Monte Carlo Simulations of a Random Rough Surface at Near Grazing Incidence

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Introduction

Monte Carlo simulations of random rough surface scattering using the conventional full-matrix inversion in the context of the method of moments are usually limited to a surface length of a few hundred wavelengths. Over the past few years, a rigorous, fast Fourier transform (FFT) based iterative technique for the analysis of a large-scale random rough surface has been systematically developed [1]-[11]. This technique provides a speedy solution for rough surfaces up to several thousands wavelengths involving tens of thousand of unknowns on a typical workstation and therefore allows us to tackle the near-grazing incidence problem. For random rough surface scattering, a tapered incidence wave is employed to minimize the edge effects from the two ends of the surface. The minimum surface length is inversely proportional to cos q_i, where q_i is the incident angle measured from normal. Therefore, rough surface scattering at near-grazing incidence possesses additional challenges.

In a recent paper [10], we have demonstrated that the BMIA/canonical grid method can be applied to near-grazing incidence for the TE case in which the electric field is perpendicular to the plane of incidence. Numerical results were presented for a surface length of 2,500 wavelengths with 25,000 surface unknowns. In the BMIA/canonical grid method, the impedance matrix is decomposed into a strong near-interaction part and a weak far-interaction part. The near-interaction part is stored in banded matrix while the kernel for the far-interaction part is translated to a canonical grid by Taylor series expansion. A flat surface is chosen as the canonical grid for the rough surface. The use of the flat surface facilitates the summation of all the far interactions simultaneously using FFTs and therefore improves the efficiency of the iterative procedure. The translational invariant nature of the interactions on the canonical grid significantly reduces the computer memory requirement. For the TE case, the impedance matrix is symmetric and only half of the banded matrix is stored. For the TM case (magnetic field perpendicular to the plane of incidence), the matrix symmetry existing in the TE case is destroyed. The memory requirement for storing the banded matrix is roughly doubled. In the TE case, we employed a direct symmetric banded matrix solver [13] that requires $O(b^2N/2)$ operations for the LU decomposition and O(2bN) operations for the backsubstitution. Here N is the number of unknowns while b is the half bandwidth of the banded matrix representing the near interactions. The BMIA/canonical grid method only requires one LU decomposition in the first iteration. In contrast, this symmetric banded matrix solver cannot be employed for the TM case, and therefore, we replace the BMIA with the conjugate gradient method (CGM) with the far interactions computed by FFTs.

Iteration Based on the CG-FFT Method

For the TM incidence with a uniform discretization along the surface length, the matrix equation resulting from the Fredholm equation of the second kind reads:

$$\{\Psi_{in}\} = [Z]\{\Psi\} \tag{1}$$

where $\{\Psi_{in}\}$ is the incident field vector and $\{\Psi\}$ is the scattered field on the random surface. The elements in the matrix [Z] are denoted by z_{mn} where it is given by:

$$z_{mn} = \frac{1}{2} - \frac{f''(x_m)\Delta x}{4\pi\gamma_m^2} \text{ when } m = n$$

$$= \Delta x \frac{ik_o}{4} \frac{(x_m - x_n)f'(x_n) - (f(x_m) - f(x_n))}{r_{mn}} H_1^1(k_o r_{mn}) \quad m \neq n \qquad (2)$$

$$\gamma_m = \sqrt{1 + f'(x_m)} \text{ and } r_{mn} = \sqrt{(x_m - x_n)^2 + (f(x_m) - f(x_n))^2}$$

Here f(x) denotes the surface height at x, and f' and f'' are the slope of the surface and the derivative of the slope, respectively. Similar to the TE case, the Green's function kernel $H_1(k_o r_{mn})$ can be expanded in a Taylor series about f(x) = 0. Once again, we decompose the impedance matrix [Z] into a strong banded matrix $[Z^s]$ for near interactions and the remaining portion of the matrix $[Z^w]$ for far interactions. $[Z^w]$ is expanded in Taylor series resulting in the following form:

$$[Z^w] = \sum_{m=0}^{M} [Z_m^w] \qquad (3)$$

Note that each term on the rhs of Eq. (3) can be written as products of a diagonal matrix $[T_r]$, a Toeplitz matrix $[Z_d]$ and a diagonal matrix $[T_s]$. The physical interpretation of these matrices are as follows. Matrix $[T_s]$ projects all the source points on f(x) to a uniform grid at f(x) = 0. The interactions among all the grid points are computed via the multiplication of $[Z_d]$. Finally, $[T_r]$ translates the far interactions back to the rough surface. Since $[Z_d]$ is a Toeplitz matrix, the matrix vector multiplication can be evaluated efficiently using FFTs. Therefore, the matrix solution to Eq. (1) can be obtained using a modified CG-FFT method to include the contributions of the strong near-interaction matrix.

Numerical Results and Discussions

To illustrate the speed of the presented method, we consider a 1-D perfectly conducting random Gaussian rough surface with an rms height of 0.1 wavelength and a correlation length of 1 wavelength. The incident angle is 85° from normal. The surface is 1000 wavelengths long with 10,000 surface unknowns. Figure 1 shows the convergence of the modified CG-FFT method. A fast convergent solution is obtained. Figure 2 shows the scattering coefficients for 1 realization. The power conservation is 0.9918. For this result, we employ the tapered incident wave suggested in [12]. For rougher surfaces such as power law surfaces, we find that the power conservation is not acceptable. This may be due to the tapered incident chosen as it does not satisfy wave equation at near-grazing incidence. To alleviate this difficulty, we have implemented a Gaussian tapered incidence that satisfies wave equation and new results will be presented. While the matrix solution is quite efficient, e.g., this 10,000 unknowns problem only requires 30 minutes on a SPARC 10 workstation, a significant portion of the CPU time is required to compute the scattering coefficients. This is because a fine increment in the scattering angle is required to obtain a good power conservation for this long surface. However, this process can be speeded up by the use of FFTs as we can also translate the field points on the rough surface onto a canonical grid. Improvement on the scattering coefficient computation will be presented also.

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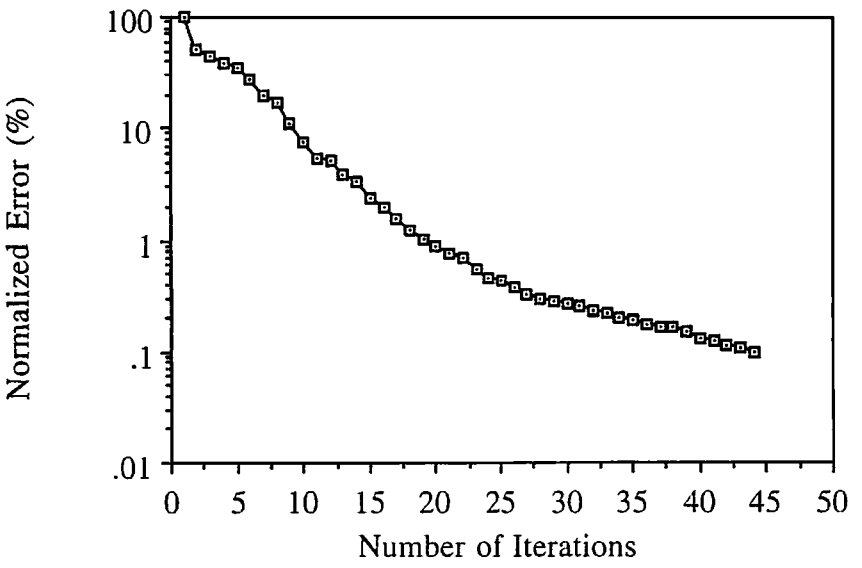

Figure 1. Rate of convergence for the modified CG-FFT for 10,000 unknowns.

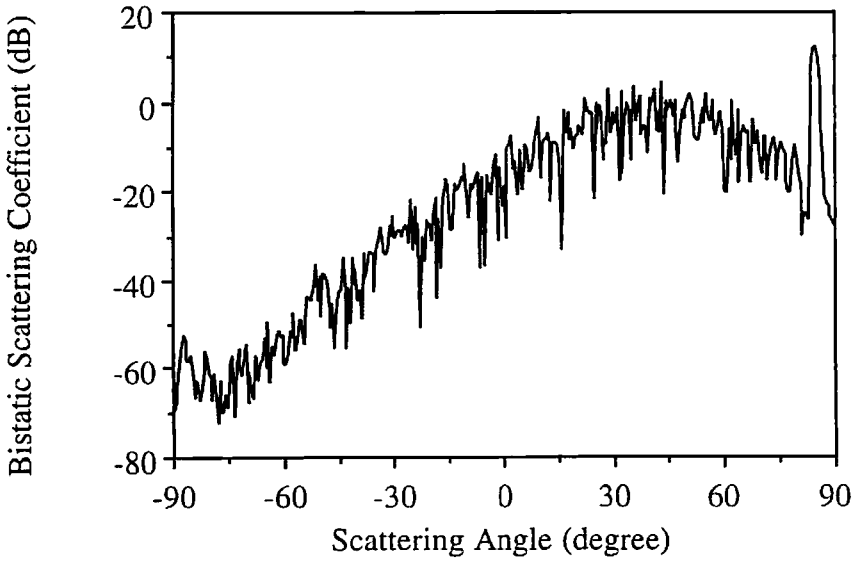

Figure 2. Bistatic scattering coefficient versus scattering angle.