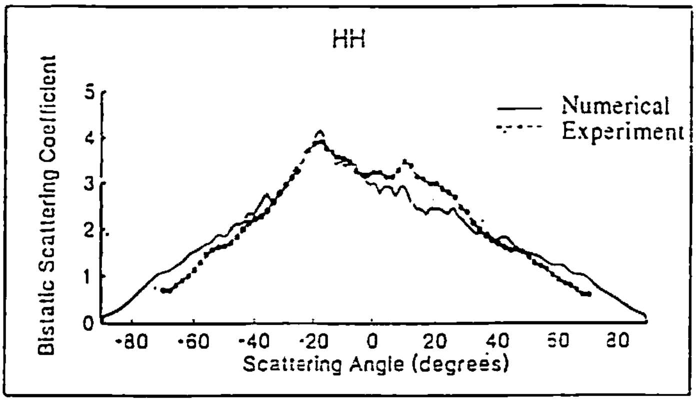

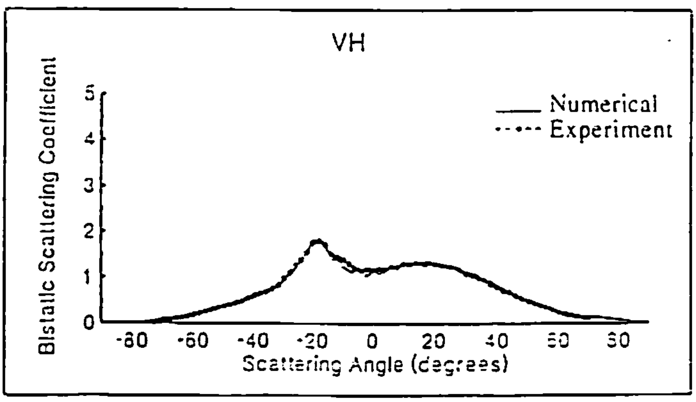

Figure 1. Comparison of Monte Carlo simulation with controlled experiment.