STATISTICAL REFLECTION PROPERTIES OF ELECTROMAGNETIC PULSE BY BURIED OBJECTS IN RANDOM MEDIUM USING FD-TD

Yasumitsu MIYAZAKI Jun SONODA and Yuki JYONORI
Toyohashi University of Technology
1-1,Hibarigaoka,Tempaku-cho,Toyohashi-shi,441,Japan

1. Introduction

Recently microwave techniques incorporating advances in signal processing have been developing rapidly. In particular mapping of sub-surface random ground and detection of underground objects are important applications. As a consequence, underground radar systems which display the scattering field patterns of electromagnetic waves have become practical [1]-[4]. However, due to randomness and inhomogeneous nature of the underground media, it is very difficult to analyse their characteristics accurately. In principle, it is necessary to study electromagnetic wave reflection and scattering under various conditions of underground media.

One of the most powerful methods of computational analysis applicable to such problems is the Finite Difference Time Domain (FDTD) method. In this paper we apply FDTD method to simulate the scattering characteristics of an underground media consisting of an air gap target region in the presence of randomly located obstacles. Numerical results are given for various cases of the medium properties like the medium electric permittivity and the number, size and location of obstacles.

2. Analysis Model

The two dimensional analysis model is schematically displayed in Fig.1 and a general FDTD modelling in Fig.2. We consider a rectangular region consisting of a target air-gap and obstacles located randomly relative to it. The figure shows that the origin is located at the ground level, implying that the mapping is conducted at a short distance above the ground. The shapes of air-gap as well as obstacles are rectangles. Typical numerical values considered are: 2.56m along x-axis, 0.44m above the ground, 2.56m below ground for the analysis region. The size of the air-gap is 1m x 0.5m and is located 0.5m below the ground.

The analysis corresponds to illuminating the region by an electromagnetic monopulse having spatio-temporal distribution, and analysing the reflected fields. This can be practically done by placing a planar antenna along the x-axis above the ground. The length of the antenna is typically 0.3m and is placed 0.1m above the ground. Except for the target air-gap and the obstacles which are random, the medium is considered non-dispersive and homogeneous with a relative permittivity, $\varepsilon_{rg}=4.0$.

3. Source Modelling

Source modelling essentially consists of choosing an appropriate current distribution function corresponding to illumination of the analysis region by a pulse wave from a planar antenna. This current distribution is the source term in the Maxwell's equations which determine the reflected and scattered fields along with the boundary conditions on the region. In order to illuminate a wide subsurface region of 0.3m long, the current source distribution is characterized by sinusoidal function in the spatial domain (x-direction) (Fig.3), and by a Gaussian function in the time domain (Fig.4).

$$J_y(x,t)=\begin{cases} A\sin\left(\frac{\pi}{L}x\right)\exp\left\{-a\left(\frac{t-3}{t_w}\right)^2\right\} & 0\le t\le t_w, 0\le x\le L \\ 0 & \text{otherwise}\end{cases} \qquad (1)$$

where A, a, t_w and L are 100, 30, 5nsec, 0.3m respectively.

4. FD-TD Method

Characteristics of electromagnetic waves are expressed by Maxwell's curl-equations. FD-TD method proposed by K.S.Yee[5] is used, in which these partial differential equations are approximated by difference equations first, and then the electric and magnetic fields are calculated mutually from initial values at every incremental time Δt. For the two-dimensional model shown Fig.1, the Maxwell's curl-equations are transformed to the following difference equations.

$$H_x^{n+\frac{1}{2}}\left(p,q+\frac{1}{2}\right)=H_x^{n-\frac{1}{2}}\left(p,q+\frac{1}{2}\right)+C_1\left\{E_y^n(p,q+1)-E_y^n(p,q)\right\} \tag{2}$$

$$H_z^{n+\frac{1}{2}}\left(p+\frac{1}{2},q\right)=H_z^{n-\frac{1}{2}}\left(p+\frac{1}{2},q\right)+C_1\left\{E_x^n(p+1,q)-E_x^n(p,q)\right\} \tag{3}$$

$$E_y^n(p,q)=C_2E_y^{n-1}(p,q)-C_3J^{n-\frac{1}{2}}(p,q)+C_4\left\{H_x^{n-\frac{1}{2}}\left(p,q+\frac{1}{2}\right)\right.$$
$$\left.-H_x^{n-\frac{1}{2}}\left(p,q-\frac{1}{2}\right)-H_z^{n-\frac{1}{2}}\left(p+\frac{1}{2},q\right)+H_z^{n-\frac{1}{2}}\left(p-\frac{1}{2},q\right)\right\} \tag{4}$$

$$C_1=\frac{\Delta t}{\Delta s\,\mu(p,q)} \tag{5}$$

$$C_2=\frac{1-\dfrac{\Delta t\,\sigma(p,q)}{2\,\varepsilon(p,q)}}{1+\dfrac{\Delta t\,\sigma(p,q)}{2\,\varepsilon(p,q)}} \tag{6}$$

$$C_3=\frac{\dfrac{\Delta t}{\varepsilon(p,q)}}{1+\dfrac{\Delta t\,\sigma(p,q)}{2\,\varepsilon(p,q)}} \tag{7}$$

$$C_4=\frac{\dfrac{\Delta t}{\Delta s\,\varepsilon(p,q)}}{1+\dfrac{\Delta t\,\sigma(p,q)}{2\,\varepsilon(p,q)}} \tag{8}$$

Where p and q are space coordinates, n is a time step number. And Δx=Δz=Δs and Δt denote the spacial increment and temporal increment, respectively. These increments have to satisfy computational stability condition. So, in this study, we set Δs=0.01m, Δt=0.02nsec.

We adopt the following absorbing boundary conditions due to G.Mur'[6], in order to replace the actual situation of infinite underground region by a finite sized rectangular region.

$$\left(\frac{\partial}{\partial n}+\frac{1}{v}\frac{\partial}{\partial t}\right)E_y=0 \tag{9}$$

where n is out ward pointing normal on the boundary. And v is propagation velocity.

So, electric field Ey on the boundary x=0 is

$$E_y^n(0,q)=E_y^{n-1}(1,q)+C_5\left\{E_y^n(1,q)-E_y^{n-1}(0,q)\right\} \tag{10}$$

where

$$C_5=\frac{v\Delta t-\Delta s}{v\Delta t+\Delta s}. \tag{11}$$

5. Statistical Representation of Random Medium

Statistically a random medium is represented by a multivariable random function with parameters representing the constants and variables of the medium including the obstacles and the target bodies within the region. In the present study, the parameters are the electric permittivity, size and location of the obstacles which are random variables. Fourier analysis can be applied to this statistical representation by considering the complexity of random medium as an input data. If the input data is considered as a two dimensional random function $\varepsilon(p,q)$, where the integer variables p and q correspond to spatial and temporal coordinates in physical space, the Fourier spectrum that formally represents the complexity of random medium is given by

$$G_\varepsilon(k,l)=\sum_{p=0}^{M-1}\sum_{q=0}^{N-1}\varepsilon(p,q)\exp\left(\frac{-2\pi jkp}{M}\right)\exp\left(\frac{-2\pi jlq}{N}\right). \tag{12}$$

Here $j=\sqrt{-1}$ and M and N denote the matrix size of the random data. k and l correspond to the two dimensional spatio-temporal frequency domain.

This spectrum can be normalized as follows:

$$G_{or}(k,l)=\frac{G_{\varepsilon}(k,l)}{\sum_{r=0}^{M-1}\sum_{s=0}^{N-1}G_{\varepsilon}(r,s)} \tag{13}$$

A limiting value of normalization can be obtained by considering an appropriate standard random medium. Thus a Gaussian distribution for the function $\varepsilon(p,q)$ is chosen as a suitable standard. Then

$$G_{g\varepsilon}=G_{g\varepsilon}(k,l)=\frac{1}{MN} \qquad (k=0\cdots M-1, l=0\cdots N-1). \tag{14}$$

For an arbitrary random function $\varepsilon(p,q)$, a formal estimation of the randomness can be given by the variance σ_f given as a deviation from the Gaussian case

$$\sigma_f=\sqrt{\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}\left\{G_{or}(k,l)-G_{g\varepsilon}\right\}^2}\ . \tag{15}$$

Another equivalent statistical representation is given by the two dimensional auto-correlation function defined by

$$\begin{aligned}R_{\varepsilon}(m,n)&=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\varepsilon(x,y)\varepsilon(x+m,y+n)\\&=F^{-1}\left\{|G_{\varepsilon}(k,l)|^2\right\}\\&=\frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}\left|G_{\varepsilon}(k,l)\right|^2\exp\left(\frac{2\pi jkm}{M}\right)\exp\left(\frac{2\pi jln}{N}\right),\end{aligned} \tag{16}$$

where F^{-1} indicates the inverse Fourier transform.

In this case an estimation of the medium randomness can be obtained from the Fourier transform of the auto-correlation function, by defining $1/e$ value of spectral intensity distribution as the indicative auto-correlation value.

Thus using Fourier analysis, it is convenient to represent statistically various important features of the random medium such as the electrical permittivity of the obstacles, their size, spacing and so on.

6. Analysis Result

As a specific example of analysis, we consider a ground medium of relative permittivity $\varepsilon_{rg}=4$, conductivity $\sigma_g=10^{-7}$[s/m], relative permeability $\mu_{rg}=1$, N = 400 obstacles each of size d < 10cm and $\varepsilon_{ro}<8$.

The model considered is shown in Fig.5 with their corresponding parameters displayed in Table1. The result of FDTD analysis of received signal is shown in Fig.6. Fourier power spectrum of this analysis model is shown Fig.7. The indicative auto-correlation value for this case is about 10cm.

7. Conclusion

Statistical representation of a random medium is done, and the relationship between the received signal and the medium is established. The analysis may be further developed in many ways like increasing the accuracy and speed of the FDTD algorithm, extending to three dimensional cases, incorporating more randomness and parameters and considering the cases much closer to practical conditions. Improvements can also be accomplished through more sophisticated signal processing of the received signal patterns.

References

[1] T.Suzuki:J.IEICE,vol.37,No.3,pp308-311,(1984-3)

[2] Y.Nagashima et al:IEICE of Japan,Trans.vol.J74-C-II,No.5,pp317-324,(1984-3)

[3] H.Dobashi et al:T.IEE Japan,vol.109-D,No.12,pp937-943,(1989-12)

[4] Y.Michiguchi et al:IEEE Trans on GRS,vol.26,No.6,pp733-740,(1988-11)

[5] K.S.Yee:IEEE Trans on AP,vol.14,No.3,pp302-307,(1966-5)

[6] G.Mur:IEEE Trans on EMC,vol.23,No.4,pp1073-1077,(1981-11)

[7] Y.He et al:IEICE of Japan.Trans.vol.J76-B-II,No.4,pp245-252,(1993-4)

[8] K.Taketomi and Y.Miyazaki:Proc.ISAP,Sapporo,Japan,pp1249-1252

[9] J.Sonoda and Y.Miyazaki: Optical Fiber Science and Electromagnetic Theory.(1995-10)

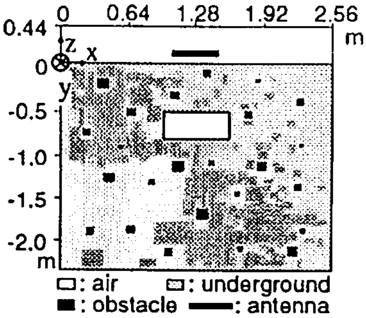

Fig.1 Analysis model

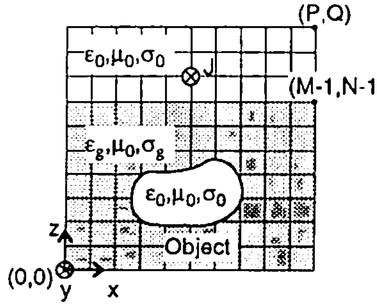

Fig.2 Divide of feild

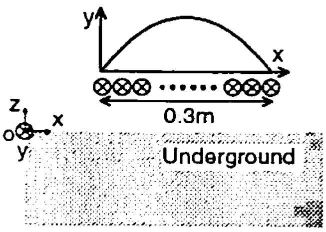

Fig.3 Source model

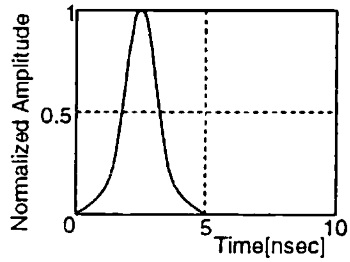

Fig.4 Incident pulse

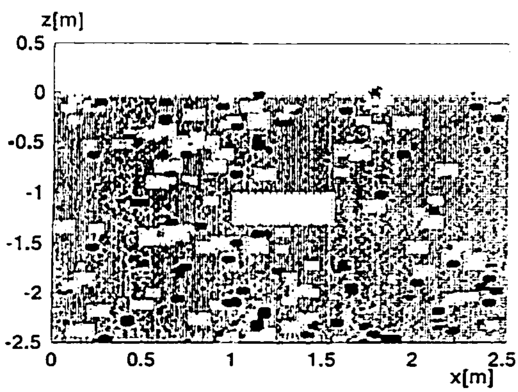

Fig.5 Permittivity $\varepsilon(p,q)$ at the underground

Table1 Position of obstacles

	X axis	Z axis	Size	ε_{ro}
1	0.02	-1.37	0.080	3.23
2	0.80	-2.30	0.080	2.38
⋮	⋮	⋮	⋮	⋮
400	1.96	-0.75	0.020	3.04
Max	2.45	-0.01	0.090	8.99
Min	0.01	-2.56	0.010	1.01
Mean	——	——	0.052	4.89

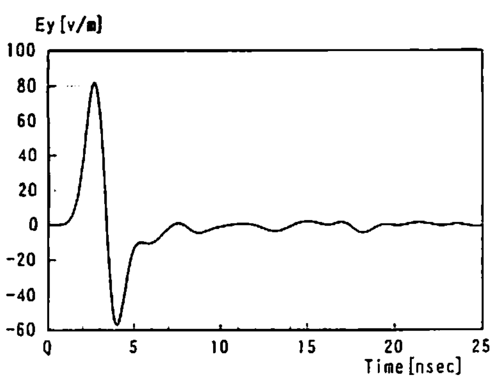

Fig.6 Time responce of Ey at (1.28,0.1)

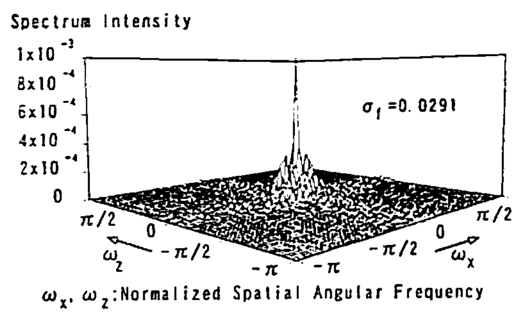

Fig.7 Spectrum intensity of $\varepsilon(p,q)$