THE EFFECTIVE DIELECTRIC CONSTANT OF A DISCRETE RANDOM MEDIUM CONTAINING DIELECTRIC CYLINDERS FOR E-WAVE AND H-WAVE INCIDENCES

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1. Introduction

Studies on the precise analysis of the effective propagation constant (K_{eff}) of a discrete random medium have become of great interest in microwave remote sensing and communication technologies[1]–[3]. When a coherent wave propagates through a discrete random medium, it is attenuated by scattering even if scatterers in the medium are lossless ones. Therefore K_{eff} of the medium becomes complex. This attenuation is usually called the coherence attenuation. Using K_{eff}, we obtain the effective dielectric constant (ε_{eff}): $\varepsilon_{eff} = K_{eff}^2/k_0^2$, where k_0 is the wavenumber in free space. As well known, multiple scattering by many scatterers plays a central role in calculating K_{eff}, and the following multiple scattering methods have been widely used: the Foldy's approximation, which is often called the effective field approximation(EFA), the quasi-crystalline approximation (QCA) and QCA with coherent potential (QCA-CP) [2].

Their conventional methods have been indicated to become invalid for high-dielectric particles; and we have presented a new method valid for the high-dielectric ones[4]. The method permits one to analyze electromagnetic wave scattering in a medium whose particles are randomly displaced from a uniformly ordered distribution[5],[6]. Therefore we can treat the case where the distribution of particles changes from total uniformity to complete randomness. Because the method has been shown to be more powerful than the conventional methods even for discrete random media, it is applicable to many cases where the interaction of waves with many particles plays a leading role in solving problems.

Although a general vector-analysis of K_{eff} is not easy at this time for complexity, we can obtain a piece of information on the polarization effects on K_{eff} by analyzing the K_{eff} of a medium whose dielectric circular cylinders are randomly distributed for both E-wave and H-wave incidences. We showed clearly the difference of the real part of ε_{eff} ($Re[\varepsilon_{eff}]$) between E-wave and H-wave incidences and also showed the difference of $Re[\varepsilon_{eff}]$ between our method and EFA[7]. However, we have not yet analyzed the imaginary part of ε_{eff} ($Im[\varepsilon_{eff}]$) closely related to the coherence attenuation by our method. In this paper, we have computed the $Im[\varepsilon_{eff}]$ as well as $Re[\varepsilon_{eff}]$ for a random medium containing dielectric cylinders, changing the volume fraction of cylinders. These computed ε_{eff} have been compared with ε_{eff} of EFA and ε_{eff} of a random medium of dielectric spheres, and have made clear the polarization effects on ε_{eff}.

2. The effective propagation constant of a random medium

Consider the 2-dimensional problem of E-wave and H-wave scattering by a random medium containing N cylinders embedded in free space (see Figure 1). The center position of the nth cylinder in a region S, written as $r_n = (x_n , y_n)$; $n = 1, 2, \cdots, N$, is assumed to be a Gaussian random vector with the mean $\langle r_n \rangle = a_n = (\ell_{xn} a_x , \ell_{yn} a_y)$ and the variance $\sigma_x^2 = \langle (x_n - \ell_{xn} a_x)^2 \rangle$ and $\sigma_y^2 = \langle (y_n - \ell_{yn} a_y)^2 \rangle$, where ℓ_{xn} and ℓ_{yn} are any integers indicating each cylinder and a_x and a_y are the mean length between each cylinder in the x and y directions, respectively.

For simplicity, we assume that each cylinder is a circular dielectric cylinder of radius b and dielectric constant $\varepsilon_s\varepsilon_0$. In addition, we assume that the mean length between each cylinder is a in the x and y directions: $a_x = a_y = a$ and that the variance from the uniformity is homogeneous and isotropic in the x and y directions: $\sigma_x^2 = \sigma_y^2 = \sigma^2$. The volume fraction of cylinders is defined by $f = N_0\pi b^2 = \pi b^2/a^2$, where N_0 is the number of cylinders per unit area.

When we analyze the mean Green's function in the medium, then the effective propagation constant of the random medium K_{eff} can be approximately expressed in the following form[4] under the condition of $(k_0 b)^2|\varepsilon_d| \ll k_0 a$; $\varepsilon_d = \varepsilon_s - 1$ [8].

$$K_{eff}^2 = k_e^2 + \frac{(-4i)}{a^2}\cdot\Big\{F_1(i_P, i_P) - F_2(i_P, i_P)\Big\}\,; \tag{1}$$

$$k_e^2 = k_0^2(1+\varepsilon_d f) + i\,Im\left[\frac{(-4i)}{a^2}\cdot\Big\{F_2(i_P, i_P) - F_1(i_P, i_P)\cdot[1-D(f)]\Big\}\right] \tag{2}$$

where i_P is the unit vector in the direction of wave propagation, k_e is the wavenumber in the continuous medium of which dielectric constant is the average of ones of the cylinder and the surrounding free space in the rate of each space occupation, and $D(f)$ is a monotonously decreasing function of f with the characteristics of $D(f)|_{f\to 1} = 0$ and $D(f)|_{f\to 0} = 1$. In this case, $F_1(i_P, i_P)$ in Eq.(1) denotes the forward scattering amplitude from a homogeneous circular cylinder of relative dielectric constant $[1 + (k_0^2/k_e^2)\varepsilon_d]$ and radius b in the medium of wavenumber k_e and is expressed as follows:

$$F_1(i_P, i_P) = \begin{cases} -\displaystyle\sum_{m=-\infty}^{\infty} \frac{n_1 J_m(k_e b) J_m'(n_1 k_e b) - J_m'(k_e b) J_m(n_1 k_e b)}{n_1 H_m^{(1)}(k_e b) J_m'(n_1 k_e b) - H_m^{(1)\prime}(k_e b) J_m(n_1 k_e b)} & \text{for E-wave incidence} \\ -\displaystyle\sum_{m=-\infty}^{\infty} \frac{n_1 J_m'(k_e b) J_m(n_1 k_e b) - J_m(k_e b) J_m'(n_1 k_e b)}{n_1 H_m^{(1)\prime}(k_e b) J_m(n_1 k_e b) - H_m^{(1)}(k_e b) J_m'(n_1 k_e b)} & \text{for H-wave incidence} \end{cases} \tag{3}$$

where $J_m(z)$ is the Bessel function of the first kind, $H_m^{(1)}(z)$ is the Hankel function of the first kind, the prime denotes the differentiation with respect to the argument, and $n_1 = [1 + (k_0^2/k_e^2)\varepsilon_d]^{1/2}$. On the other hand, $F_2(i_P, i_P)$ in Eqs.(1) and (2) denotes the forward scattering amplitude from an inhomogeneous circular cylinder of relative dielectric constant $[1 + (k_0^2/k_e^2)\varepsilon_e(r)]$ in the medium of wavenumber k_e. Here $\varepsilon_e(r)$ is the inverse transform of $\hat{\varepsilon}_e(\kappa) = \hat{\varepsilon}(\kappa)\exp[-Q(\kappa)]$ in which $Q(\kappa)$ is the cumulant function of $(r_n - a_n)$ and denotes the degree of random displacement from the uniform distribution. Because the displacement is Gaussian random, we obtain $\exp[-Q(\kappa)] = \exp[-\sigma^2\kappa^2/2]$. Here $\hat{\varepsilon}(\kappa)$ denotes the Fourier transform of $\varepsilon(r) = \varepsilon_d$ for $|r| \le b$ and $\varepsilon(r) = 0$ for $|r| > b$. If $\sigma = 0$, which means $f \to 1$, then $F_1(i_P, i_P) = F_2(i_P, i_P)$ because $\varepsilon_e(r) = \varepsilon(r)$.

On the low frequency assumption of $k_0 b \ll 1$, $F_2(i_P, i_P)$ can be approximately expressed by using the forward scattering amplitude from an homogeneous circular cylinder of relative dielectric constant $[1 + (k_0^2/k_e^2)\varepsilon_e(0)]$ and radius $b_e = b[\varepsilon_d/\varepsilon_e(0)]^{1/2}$ in the medium of wavenumber k_e:

$$F_2(i_P, i_P) = F_1(i_P, i_P)\Big|_{b\,\to\, b_e,\ n_1 \,\to\, n_e} \tag{4}$$

where $n_e = k_e/k_0 = [1 + (k_0^2/k_e^2)\varepsilon_e(0)]^{1/2}$, in which $\varepsilon_e(0) = \{1 - \exp[-b^2/(2\sigma^2)]\}\,\varepsilon_d$.

We can not give a definite relation of σ/a and f in our method because the relation depends on the physical state of medium. However, the relation expressed as $\sigma/a = 1 - f$ is assumed here because $Re[\varepsilon_{eff}]$ did not alter appreciably with changing this relation and was almost equal to that of QCA-CP for media containing randomly distributed spheres of a fairly low-dielectric constant[4].

In order to make a comparison with ε_{eff} of our method, we present K_{eff} of the Foldy's approximation. The Foldy's approximation yields

$$K_{eff}^2 = k_0^2 + \frac{(-4i)}{a^2}\cdot F_1(i_P, i_P)\Big|_{k_e\,\to\, k_0,\ n_1\,\to\, n_s} \tag{5}$$

where $n_s = \sqrt{\varepsilon_s}$. For very low frequencies, the ε_{eff} can be expressed as $1 + \varepsilon_d f$ for the E-wave case and $1 + [2\varepsilon_d f/(\varepsilon_d + 2)]$ for the H-wave case.

3. Numerical results

The calculation of ε_{eff} from Eqs.(1) and (2) requires a functional form of $D(f)$. We can choose $D(f)$ free under the condition that $D(f)$ is a monotonously decreasing function of f with $D(0) = 1$ and $D(1) = 0$, because our method is quite different from the conventional methods and does not need explicitly the pair distribution function[2]. In this paper, we assume $D(f) = (1-f)^4/(1+2f)^2$ which is derived by letting ε_{eff} of our method be equal to ε_{eff} of QCA-CP with the Percus-Yevick pair distribution function (QCA-CP-PY) for a random medium of low-dielectric spheres, because QCA-CP-PY is valid for the random medium.

Figure 2 shows the numerical results of $Re[\varepsilon_{eff}]$ and $Im[\varepsilon_{eff}]$ for the frequency 10[GHz] as a function of f for $\varepsilon_s = 3.2$ and $b = 1$[mm]. These parameters are chosen in order to compare with ε_{eff} of a medium containing randomly distributed dielectric spheres[4]. When increasing f from zero, the difference of $Re[\varepsilon_{eff}]$ between E-wave and H-wave incidences becomes large at first. After that, it becomes small gradually and zero finally. This result is physically reasonable and quite different from that of the Foldy's approximation. In case of $f > 0.785$ ($b > 0.5a$), our method seems to be invalid because soft cylinders are deformed and the cross-sections become non-circular, but ε_{eff} calculated here take normal values for $f \to 1$ which means complete occupation by dielectric material.

Let us compare ε_{eff} of our method with that of EFA. The ε_{eff} of both methods are almost equal to each other for quite low volume fraction. For the H-wave case, the difference of ε_{eff} between both methods becomes remarkable for high volume fraction, because EFA becomes invalid as well known.

Let us compare ε_{eff} of the medium containing dielectric cylinders with that of the medium containing spheres. The $Re[\varepsilon_{eff}]$ for H-wave case is almost equal to one of the random medium containing many spheres of the same dielectric constant and radius that the cylinders have. On the other hand, the $Im[\varepsilon_{eff}]$ for both cases of E-wave and H-wave are very larger enough than one of the random medium of spheres.

4. Conclusion

We calculated the effective dielectric constant (ε_{eff}) of a medium where many circular dielectric cylinders are randomly distributed in free space for both E-wave and H-wave incidences, by using the multiple scattering method presented by us. From the results, we showed clearly the difference of ε_{eff} between E-wave and H-wave incidences and also showed the difference of ε_{eff} between our method and the Foldy's approximation. When increasing the volume fraction of cylinders from zero, the difference of the real part of ε_{eff} due to the polarization of waves becomes large at first, small gradually and zero finally. This result is quite different from that of the Foldy's approximation. For the H-wave case, the real part of ε_{eff} is almost equal to one of the random medium containing many spheres of the same dielectric constant and radius that the cylinders have. However, the imaginary part of ε_{eff} depends largely on the polarization and the scatterer shape.

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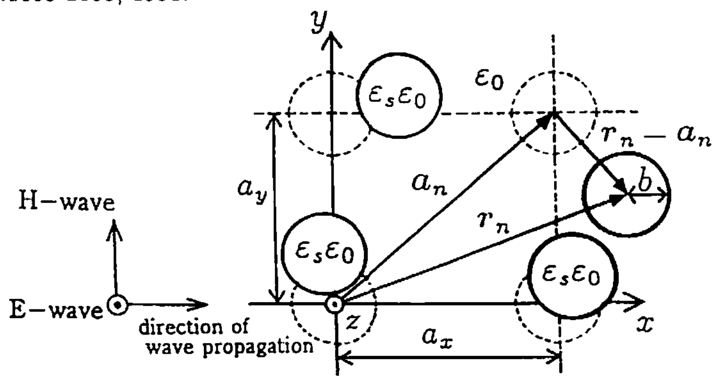

Fig. 1 A random medium whose dielectric cylinders are randomly displaced from a uniformly ordered distribution.

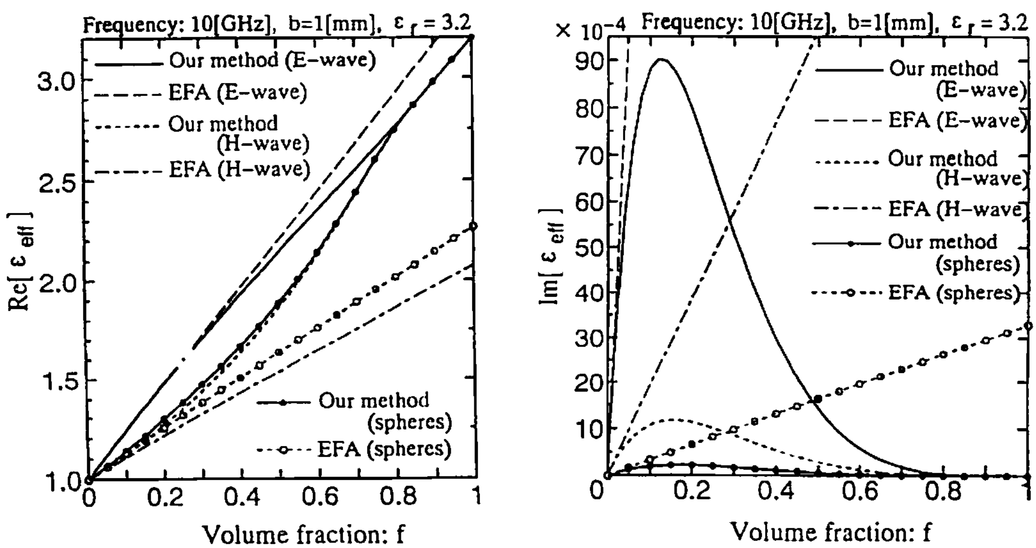

Fig. 2 The effective dielectric constant (ε_{eff}) as a function of the volume fraction (f) where $f = \pi b^2/a^2$ for cylinders and $f = [4\pi b^3/3]/a^3$ for spheres.