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PROPAGATIONAL ANALYSIS OF INHOMOGENEOUSLY LOADED WAVEGUIDES USING NODAL FINITE ELEMENTS

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Abstract *- A nodal finite element method is presented to analyze the propagational characteristics of inhomogeneously loaded waveguides. The propagation constant is treated as an eigenvalue. The bisection method is modified in order to be applicable to nonpositive definite matrices. Owing to the sparse matrix techniques and the iterative solver applied, complicated structures can be analyzed.*

I. INTRODUCTION

Due to the development in the computer techniques and the finite element analysis, the propagational analysis of inhomogeneously loaded waveguides is becoming a routine task in the design of microwave systems. In contrast to the great hardware and software improvement, there are still important problems to be solved.

The problem of the spurious modes seems to have been overcome, when the wavenumber is treated as an eigenvalue. In this case, there is no much difference between the 2D and 3D applications. The key is the treatment of the undesired gradient field which occures at zero wavenumber. The solution is either to impose the solenoidality condition or to to restrict the approximation of the gradient fields to a subspace of finite dimension.

The imposition of the solenoidality condition is usual in the case of a nodal element approximation [1]. A special penalty factor method is described in [3], introducing a scalar potential to incorporate the Coulomb gauge which coincides with the solenoidality condition. Special elements are used in [5] to filter out the undesired gradient field.

Recently, vector tangential finite elements or edge elements are used to avoid the spurious solutions [2,4]. A decisive property of these elements is that a finite number of gradient fields can be described exactly by them.

Other problems arise when the propagation constant is treated as an eigenvalue. The first practical problem is that a second order polynomial eigenvalue problem is obtained. There is a possibility to reduce it to a usual, generalized eigenvalue form, but it leads to indefinite matrices. The second problem is how to use an iterative solver which yields only the desired few eigenvalues with the sparsity of the matrices fully utilised. The most of the iterative solvers utilise the positive definiteness of the coefficient matrix of the propagational constant. If a formulation leads to symmetric, sparse but not positive definite matrices, the question remains, how to solve it in an efficient way.

Cendes [4] uses a transformation for the variables which results in the usual form with symmetrical matrices, but the coefficient matrix of the eigenvalue is not positive definite. A modification of the Lanczos method is proposed for the solution.

The aim of the paper is to present a nodal element based method for the treatment of the propagation constant as an eigenvalue which leads to symmetrical matrices. Lossless waveguides with isotropic material characteristics are discussed. Since the propagation constant is the eigenvalue, the applied formulation has to be valid for both imaginary and real case. Though, a second order polynomial eigenvalue problem is obtained, and the 'key' matrix is not positive definite, a modified version of the bisection method results in an efficient iterative solver. This is due to the fact that the order of the negative definite block of the coefficient matrix of the propagation constant is known and it is equal to the number of the nodes. Sparse matrix techniques are used for the solution.

Numerical examples are presented to demonstrate the efficiency of the proposed method.

II. BASIC EQUATIONS

A waveguide inhomogeneously loaded with isotropic dielectric and magnetic materials is considered. The cross section, Ω, of arbitrary shape is in the x - y plane and its boundary is Γ. Using the magnetic vector and electric scalar potentials and the fact that every variable varies in z-direction as $e^{\gamma z}$, the following wave equations and boundary contitions have to be satisfied ([3]):

$$\nu\nabla_{tr}\times\nabla_{tr}\mathbf{A}_{tr} - \nu\nabla_{tr}\nabla_{tr}.\mathbf{A}_{tr} - \gamma^2\mathbf{A}_{tr} - k_0^2\varepsilon_r(\mathbf{A}_{tr}+\nabla_{tr}V) = 0 \qquad (1)$$

$$-\nu\nabla_{tr}.\nabla_{tr}A_z - \nu\gamma^2A_z - k_0^2\varepsilon_r(A_z+\gamma V) = 0 \qquad (2)$$

$$k_0^2(\nabla_{tr}.\varepsilon_r\mathbf{A}_{tr} + \gamma\varepsilon_rA_z + \nabla_{tr}.\varepsilon_r\nabla_{tr}V + \gamma^2\varepsilon_rV) = 0 \qquad (3)$$

$$\mathbf{A}_{tr}\times\mathbf{n} = 0,\quad A_z = 0,\quad V = 0,\quad \nu\nabla_{tr}.\mathbf{A}_{tr} = 0, \qquad \text{on electric walls} \quad (4)$$

$$\nu\nabla_{tr}\times\mathbf{A}_{tr}\times\mathbf{n} = 0,\ \varepsilon_r(\mathbf{A}_{tr}+\nabla V).\mathbf{n} = 0,\ \nabla_{tr}A_z.\mathbf{n} = 0,\ \mathbf{A}_{tr}.\mathbf{n} = 0 \quad \text{on magnetic walls} \quad (5)$$

where the subscript "tr" and "z" mean the transversal and the longitudinal components, respectively.

III. DISCRETIZATION

For the solution of the differential equations (1) - (3), the following functional has to be extremized:

$$F(\mathbf{A}^*,V^*) = \int_\Omega\left[\left(\nabla\times\mathbf{A}^*\nu\nabla\times\mathbf{A} + \nabla.\mathbf{A}^*\nu\nabla.\mathbf{A}\right) - k_0^2\left(\mathbf{A}^*+\nabla V^*\right)\varepsilon_r(\mathbf{A}+\nabla V)\right]d\Omega, \qquad (6)$$

where the superscript asterisk denotes the test function and conjugate complew quantities. Since there is no integration in z-direction, the integrandus has to be independet of z. This can be attained , if in the test functions , $\gamma^* = -\gamma$. It can be shown [3], that when the first variation of the functional is zero, both the differential equations (1)-(3) and the natural boundary conditions are satisfied. Partitioning the variables into transversal and longitudinal parts, we get:

$$F(\mathbf{A}_{tr}^*,A_z^*,V^*) = \int_\Omega [(\nabla_{tr}\mathbf{A}_{tr}^* + \gamma^*\mathbf{e}_z\times\mathbf{A}_{tr}^* + \nabla_{tr}A_z^*)\nu(\nabla_{tr}\mathbf{A}_{tr} + \gamma\mathbf{e}_z\times\mathbf{A}_{tr} + \nabla_{tr}A_z) +$$

$$(\nabla_{tr}\mathbf{A}_{tr}^* + \gamma^*A_z^*)\nu(\nabla_{tr}\mathbf{A}_{tr} + \gamma A_z) -$$

$$k_0^2\left(\mathbf{A}_{tr}^* + \nabla_{tr}V^* + \mathbf{e}_z(A_z^*+\gamma^*V^*)\right)\varepsilon_r\left(\mathbf{A}_{tr} + \nabla_{tr}V + \mathbf{e}_z(A_z+\gamma\ V)\right)]d\Omega \qquad (7)$$

Using the usual nodal finite element discretization (8-noded isoparametric elements were used), the structure of the matrix equation is:

$$\mathbf{A}\mathbf{x} + \gamma^2\mathbf{B}(\gamma)\mathbf{x} = \mathbf{0}; \qquad \mathbf{x}^T = \begin{bmatrix} \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z & \mathbf{V} \end{bmatrix} \tag{8}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{xx} & \mathbf{A}_{xy} & \mathbf{0} & \mathbf{A}_{xv} \\ \mathbf{A}_{xy}^T & \mathbf{A}_{yy} & \mathbf{0} & \mathbf{A}_{yv} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{zz} & \mathbf{0} \\ \mathbf{A}_{xv}^T & \mathbf{A}_{yv}^T & \mathbf{0} & \mathbf{A}_{vv} \end{bmatrix}; \qquad \mathbf{B} = \begin{bmatrix} \mathbf{B}_{xx} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{yy} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{zz} & \frac{-k_0^2}{\sqrt{-\gamma^2}}\mathbf{B}_{zv} \\ \mathbf{0} & \mathbf{0} & \frac{-k_0^2}{\sqrt{-\gamma^2}}\mathbf{B}_{zv}^T & -k_0^2\mathbf{B}_{vv} \end{bmatrix}; \tag{9}$$

IV. BISECTION SOLVER

The bisection method consists of two main steps. The first step is to localize the eigenvalues finding lower and upper bounds for them by a factorization process. The sufficient condition for the factorization is that the matrix **B** is positive definite. This fact ensures that there exist a lower bound of the eigenvalues at which all the diagonal elements of the factorized matrix will be negative. If the matrix **B** is not positive definite, there remain positive elements in the factorized matrix at arbitrary low eigenvalues. In our case, the number of these elements is equal to the number of the nodes, which is the order of the negative definite block of the **B** matrix. This means that the detection of the lower bound is shifted by this number. If γ becomes real, the off-diagonal blocks of the **B** matrix are neglected at the calculation of the approximate bounds of the eigenvalues.

The second step is to calculate correct eigenvalues and eigenvectors by inverse iteration. During the inverse iteration, the eigenvalue is treated as a complex variable (real or imaginary). The procedure needs a very little modification of the bisection solver described in [7]. There was no significant difference in the computation time compared to the case when the wave number was the eigenvalue. Since sparse matrix techniques are used and only the desired eigenvalues are sought, the number of the unknowns can be high, without the computation time becoming too long.

V. EXAMPLES

Two numerical examples are presented to test the method. One is the well know half filled waveguide, the other is an image waveguide. Fig. 1 shows the first two dispersion curves of the half filled waveguide. The results show a good agreement with the analytical solutions. Fig. 2 and 3 show the dispersion curves and the power distributions of an image waveguide, respectively. The curves fit well to the curves taken from [4].

CONCLUSIONS

The nodal finite element formulation of inhomogeneously loaded waveguides yields the possibility to treat the propagational constant as an eigenvalue, using a modified version of the iterative bisection method for the solution. This is due to the fact that the order of the negative definite block of the coefficient matrix of the propagation constant is known. Sparse matrix techniques are applied for the numerical realization.

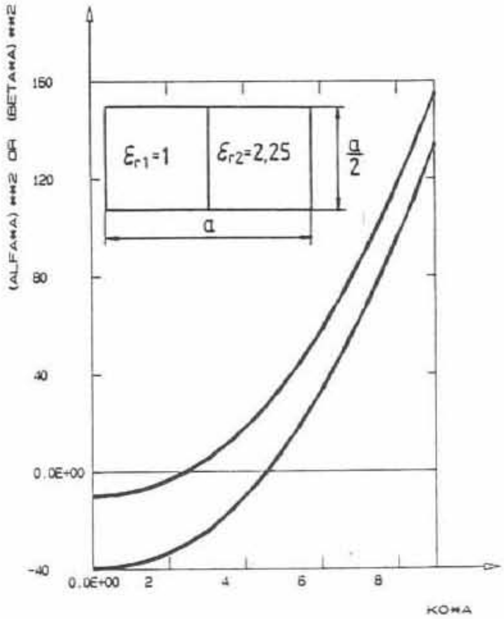

Fig. 1 : The dispersion characteristics of the half filled waveguide

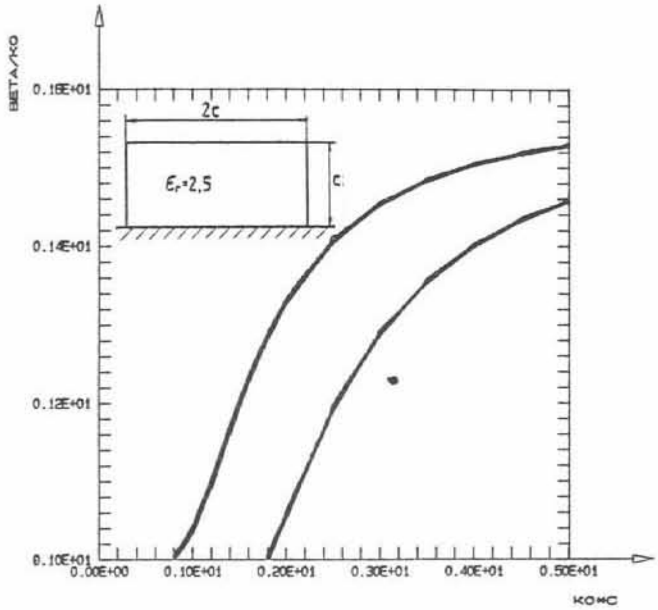

Fig. 2 : The dispersion characteristics of the image waveguide

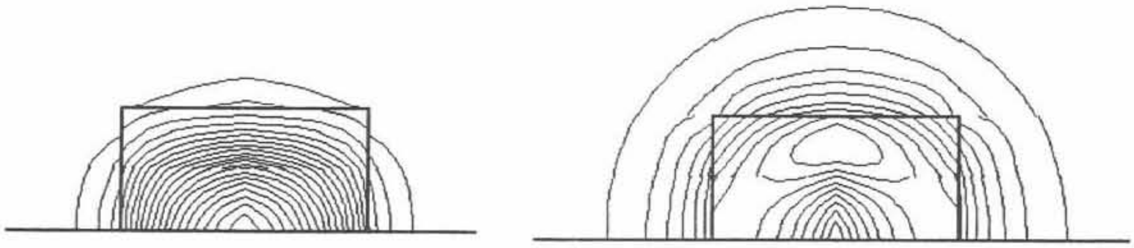

Fig. 3 : The power distribution for the first and second modes in the image waveguide at $k_0c=2.0$

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