THE DEPENDENCE OF BEAM SPREAD ON OBSERVATION DURATION

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1 Introduction

The theory of wave propagation in random medium has been greatly developed in the past three decades[1][2], It has been widely used in scientific research field, such as : atmospheric optics, radio wave propagation, oceanic sounding and so forth. It also provided the theoretical basis for the selection of the manner of remote sensing and communication as well as data processsing.

However, the theory is far from perfect. There is distance to explain all experimental phenomena completely. As pointed out by strohbehn[3] that "there does not exist any rigorouly derived theory that adequately explain all the experimental data". At least, we think that there are following problems for wave propagation in turbulence need to be studied further: (1) The dependence of wave statistics on the duration of observation. Although Fante[4] et al has defined "short—term beam spread" and "long—term beam spread" and try to explain it, the relation between time scale of wave beam and medium, and the beam spread for strong fluctuation at any time period have not been given. (2) The physical mechanism for the suturation of scintillation index is not clear even though the forth moment of wave field has been solved analytically and numerically. (3) The probability distribution of wave intensity at intermediate fluctuation deviates from log—normal distribution or Rice—Nakagami distribution.

As we know, until present time the statistical theory of wave propagation in turbulence is established on the basis of supposing the medium to be homogeneous or local homogeneous in space and stationary in time statistically. The moments of wave field are obtained theoretically by solving random wave equation under various approximations. The explanation to experiment is also based on the ergodicity hypathesis, which the timporal average is used to represent the ensemble average. The ergodicity hypothesis, however, requirs that the random process is stationaty and the duration of observation is long enough so that the number of independent event is large, which is difficult to be satisfied for random wave propagation.

In order to explain the dependence of wave statistics on the duration of observation , we propose the multi—scale model for turbulence and think that only a portion scale of the inhomogeneity is romdom in the measurement period, which contribute to the wave fluctuation. By using the multi—exponential model, the beam spread for wave propagation in turbulent medium is calculated for variaus parameters of time and space, which shows that the beam spread strongly depends on the duration of observation and average scale of inhomogeneity.

2. Multi—scale Model of Turbuleuce

The description of turbuleuee is basis of the study of wave propagation in turbulent medium. The statistical property of turbulence is generally described by speetral density. The exponent, power, Gaussion and Kolmogorov spectrum are widely used. The natural process in atmosphere, however, is time space correlate and not stationary in limited duratioon of observation. So the spectrum description is not strict and need to be improved. The fractal concept has been adopted to study the problem by B. J. west[5].

It can be imagined that there are various different scale inhomogeneity in the turbulent medium. Their characteristic scales are l_i and the correlation functions of the fluctuation of dielectric constant for two point are $B(r.\ l_i)$, respeetively. The portion of one scale l_i is $p(l_i)$ which can be understood as the distribution densety of seale. So the correlation function of dielectric contant is sum of that for all scales, i. e.

$$B_\varepsilon(\vec{r}) = < \varepsilon_1(\vec{r})\varepsilon_1{}^*(\vec{r}) > = \sum_{i=1}^{\infty} B_\varepsilon(\vec{r}, li)p(li) \tag{1}$$

which can be written in integral

$$B_\varepsilon(\vec{r}\ ) = \int_0^\infty B_\varepsilon(\vec{r}, l)p(l)dl \tag{2}$$

since the variations of turbulenec for time and space are correlated. The characteristic scale can be function of time $l(t)$. So the eq(2) can be changed into integral of time

$$B_\varepsilon(\vec{r}) = \int_0^\infty B_\varepsilon(\vec{r}, t)p(t)dt \tag{3}$$

Actually, the period of measurament is limited. If the duration of observation is T, the movement velocity of inhomogeneity is V. The large inhomogeneity ($l >> VT$)change not too much and is almost determination in such period, which action to wave propagation is mainly refraction. the small inhemogeneity ($l << VT$) change rapidly and is random, which scatter the wave, They cause the random functuation of wave field and singal scintillation. The main contributions to the statistics of wave feuctuation come from the multi—scattering by the part of inhomageneities which scale is small ($l < L_T, L_T - VT$). Considering eqs. (2) and (3), we obtain the correlation function of dielectric constant in the period T.

$$B_\varepsilon(\vec{r}, T) = \int_0^{L_T} B_\varepsilon(\vec{r}, l)p(l)dl \tag{4}$$

thus, the correlation function of turbulent medium depends on the duration of observation and the selection of the distribution density of scale $p(l)$ as well as the correlation function for sinle scale inhomoggeneity.

The analytical solutions for the moments of wave field are generally obtained under approximation of Markov process of the correlation function of medium in the direction of propagation. It is assumed here that the correlation function in limited period is also delta function in longitudial direction

$$B_\varepsilon(\vec{r}, T) = A_\varepsilon(\vec{\rho}, T)\delta(z - z') \tag{5}$$

where $A_\varepsilon(\vec{\rho}, T)$is the transverse correlation function of the dieletric constant, which can be obtained by integral of three dimension correlation function in longitudal direction. From eq(4), The transvers correlation function is obtained

$$A_\varepsilon(\vec{\rho}, T) = \int_0^{L_T} A_\varepsilon(\vec{\rho}, l)p(l)dl \tag{6}$$

where $A(\vec{\rho}, l)$ means the transverse correlation function for single scale inhomogeneity.

3. Beam Spread

When the scale of inhomogencity is much larger than the wavelength, the time change of the medium is much slower than the frequency of carrier wave. The parabalic approximation of wave equation is tenable and the complex amplitude of random wave field saticfies the equation

$$2ik\frac{\partial u}{\partial z} + \nabla_t^2 u + k^2\varepsilon_1 u = 0 \tag{7}$$

where k is the propagation constant, ε_1 is the random fluctuation of dielectric contant.

The solution of eq (8) has been widely studied. The satistical moments have been obtained analytically and numerally. For incident wave beam propagate through the random medium with distance z the average intensityof wave beam is [1]

$$< I(\vec{\rho},z) > = \frac{w_0^2}{8\pi}\int d\vec{K} exp[-bK^2 - i\vec{K}\cdot\vec{\rho} - H(\overline{K})] \tag{8}$$

This result is extended to the problem of wave beam propagation in multi—scale and non—slationary turbulence. i. e. the eq (8) is tenable formally, the stucture function and average intensity are functions of the duration T. For isotropic turbulence, the eq. (8) can be re—written is

$$< I(\vec{\rho}. z;T) > = \frac{w_0^2}{4}\int_0^\infty J_0(KP)exp[-bK^2 - H(\vec{K},T)]KdK \tag{9}$$

The structure function $H(\vec{K},T)$depends an the transverse correlation function $A_t(\vec{K},T)$ in limited period.

Suppose that both the transverse correlation function for single scale and the distribution density of scale l are exponenntial. By substituting them into eq. (6), we obtain

$$A_t(\rho,T) = \sigma^2\int_0^{L_T} l^2 exp(-al - \rho/l)dl \tag{10}$$

where the parameter σ^2 is the fluctuation variance, a is proportional to the inverse of average scale. and the stucture function

$$H(\vec{K},T) = \frac{k^2\sigma^2}{4}\int_0^{L_T} l^2 exp(-al)[z - \frac{kl}{K}(1 - exp(-\frac{Kz}{kl}))]dl \tag{11}$$

The average intensity of wave beam in the duration of observation is obtained from the substitution of eq. (11) into eq. (9). Suppose that incident optical beam with wavelength 6328 Å and $w_0 = 0.05$ meter. the parameters of turbulence are $\sigma^2 = 10^{-14}$, $a = 1$, The normalized average intensity is given in Fig. 1, which is obtained from the numerical integral of eq(9). It can be seen that the shorter the duration of observation, the less the beam spread. The beam spread for large value of T is significant.

The dependence of beam spread on the duration of observation can be seen further in Fig. 2, in which the parameters used are same as in Fig. 1 except for a. The left ordinate is the centre averager intensity of wave beam, right oridinate is the beam width correspounding to the relative intensity $\frac{< I(\vec{\rho},T) >}{< I(0,T) >} = 0.707$. It shows that the centre intensity decrease and the width of wave beam increase as the duration increases. This is because the refraction of wave by large inhomogeneity appear in beam spread only in long period. The beam spread also depends on the parameter a, For $a = 0.5$ the wave beam change not too much until the duration of 0. 1 sec. But for $a = 5$ the beam spread has smaller time scale. The reason of this is that large value of a means the average scale small, which is of short characteristic time. The time for wave beam changing sharply at order of 0. 1 sec agrees with the experiment result. The flat of the curve at left may be in correspondence with the "short—term beam spread"and that at right may be in correspondence with the "long—term beam spread".

4. Conclusion and Discussion

The beam spread obtained from multi—scale model of turbulence and statistical theory of wave propagation is of strong dependence on the duration of observation. The characteristic time of wave beam depends on the selection of correlation function for single scale inhomogeneity and the distribution density of scale. This make it possible that the spread phenomenon of wave beam propagation in turbulent medium is explained theoretically. The statistics of other wave parameters (such as pulse distortion, time delay, Scintillation et al)in limited period can be studied in same way.

It must be pointed out that although the non—stationariness of turbulent medium and wave beam have been involved in this paper, the classical result of solution also be adopted. In fact, this is a extension of former statistical theory as a try. The more strict and complete method need to be studied further.

Reference

1. A. Ishimaru, Wave Propagation and Scattering in Random Media, Academic, New York, 1978
2. V. I. Tatarski. The Effects of the Turbulent Atmosphere on wave Propagation. Springfield. 1971
3. J. W. Strohbehn, Laser Beam Propagation in the Atmosphere. Springer—verlag. Berlin. 1978
4. R. L. Fante, Proc, IEEE vo163, No. 12, P1669—1692, 1975
5. B. J. West. J. Opt. Sol. Am. A/vol. 7. NO. 6, P1074—1100

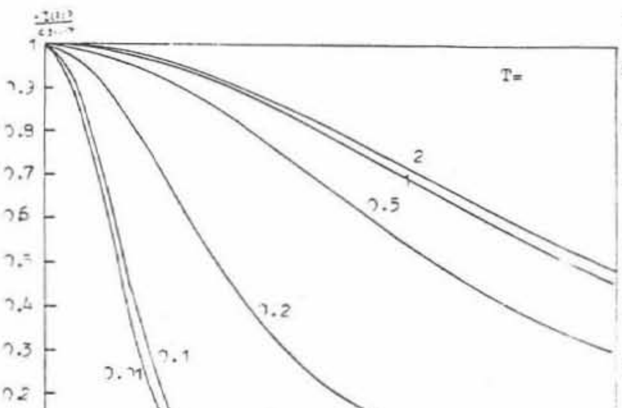

Fig.1 The normalized intensity of wave beam in turbulence

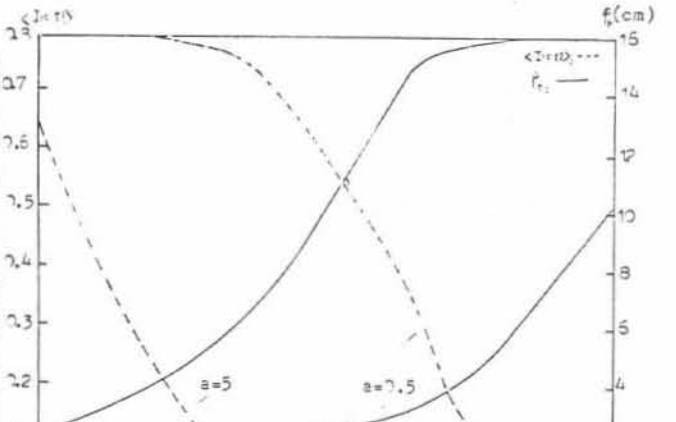

Fig.2 The centre intensity and the width of wave beam as the duration of observation