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FULL-WAVE ANALYSIS OF INTEGRATED WAVEGUIDES WITH SHARP METAL EDGES USING HYBRID EDGE/NODAL FINITE ELEMENTS

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1. Introduction

In the microwave integrated circuits, inhomogeneous waveguides with sharp metal edges are randomly used: most common examples are microstrip lines, slotlines, finlines, and ridge waveguides. The accurate prediction of dispersion characteristics in such waveguides is very important for the design of efficient integrated circuits. However, a proper modeling of such waveguides should include two considerations: hybrid mode propagation and field singularity occuring at sharp edges are of great concern. Of the methods available for the analysis of those waveguides, there has been some success with the finite element method (FEM). The recent development with the vector finite element method for the full-wave analysis of inhomogeneous waveguides is with hybrid elements [1]–[6] which suppress spurious solutions. The hybrid elements are composed of edge and nodal elements, where edge elements model the transverse field ensuring tangential continuity along the element interfaces and nodal elements model the axial fields. As the edge elements assign the degrees of freedom to the edges, they allow the field to change it's direction abruptly and thus are capable of modeling the field properly at sharp edges at which singularity occurs. With hybrid elements, the finite element method overcomes all the shortcomings, which previously clouded many of the analyses. At present, two different approaches using hybrid edge/nodal elements are present: (1) electric or magnetic field approach [1]–[5] and (2) scalar and vector potential approach [6].

In this paper, the electric or magnetic field approach with the high-order hybrid element which is composed of linear edge and quardratic nodal elements [3] is applied to analyzing propagation characteristics of integrated waveguides with sharp metal edges on achiral and chiral substrates. In the present approach the frequency is specified as the input parameter and the system is solved for the propagation constant as an eigenvalue. Therefore, it is possible to handle lossy waveguides. Taking advantage of the sparsity of finite element matrices, we have implemented the subspace iteration method [7] in the hybrid element algorithm and successfully solved the eigenvalue problem for matrices of dimension in several thousands, where M and $2M$ dimensional sparse matrices with M being the number of degrees of freedom are obtained for achiral and chiral waveguides, respectively.

2. Finite Element Method with Hybrid Edge/Nodal Elements

The finite element method with hybrid edge/nodal triangular elements is an accurate simulation technique and has become very popular for the analysis of planar-type transmission lines, because this technique ensures full-wave analysis and all nonzero eigenvalues obtained correspond to true waveguide modes. Moreover, with great advantage, one can easily employ the magnetic field H or the electric field E as the working variable for the analysis. The use of

E as the working variable can substantially reduce the size of the system by removing the edges and nodes lying on conducting surfaces, and thus the computation time can also be greatly reduced.

By discretizing the functional for the vector wave equation using the standard finite element technique, an eigenvalue equation can be derived with the propagation constant as the eigenvalue; this is called the propagation constant formulation, where the propagation constant can be directly determined for a given frequency, and thus the propagation constant formulation can be easily applied to lossy waveguiding problems. In order to reduce the number of unknowns, an equation in terms of the transverse unknowns only can be obtained [2], [3]. In this case, however, the inversion of a matrix is needed and so the sparsity is destroyed. Such a formulation leads to a full-matrix eigensystem.

3. Sparse Generalized Eigenvalue Problems

A recent trend is a variable transformation [1], [8], [9], which is often used to get an equation in terms of both the transverse and the axial unknowns, and then it is also possible to fully exploit the sparsity.

Introducing the variable transformation as

$$\left.\begin{matrix} E_t \\ H_t \end{matrix}\right\} = \phi_t, \qquad \left.\begin{matrix} E_z \\ H_z \end{matrix}\right\} = j\beta\,\phi_z \tag{1}$$

the resultant matrix equation has the form

$$[K]\{\phi\} - j\beta[L]\{\phi\} - \beta^2[M]\{\phi\} = \{0\} \tag{2}$$

with

$$[K] = \begin{bmatrix} [K_{tt}] & [0] \\ [0] & [0] \end{bmatrix}, \quad [L] = \begin{bmatrix} [L_{tt}] & [L_{tz}] \\ [L_{zt}] & [0] \end{bmatrix}, \quad [M] = \begin{bmatrix} [M_{tt}] & [M_{tz}] \\ [M_{zt}] & [M_{zz}] \end{bmatrix} \tag{3}$$

where the subscripts t and z denote the transverse and axial components, respectively, β is the phase constant in the axial direction, the components of the $\{\phi\}$ vector are composed of edge and nodal variables, $\{0\}$ is a null vector, $[K]$, $[L]$, and $[M]$ are the finite element matrices, and $[0]$ is a null matrix. In lossy cases, β is replaced by $\beta - j\alpha$ with α being the attenuation constant.

For achiral waveguides, $[L]$ becomes a null matrix, and then (2) is reduced to an M-by-M sparse generalized eigenvalue equation with M being the number of degrees of freedom. For chiral waveguides, on the other hand, (2) can be transformed into the following $2M$-by-$2M$ generalized eigenvalue equation:

$$\begin{bmatrix} [0] & [I] \\ [K] & -[L] \end{bmatrix} \begin{bmatrix} \{\phi\} \\ j\beta\{\phi\} \end{bmatrix} - j\beta \begin{bmatrix} [I] & [0] \\ [0] & -[M] \end{bmatrix} \begin{bmatrix} \{\phi\} \\ j\beta\{\phi\} \end{bmatrix} = \{0\} \tag{4}$$

where$[I]$ is a unit matrix. The band of the global matrices in (4) can be reduced by reordering the components of the eigenvector as [8]

$$\begin{bmatrix} \{\phi\} \\ j\beta\{\phi\} \end{bmatrix} = [\phi_1,\ j\beta\phi_1,\ \phi_2,\ j\beta\phi_2,\ \cdots\cdots,\ \phi_M,\ j\beta\phi_M]^T \tag{5}$$

where T denotes a transpose. The subspace iteration method is used to handle the large sparse generalized complex eigenvalue problems with M or $2M$ dimensional matrices.

4. Numerical Examples

Any arbitrary cross section transmission lines can be analyzed with the present approach.

Table 1 is comparing the FEM results to the method of moment results by Ma *et al.* [10] for an achiral microstrip line with a trapezoidal conductor strip as shown in Fig. 1, where

$W = 3.0$ mm, $t = 0.3$ mm, the tilted angle $\theta = 135°$, $a = 10.0$ mm, $b = 6.985$ mm, $c = 0.635$ mm, the relative permittivity of substrate $\epsilon_r = 9.8$, and the normalized phase constant is defined as β/k_0 with k_0 being the free-space wavenumber. We can see that the results of the method of moment [10] lie between the E-FEM and H-FEM results.

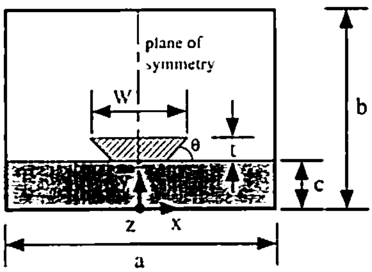


Fig. 1: Cross section of a shielded microstrip line.

Table 1: Normalized phase constant of the dominant mode in a shielded microstrip line with a trapezoidal conductor strip ($\theta = 135°$).

Frequency [GHz]	FEM E-formulation	FEM H-formulation	Ma, Yamashita, and Xu [10]
10	2.9065	2.9018	2.90346
20	2.9958	2.9919	2.99371
30	3.0420	3.0387	3.04058

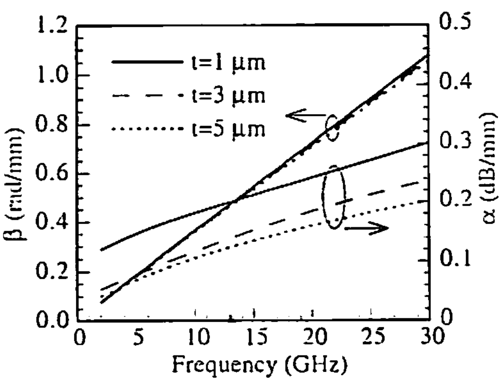


Fig. 2: Phase and attenuation constants of a lossy microstrip line on a uniaxial anisotropic substrate.

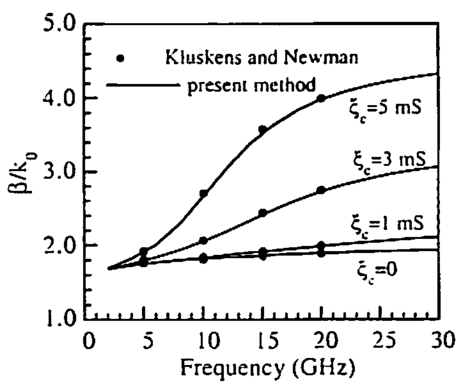
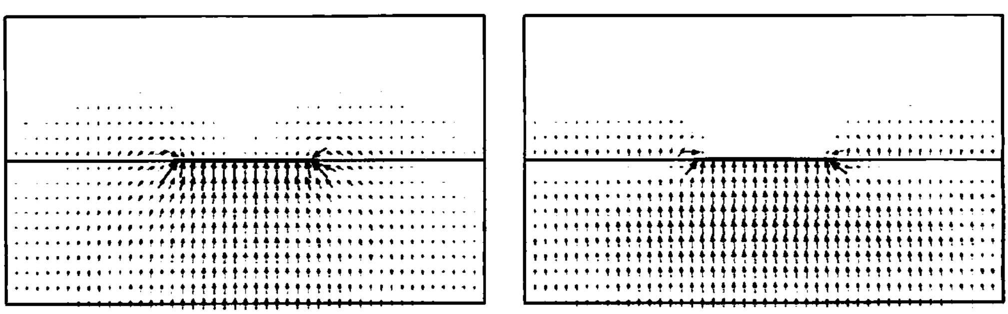


Fig. 3: Normalized phase constant of a microstrip line on a chiral substrate.



(a) (b)

Fig. 4: Transverse electric field distributions at 20 GHz. (a) $\xi_C=0$, (b) $\xi_C=5$ mS.

Figure 2 shows the phase and attenuation constants of a micron-sized lossy microstrip line on a uniaxial anisotropic substrate with the strip thickness t as a parameter, where $W = 22\mu m$, $\theta = 90°$, $a = 400\mu m$, $b = 200\mu m$, $c = 10\mu m$, the conductivity of gold strip is 3.9×10^7S/m, and the relative permittivities of substrate $\epsilon_{rx} = \epsilon_{rz} = 5.12$ and $\epsilon_{ry} = 3.4$. We can see that with increasing strip thickness, the attenuation constant decreases over the whole frequency range.

Figure 3 shows the normalized phase constant of a microstrip line on a chiral substrate with the chirality admittance ξ_c as a parameter, where $W = 3.0$ mm, $t = 0$, $a = 40.0$ mm, $b = 8$ mm, $c = 3$ mm, and the relative permittivity of substrate $\epsilon_r = 4.0$. The results of FEM denoted by solid lines are in good agreement with those of spectral domain method by Kluskens and Newman [11]. At lower frequencies the effect of ξ_c is very small, but, at higher frequencies, it greatly affects the propagation characteristics. Real parts of the transverse electric field distributions are shown in Fig. 4. It is found that chirality causes asymmetric field distributions.

5. Conclusions

The vector FEM with hybrid edge/nodal triangular elements is effectively used for the full-wave analysis of many inhomogeneous waveguides with sharp metal edges on achiral and chiral substrates. Sparse eigensystems with M-by-M and $2M$-by-$2M$ matrices are obtained for achiral and chiral waveguides, respectively. The subspace iteration method is used to handle large sparse generalized complex eigenvalue problems. In this approach, spurious solutions do not occur, and ohmic loss and dielectric loss are treated under the full-wave regime.

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