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ANALYSIS OF NONLINEAR GRATING COUPLERS

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1. INTRODUCTION

Recently, nonlinear guided-optical phenomena have attracted the attention of many workers to establish the all-optical signal processing. As one of the nonlinear media, the Kerr-type medium whose refractive index depends on the light intensity is usually used. This medium shows the interesting phenomena such as self-focusing, generation of self-phase modulation and optical bistability [1], [2]. The grating couplers are one of the most important elements for the optical integrated circuits. Under some approximations, the mode conversions [3] and the beam scanner [4], [5] are examined. For the more complete interpretation of the properties of the nonlinear grating couplers, the precise analysis is needed.

In this paper, the nonlinear grating couplers are analyzed by using the singular perturbation technique with the multiple space scales [6]–[8]. The perturbation is carried up to the second order. As a numerical example, the properties of the output coupler and the input one are examined. The case of TE polarized wave is treated and the time factor $\exp(j\omega t)$ is suppressed.

2. FORMULATION OF THE PROBLEM

Let us consider the nonlinear grating couplers as shown in Fig. 1. The relative dielectric constant in the film is $\varepsilon_1 + \alpha|E_y|^2$. The coefficient α is the nonlinear coefficient and E_y is the electric field in the film. The relative dielectric constant in the cladding is ε_2. We assume that $\varepsilon_1 > \varepsilon_2$ and that the nonlinear medium in the film is self-focusing, i.e., $\alpha > 0$. The equation for the grating profile is given as follows:

$$x = \xi(z) = d + h\cos Kz\,, \tag{1}$$

where $K(= 2\pi/\Lambda)$ is the wave number of the grating. For TE wave, the governing equation is written as follows:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\varepsilon(x)\right)E_y + k^2\alpha(x)|E_y|^2E_y = 0\,, \tag{2}$$

where

$$\varepsilon(x) = \begin{cases} \varepsilon_1\,, & -d < x < \xi(z) \\ \varepsilon_2\,, & x < -d, \quad x > \xi(z) \end{cases}, \quad \alpha(x) = \begin{cases} \alpha\,, & -d < x < \xi(z) \\ 0\,, & x < -d, \quad x > \xi(z) \end{cases}, \tag{3}$$

and $k(= 2\pi/\lambda)$ is the wave number in free space.

We assume that the nonlinearity is weak and the maximum grating depth is small compared to the average thickness $2d$ of the slab waveguide. The orders of $\alpha|E_y|^2$ and h/d are the same and these smallness are indicated by the parameter δ. The different space scales in the z direction $z_0 = z, z_1 = \delta z, z_2 = \delta^2 z$ are introduced and expansion of the electromagnetic field components are used:

$$\psi(x,z) = \sum_{m=0}^{2} \delta^m\,\psi^{(m)}(x, z_0, z_1, z_2)\,, \tag{4}$$

where $\psi(x,z)$ denotes the electromagnetic field components H_x, E_y, H_z. Substituting Eq. (4) into Eq. (2) and equating coefficients to equal of δ to zero, we can get the governing equations for each order [8].

Since the maximum depth of the grating is very small compared to the thickness of the slab waveguide, the equivalent boundary conditions are derived by expanding the electromagnetic field components in a Taylor series about $x = d$. The boundary conditions at $x = -d$ require the continuity of the tangential electric and magnetic fields.

3. PERTURBATION SOLUTIONS

$O(\delta^0)$ solutions : From the consideration of the governing equation and the equivalent boundary condition, the zero-order solutions correspond to the symmetric linear slab waveguide. In this paper, the following even guided wave is considered.

$$E_y^{(0)} = \exp(-j\beta_0 z_0) N \begin{cases} A_g(z_1, z_2)\cos\alpha_0 d \exp\{-\gamma_0(|x|-d)\}, & |x| > d \\ A_g(z_1, z_2)\cos\alpha_0 x, & |x| < d \end{cases}, \tag{5}$$

where

$$\alpha_0^2 = \varepsilon_1 k^2 - \beta_0^2, \qquad \gamma_0^2 = \beta_0^2 - \varepsilon_2 k^2, \tag{6}$$

and $A_g(z_1, z_2)$ is the complex amplitude of the guided wave. The normalized coefficient N is defined in such way that $|A_g|^2$ equals to the power carried by the guided wave. From the boundary condition, the dispersion equation $\tan\alpha_0 d = \gamma_0/\alpha_0$ is obtained.

$O(\delta^1)$ solutions : From the boundary conditions, it can be shown that the first-order solutions are composed of the first harmonics possessing the wave numbers $\beta_0 - K$ and $\beta_0 + K$ in the z_0 scale. We assume that only the $\beta_0 - K$ wave is a fast wave, i.e. a radiated wave. Consequently, we seek the solutions in the following form:

$$\begin{aligned} E_y^{(1)} &= N_r e^{-jK_{-1}z_0}\left\{a_1 e^{j\gamma_{-1}(x-d)} + b_1 e^{-j\gamma_{-1}(x-d)}\right\} + c_1 e^{-jK_1 z_0 - \gamma_1(x-d)} \\ &\quad + e^{-j\beta_0 z_0 - \gamma_0(x-d)}\left\{G_1 - \frac{j\beta_0 N}{\gamma_0}\cos\alpha_0 d\,\frac{\partial A_g}{\partial z_1}\,x\right\}, \quad x > d, \end{aligned} \tag{7}$$

$$\begin{aligned} E_y^{(1)} &= e^{-jK_{-1}z_0}\left(A_1 \sin\alpha_{-1}x + B_1\cos\alpha_{-1}x\right) + e^{-jK_1 z_0}\left(C_1\sin\alpha_1 x + D_1\cos\alpha_1 x\right) \\ &\quad + e^{-j\beta_0 z_0}\left[E_1\sin\alpha_0 x + F_1\cos\alpha_0 x + \frac{j\beta_0 N}{\alpha_0}\frac{\partial A_g}{\partial z_1}\,x\sin\alpha_0 x \right. \\ &\quad \left. - \frac{k^2\alpha N^3}{32\alpha_0^2}|A_g|^2 A_g(12\alpha_0 x\sin\alpha_0 x - \cos 3\alpha_0 x)\right], \quad -d < x < d, \end{aligned} \tag{8}$$

$$\begin{aligned} E_y^{(1)} &= \overline{N}_r e^{-jK_{-1}z_0}\left\{\overline{a}_1 e^{-j\gamma_{-1}(x+d)} + \overline{b}_1 e^{j\gamma_{-1}(x+d)}\right\} + \overline{c}_1 e^{-jK_1 z_0 + \gamma_1(x+d)} \\ &\quad + e^{-j\beta_0 z_0 + \gamma_0(x+d)}\left\{\overline{G}_1 + \frac{j\beta_0 N}{\gamma_0}\cos\alpha_0 d\,\frac{\partial A_g}{\partial z_1}\,x\right\}, \quad x < -d, \end{aligned} \tag{9}$$

where

$$\begin{aligned} K_{-1} &= \beta_0 - K, \qquad \alpha_{-1}^2 = \varepsilon_1 k^2 - K_{-1}^2, \qquad \gamma_{-1}^2 = \varepsilon_2 k^2 - K_{-1}^2 \\ K_1 &= \beta_0 + K, \qquad \alpha_1^2 = \varepsilon_1 k^2 - K_1^2, \qquad \gamma_1^2 = K_1^2 - \varepsilon_2 k^2 \end{aligned} \tag{10}$$

In Eq. (7), a_1, b_1 and N_r are the normalized amplitudes of the incident and radiated waves in the region $x > d$ and the normalized coefficient. The coefficients $\overline{a}_1, \overline{b}_1$ and $\overline{N}_r$ in Eq. (9) are the normalized amplitudes of the incident and radiated waves in the region $x < -d$ and the normalized coefficient. a_1 and $\overline{a}_1$ is assumed to be known. The unknown coefficients $b_1, \overline{b}_1, c_1, \overline{c}_1$, $A_1, B_1, C_1, D_1, E_1, F_1, G_1, \overline{G}_1$, which are functions of z_1, z_2, are determined by the boundary conditions. Applying the boundary condition using the zero-order dispersion equation, we can get the solvability conditions to have nontrivial solutions as follows:

$$\frac{\partial A_g}{\partial z_1} = j\eta^{(1)}|A_g|^2 A_g, \tag{11}$$

where $\eta^{(1)}$ is the coefficient including α. In the linear case, the amplitude A_g is not a function of z_1 [6], [7]. However, it is found that A_g depends on z_1 in the nonlinear case.

$O(\delta^2)$ solutions : An examination of the second-order boundary conditions shows that only the second-order solutions having a z dependence $\exp(-j\beta_0 z_0)$ will participate the coupling between the guided waves and the scattered first-order waves. Therefore, we shall seek the solutions having $\exp(-j\beta_0 z_0)$. The concrete expressions of the solutions are omitted on an account of space consideration. By similar calculation in the $O(\delta^1)$ solutions, the solvability conditions to have nontrivial solutions are obtained as follows:

$$\frac{\partial A_g}{\partial z_2} = g_1 a_1 + g_2 b_1 + g_3 A_g + j\eta^{(2)} |A_g|^4 A_g , \tag{12}$$

where $g_1 \sim g_3$ and $\eta^{(2)}$ are the coefficients including α, h and so on.

4. PROPERTIES OF NONLINEAR GRATINGS

After the multiple space scales z_0, z_1, and z_2 are transformed back into the original space scale z, and Eqs. (11) and (12) are used, it is found that the complex amplitude of the guided wave $A_g(z)$ is governed by the following equation:

$$\frac{dA_g}{dz} = g_1 a_1 + g_2 b_1 + g_3 A_g + j\eta^{(1)}|A_g|^2 A_g + j\eta^{(2)}|A_g|^4 A_g . \tag{13}$$

This equation shows the nonlinear equation to describe the power leakage of the guided wave due to the second-order coupling to the first-order wave. We can obtain the field distribution, leakage power, and the radiation angle at the arbitrary z by solving Eq. (13) numerically.

Let us consider that the grating exists over a finite length of the waveguide from $z = 0$ to $z = L$ and assume that the end effects taking place near $z = 0$ and $z = L$ are neglected. In this case, the angle for the maximum radiation θ_{rad} is given by $\cos^{-1}[(K_{-1} + \beta_g)/\sqrt{\varepsilon_2}k]$. β_g denotes the imaginary part of the complex amplitude of the guided wave A_g.

The coefficients $g_1 \sim g_3$ in Eq. (13) is the same as that of the linear grating couplers. The real coefficient $\eta^{(2)}$ is the same as that of the nonlinear slab waveguide. Therefore, the attenuation coefficient α_g, which is the real part of A_g, is constant. However, β_g is a function of the power carried by the guided wave $|A_g|^2$ and changes as propagating along the z axis. Then the angle for the maximum radiation is deviated from that of the linear case. The nonlinearity effects on the phase modulation.

In this paper, the liquid crystal MBBA is chosen as the nonlinear medium. The parameters are $\sqrt{\varepsilon_1} = 1.55$, $\sqrt{\varepsilon_2} = 1.52$, $\alpha = 0.6377 \times 10^{-11}$ [$\mathrm{m}^2/\mathrm{V}^2$], $\lambda = 0.5145\mu\mathrm{m}$, $d = 1\mu\mathrm{m}$, $\Lambda/\lambda = 0.6$, and $h/d = 0.1$.

At first, the output grating, which is not involved the incident wave in Eq. (13), is considered. Figure 2 shows the angle θ_{rad} for the maximum radiation as a function of the propagating length kz. The kinds of the lines mean the incident power and are explained in the figure. It is found that θ_{rad} becomes larger as the guided wave propagates. So, the radiated wave is focused at some plane over the grating.

Next, we consider the input grating with the length $L = 2$mm. In this paper, the incident wave is considered in the form of a uniform plane wave. Figure 3 indicates the input efficiency, which is defined as the ration of the power of the guided wave at $z = L$ to the total incident power $P_{in} = |a_1|^2 L$ on the grating, as a function of the deviation $\Delta\theta$ from the incident angle of the linear grating which equals to the radiation angle. The kinds of the lines mean the incident power and are explained in the figure. It is found that the input efficiency decreases as the input power increases and the angle for the maximum input efficiency deviates from that of the linear grating. The input efficiency is maximum when the incident angle equals to the radiation one. As explained in Fig. 2, the radiation angle depends on the power of the guided wave which is excited by the incident wave. So, the input efficiency depends on the incident power. Figure 4 shows the power of the guided wave at $z = L$ as a function of the incident power P_{in} for $\Delta\theta = 0.006°$. This figure shows that the nonlinear input grating might be useful for the bandpass power filter [9] or optical switch since the guided wave can be transmitted a large signal for input powers within a certain range, and little of the signal for the input power above or below the range.

5. CONCLUSIONS

We have analyzed the nonlinear grating couplers by using the singular perturbation technique. The properties of the output and input grating couplers are examined numerically. The results in this paper are the fundamental properties for the design of the integrated optical circuits. From now, the properties of the nonlinear grating couplers have been investigated from the practical point of view. This work was partially supported by a Scientific Research Grant-in-Aid from the Ministry of Education, Science, and Culture, Japan.

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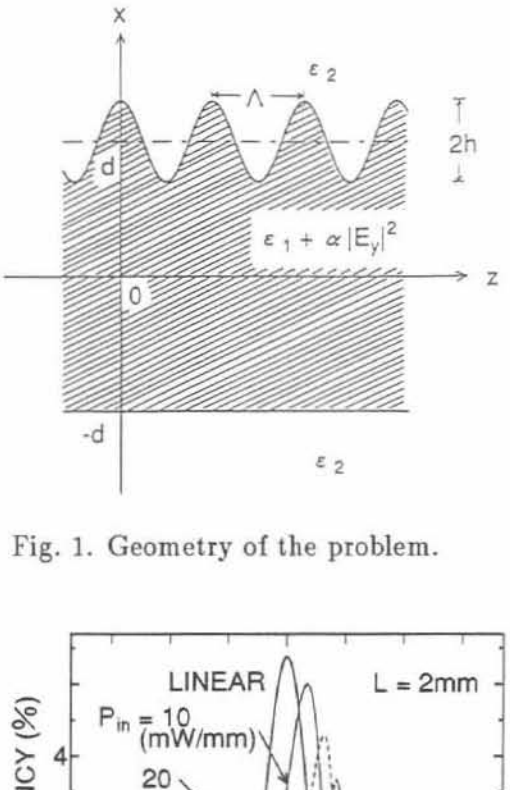

Fig. 1. Geometry of the problem.

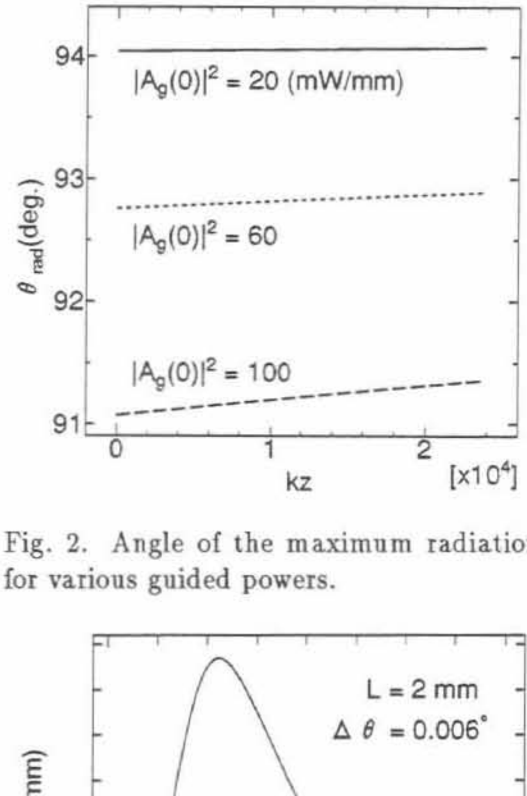

Fig. 2. Angle of the maximum radiation for various guided powers.

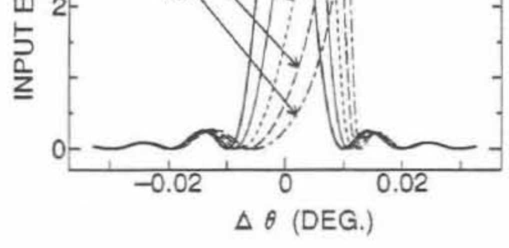

Fig. 3. Input efficiency for various input powers.

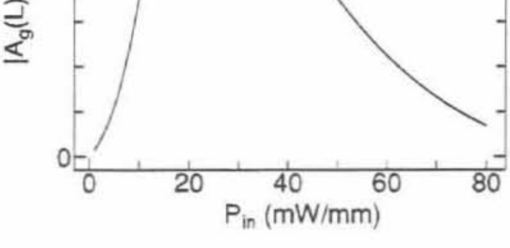

Fig. 4. Guided power at the end of the grating against the input power.