ELECTROMAGNETIC SCATTERING FROM A PERFECTLY CONDUCTING TARGET SURROUDED BY THE ATMOSPHERIC TURBULENCE

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1. Introduction

In this paper we derived general formulas of the scattered field for a target surrounded by the atmospheric turbulence. The second moment of the scattered field and average intensity are given. And the mean laser scattering cross section of a disk is obtained. Our numerical results can be reduced to Bugnol's results for the disk if the effects of atmospheric turbulence on the path from the target to the receiver are neglected, and our formulas are simpler.

2. Formulation

Considering the closed surface of a finite target disgnated by S', the scattered field at some observation point P not on the surface is given by[2]

$$\vec{E}_S(\vec{r}) = \int_{S'} [(\hat{n}' \times \vec{H}^T) \times \nabla' G_0(\vec{r},\vec{r}') - i\omega_0 \varepsilon (\hat{n}' \times \vec{E}^T) G_0(\vec{r},\vec{r}') + (\hat{n}' \cdot \vec{H}^T) \nabla' G_0(\vec{r},\vec{r}')] dS' \quad (1)$$

The integral in equation (1) is performed over the close surface S'. $\hat{n}$ is a unit vector normal to the surface at the point $\vec{r}'$, $G_0(\vec{r},\vec{r}')$ is the free space Green function, and $\vec{E}^T(\vec{r}')$ and $\vec{H}^T(\vec{r}')$ are the total wave fields at point $\vec{r}'$.

If a trasmitter, a receiver and a target are embeded with the atmospheric turbelence, we consider equation (1) right also unless making following corrections:

(1) Replacing the free space Green function $G_0(\vec{r},\vec{r}')$ with the atmospheric turbulence Green function $G(\vec{r},\vec{r}')$.

(2) Considering the effects of the atmospheric turbulence on the incident and scattering wave propagation.

The Green function $G(\vec{r},\vec{r}')$ satisfies the following equation in the atmospheric medium V except the transmitting region.

$$\left[\nabla^2 + k_0^2 n^2(\vec{r},t) \right] G(\vec{r},\vec{r}') = -\delta(\vec{r},\vec{r}') \quad (2)$$

$$n(\vec{r}) = 1 + n_1(\vec{r}) \ , \qquad \langle n_1(\vec{r}) \rangle = 0 \quad (3)$$

where $n_1(\vec{r})$ is the flunctuation of the refractive index, k_0 is wave number in free space.

Assuming the solution to equation (2) can be expressed as

$$G(\vec{r},\vec{r}') = G_0(\vec{r},\vec{r}') \exp[\psi(\vec{r},\vec{r}')] \quad (4)$$

where $\psi(\vec{r},\vec{r}')$ is the random part of the complex phase for spheric wave propagation in the atmospheric turbulence from the point $\vec{r}'$ to the point $\vec{r}$ on the basis of approximation condition $\nabla\psi(\vec{r},\vec{r}')/k_0 \ll 1$. At infrared and optical wavelengths this condition is satisfied in the atmosphere.

Under the far field approximation

$$G_0(\vec{r},\vec{r}') = \exp[ik_0(R_0 - \hat{k}_S \cdot \vec{r}')] / 4\pi R_0 \tag{5}$$

where R_0 is the radial distance from a convenient point to the observation point, $\hat{k}_S$ is unit scattering wave vector.

$$\nabla' G(\vec{r},\vec{r}') = -ik_0 \hat{k}_S G(\vec{r},\vec{r}') \tag{6}$$

If the radii of curvature at any point of the target surface are much larger than the incident wavelength, physical optics approximation can be applied. Using the boundary conditions for a perfectly conducting target

$$\hat{n}' \times \vec{E}^T = 0, \quad \hat{n}' \cdot \vec{H}^T = 0, \quad \hat{n}' \times \vec{H}^T = \begin{cases} 2\hat{n}' \times \vec{H}_i & \hat{k}_i \cdot \hat{n}' < 0 \\ 0 & \hat{k}_i \cdot \hat{n}' > 0 \end{cases} \tag{7}$$

and $\vec{H}_i(\vec{r}') = \hat{e} H_i(\vec{r}')$ is the incident wave field on the surface of the target, $\hat{k}_i$ is unit incident wave vector, then the scattered field is expressed as

$$\vec{H}_S(\vec{r}) = \frac{-ik_0 e^{ik_0 R_0}}{2\pi R_0} \int_{\hat{k}_i \cdot \hat{n}' < 0} dS' \, \{(\hat{n}' \times \hat{e}) \times \hat{k}_s\} \, H_i(\vec{r}')$$

$$\exp[-ik_0 \hat{k}_S \cdot \vec{r}' + \psi(\vec{r},\vec{r}')] \tag{8}$$

3. Moment

Using equation (8) we obtain the second moment of the scattered field

$$< \vec{H}_S(\vec{r}_1) \cdot \vec{H}_S^*(\vec{r}_2) > = \frac{k_0^2}{4\pi^2 R_0^2} \int_{\hat{k}_i \cdot \hat{n}'(\vec{r}_1') < 0} dS_1' \int_{\hat{k}_i \cdot \hat{n}'(\vec{r}_2') < 0} dS_2'$$

$$\{[\hat{n}'(\vec{r}_1') \times \hat{e}] \times \hat{k}_S\} \cdot \{[\hat{n}'(\vec{r}_1') \times \hat{e}] \times \hat{k}_S\} \tag{9}$$

$$\exp[ik_0 \hat{k}_S \cdot (\vec{r}_2' - \vec{r}_1')] < H_i(\vec{r}_1')H_i^*(\vec{r}_2')\exp[\psi(\vec{r}_1,\vec{r}_1') + \psi^*(\vec{r}_2,\vec{r}_2')] >$$

where $< \cdot >$ denotes an ensemble average.

We assume that the two paths, which are the path from a trasmitter to the target and that from the target to a receiver, are uncorrelated. The assumption is reasonable if the trasmitter and the receiver are well separated such as in a bistatic system. When propagation distance is large compared with the size of the aperture,it has been employed in the analysis of a coaxial trasmitter–receiver configuration by other reseachers[3]. Under this assumption, one can obtain

$$< H_i(\vec{r}_1')H_i^*(\vec{r}_2')\exp[\psi(\vec{r}_1,\vec{r}_1') + \psi^*(\vec{r}_2,\vec{r}_2')] > =$$

$$< H_i(\vec{r}_1')H_i^*(\vec{r}_2') > < \exp[\psi(\vec{r}_1,\vec{r}_1') + \psi^*(\vec{r}_2,\vec{r}_2')] > \tag{10}$$

When a plane wave is incident normally upon a perfectly conducting plane target through the isotropic atmospheric turbulence, the mutual coherent function of incident wave field at the surface of the target is given by [3]

$$< H_i(\vec{r}_1')H_i^*(\vec{r}_2') > = \exp[-D_1(L,\rho_d')] \tag{11}$$

where the difference coordinate $\vec{\rho}_d' = \vec{\rho}_1' - \vec{\rho}_2'$, $\vec{\rho}_1'$ and $\vec{\rho}_2'$ are two dimensional vector coordinates in the target plane at propagation distance L. The structure function for a plane wave through the isotropic atmospheric turbulence expresses as follows

$$D_1(L,\rho_d') = 8\pi^2 k_0^2 L \int_0^\infty K dK \Phi_n(K)[1 - J_0(K\rho_d')] \tag{12}$$

where $\Phi_n(K)$ is three–dimensional spectral density of the fluctuation of refractive index, $J_0(x)$ is the zero order Bessel function.

The mutual coherent function of the complex random phase at the observation plane to distance L′ far from the target can be written as[4]

$$< \exp[\psi(\vec{r}_1,\vec{r}_1') + \psi^* \ (\vec{r}_2,\vec{r}_2')] > = \exp[-\frac{1}{2}D_2(L',\rho_d,\rho_d')] \qquad (13)$$

and the structure function for spheric wave through the turbulence from the target can be obtained as follows

$$D_2(L',\rho_d,\rho_d') = 8\pi^2 k_0 L' \int_0^1 dt \int_0^\infty K dK \Phi_n(K) \ \{1 - J_0[K|t\rho_d + (1-t)\rho_d'|]\} \qquad (14)$$

where $\vec{\rho}_d = \vec{\rho}_1 - \vec{\rho}_2$, and $\vec{\rho}_1, \vec{\rho}_2$ are two dimensional coordinates at receiving plane.

When $\vec{r}_1'$ and $\vec{r}_2'$ are two points on the surface of the convex object and they are not lie in the transverse plane to propagation direction, but the distance between them is much smaller than the wave propagation distance L or L'. A correction for the mutual coherent function of plane wave is only required. We mutiply (11) by an addational phase $\exp[ik_0\hat{k}_i(\vec{r}_1' - \vec{r}_2')]$ for only to plane wave, then (11) becomes

$$< H_i(\vec{r}_1')H_i^* \ (\vec{r}_2') > = \exp[-D_1(L,\rho_d') + ik_0\hat{k}_i(\vec{r}_1' - \vec{r}_2')] \qquad (15)$$

Inserting (13) and (15) into (10), we can obtain the second moment of the scattered wave field

$$< \vec{H}_S(\vec{r}_1) \cdot \vec{H}_S^* \ (\vec{r}_2) > = \frac{k_0^2}{4\pi^2 R_0^2} \int_{\hat{k}_i \cdot \hat{n}'(\vec{r}_1') < 0} dS_1' \int_{\hat{k}_i \cdot \hat{n}'(\vec{r}_2') < 0} dS_2'$$

$$\{[\hat{n}'(\vec{r}_1') \times \hat{e}] \times \hat{k}_S\} \ \cdot \ \{[\hat{n}'(\vec{r}_2') \times \hat{e}] \times \hat{k}_S\}$$

$$\exp[-ik_0(\hat{k}_i - \hat{k}_S) \cdot (\vec{r}_2' - \vec{r}_1') - \frac{1}{2}D_1(L,\rho_d') - \frac{1}{2}D_2(L',\rho_d,\rho_d')] \qquad (16)$$

4. Numerical results

Consider backscattering, $\hat{k}_i = -\hat{k}_S$, $\hat{k}_i \cdot \hat{e} = 0$, and $L = L' = R_0, \vec{r}_1 = \vec{r}_2 = \vec{r}$, the mean intensity of the scattering field becomes form (16)

$$< \vec{H}_S(\vec{r}) \cdot \vec{H}_S^* \ (\vec{r}) > = \frac{k_0^2}{4\pi^2 R_0^2} \int_{\hat{k}_i \cdot \hat{n}'(\vec{r}_1') < 0} dS_1' \int_{\hat{k}_i \cdot \hat{n}'(\vec{r}_2') < 0} dS_2'$$

$$[\hat{k}_i \cdot \hat{n}'(\vec{r}_1')][\hat{k}_i \cdot \hat{n}'(\vec{r}_2')]\exp[-i2k_0\hat{k}_i \cdot (\vec{r}_2' - \vec{r}_1')]$$

$$\exp[-\frac{1}{2}D_1(R_0,\rho_d') - \frac{1}{2}D_2(R_0,\rho_d')] \qquad (17)$$

For example, a plane wave is incident normally on a perfectly conducting disk, the mean scattering cross section becomes

$$< \sigma > = \frac{4\pi}{\lambda^2} \int dS_1' \int dS_2' \exp[-\frac{1}{2}D_1(R_0,\rho_d') - \frac{1}{2}D_2(R_0,\rho_d')] \qquad (18)$$

If the effects of the turbulence on the path from the target to the receiver are neglected, (18) is reduced to Bugnol's result for the disk[1]. And in the absence of the turbulence, equation (18) becomes[2]

$$\sigma_0 = 4\pi A^2 / \lambda^2 \qquad (19)$$

where λ is wavelength, and $\Phi_n(K)$ is given approximately by

$$\Phi_n(K) = 0.03273 C_n^2 [K^2 + 1/L_0^2]^{\frac{-11}{6}} \qquad (20)$$

where C_n^2 is known as the structure constant, L_0 is the outer scale of turbulence. Then in equation (18) the structure functions for plane and spheric wave are given by, respectively

$$D_1(R_0,\rho_d) = 0.157 C_n^2 \pi^2 k_0^2 L_0 R_0 [1 - \frac{2^{\frac{1}{6}}}{\Gamma(\nu)} (\frac{\rho_d}{L_0})^\nu K_\nu (\frac{\rho_d}{L_0})] \quad (21)$$

and

$$D_2(R_0,\rho_d) = 0.157 C_n^2 \pi^2 k_0^2 L_0 R_0 [1 - \frac{2^{\frac{1}{6}}}{\Gamma(\nu)} \int_0^1 (\frac{\rho_d t}{L_0})^\nu K_\nu (\frac{\rho_d t}{L_0}) dt] \quad (22)$$

where $\nu = 5/6$, $K_{5/6}$ is the 5/6 order modified Bessel function, and $\Gamma(x)$ is the Gamma function.

Now,we can calculate the average cross section normalized by the scattering cross section in the absence of the turbulence, $<\sigma>/\sigma_0$, for $1.06\mu m$. Fig. 1*a* shows that with increasing the structure constant C_n^2, $<\sigma>/\sigma_0$ decreases obviously when the outer size of the turbulence and the range are given. Similarly, it also decreases when the propagation distance increases. Fig. 1b shows that the normalized scattering cross section varies very severely when the radius of the disk is of the same order of magnitude as the outer scale L_0. The results for $10.6\mu m$ are similar to those for $1.06\mu m$, but the turbulence has fewer effects on the normalized cross section than that for $1.06\mu m$.

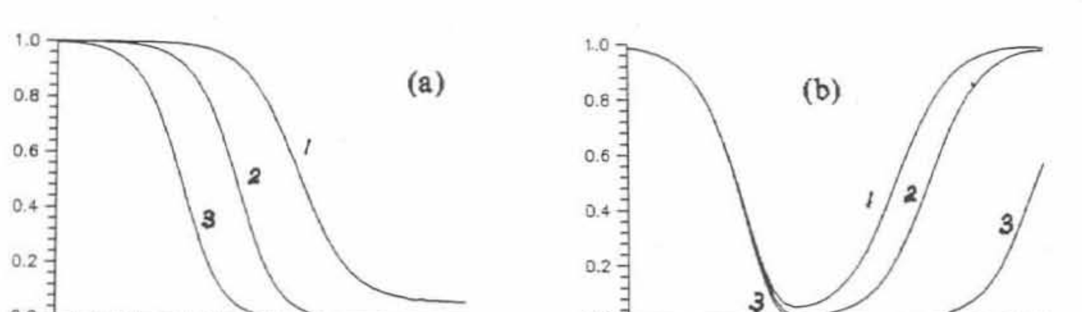

Fig.1 Affects of the turbulence on normalized scattering cross section.

(a): $L_0 = 1m, R_0 = 500m$. 1: $C_n^2 = 10^{-16}$, 2: $C_n^2 = 10^{-15}$, 3: $C_n^2 = 10^{-14}$

(b): $C_n^2 = 10^{-15}$, $R_0 = 500m$. 1: $a = 0.5m$, 2: $a = 1.0m$, 3: $a = 10m$.

(a is the radius of the disk)

References

[1] D. S. Dugnol, 'Laser radar cross section of objects immersed in the Earth's atmosphere:Theory and numerical results for a disk', SPIE Vol .663, p65–69 (1986).

[2] G. T. Ruck, et al, 'Radar Cross Section Handbook', Plenum Press, New York (1970).

[3] A. Ishimaru, 'Wave Propagation and Scattering in Random Media', Vol.2, Academic Press, New York (1978).

[4] H. T. Yura, 'Mutual coherence function of a finite crosssection optical beam propagation in a turbulence medium', Appl. Opt. , Vol.11, No.6, p1399–1460 (1972).