A PROPAGATOR MODEL FOR WAVE PROPAGATION IN DISCRETE RANDOM MEDIA

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ABSTRACT

A propagator model is presented for studying both coherent and incoherent intensitites of the electromagnetic field in a discrete random medium. Lax's quasi-crystalline approximation (QCA) with suitable averaging techniques[1,2] and the T-matrix of a single scatterer has been employed in the analysis[3,4]. Pair-correlation functions generated by Monte-Carlo simulation have been used in the computation. This model also provides a dispersion equation which is solved for both phase velocity and coherent attenuation as a function of frequency for various values of concentrations of scatterers. Numerical results obtained show excellent agreement with experimental measurements of Killey and Meeten.

Propagator Model

The random medium occupies a volume V such that $N\to\infty$, $V\to\infty$ but $\frac{N}{V} = n_o$, the number density of scatterers is finite. The volume fraction is denoted by the concentration c. In order to obtain the coherent field, a configurational average is performed over the random positions. The QCA is invoked to restrict the type of positional correlations that we are willing to consider. It is known that the QCA restricts the types of multiple scattering processes considered to include only successive scattering among distinct scatterers and requires only a knowledge of the joint probability function $p(\vec{r}_j|\vec{r}_i)$. In diagrammatic form, the coherent field may be written as

$\langle E\rangle_{QCA}$ =  + ··· (1)

where  denotes positional correlation between two scatterers and it is clear from the diagrams that each scatterer participates only once in a given term, there is no back and forth scattering, all scattering is sequential and only sequential positional correlations are allowed.

Introducing spatial Fourier transforms of the propagation matrix σ and the radial distribution functions g (given by $p(\vec{r}_j|\vec{r}_i) = \frac{1}{V} g(\vec{r}_{ij})$) which are denoted by $\bar{\sigma}(K)$ and $\bar{g}(K)$, respectively, and using the convolution theorem, we obtain

$$\langle E(\vec{r})\rangle = \vec{E}^0(\vec{r}) + \int Ou\, \vec{\psi}_n(\vec{r}-\vec{r}_1)\; T_{nn'}\; n_o \int [1-n_o\, \overline{\sigma g}\,(k)\; T]^{-1}$$

$$e^{i\vec{K}\cdot(\vec{r}_1-\vec{r}_2)}\; a^{(2)}_{n''}\; d\vec{K}\; d\vec{r}_1\; d\vec{r}_2 \qquad (2)$$

In (2), the propagation matrix σ is the translation matrix for the vector spherical functions and includes all orders of multipoles. Ou $\vec{\psi}_n$ represents the outgoing vector spherical function and the abbreviated index $n \to \tau, \ell, m\sigma$, $\tau=1,2, \ell\epsilon[0,\infty]$, $m\epsilon[0,\ell]$ and σ= even or odd. The T-matrix again includes all orders of multipoles for scatterers of arbitrary shape. $\vec{E}^0$ is the incident plane harmonic wave and $a^2_{n''}$ are the known expansion coefficients of $\vec{E}^0$ at $\vec{r}_2$.

This new form of the average field can be interpreted as an incident plane wave propagating through an effective medium of propagation constant K and propagator $[1-n_o\, \overline{\sigma g}(K)T]^{-1}$ undergoing scattering from a particle at r_1 and then propagating to the observation point r with the wave number of the host medium. The dispersion equation in the model medium can be obtained by setting the determinant of the propagator equal to zero:

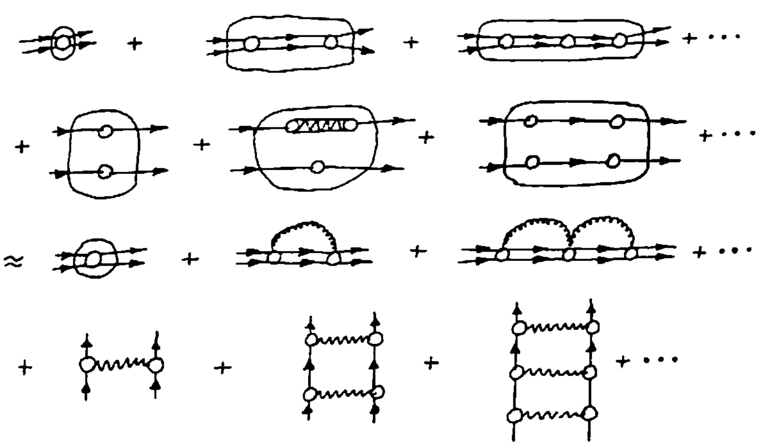
$$|\; 1-n_o\, \overline{\sigma g}(k)\; T\; | = 0 \qquad (3)$$

The field fluctuations $\Delta\vec{E}$ may now be given by

$$\Delta\vec{E} = \vec{E} - \langle\vec{E}\rangle \qquad (4)$$

Along these lines, we define the incoherent intensity or the spectral density $G_\alpha(\vec{R},\omega)$ at position $\vec{R}$ and frequency ω for field polarization in the direction α.

$$G_\alpha(\vec{R},\omega) = \frac{1}{4} \langle|\hat{\alpha}\cdot\Delta\vec{E}|^2\rangle$$



(5)

(6)

where — denotes propagation of the field from one point to the other and 0 0 denotes a scatterer. If two or more scatterers are enclosed in an area such as  arbitrary multiple scattering any number of times and in any order can go on between scatterers 1, 2 and 3.

The first set of the above diagrams represents a partial summation of QCA type terms incorporating two body correlations while the second set represents the conventional ladder diagrams. In both sets of diagrams, we can use so called "dressed propagators" obtained from (3) between scatterers instead of "bare propagators". This means that K from (3) can be used as the wave number characterizing the medium between scatterers involved in calculation of the spectral density, i.e., the other scatterers that participate in only one or other of the field lines are averaged over separately and replaced by K.

NUMERICAL RESULTS

The numerical procedure is described in detail in Refs. 3 and 4, and will not be repeated here.

In Figs. 1 and 2, the real and imaginary parts of the coherent field are compared with the experimental measurements of Killey and Meeten[5]. In Fig. 3, Calculations of the coherent intensity for a suspension of Revacryl spheres in distilled water show excellent comparison with measurements of Killey and Meeten[5].

ACKNOWLEDGEMENTS

This work was supported by the U.S. Army Research Office under contract DAAG29-83-K-0097.

REFERENCES

1. V. Twersky, Coherent electromagnetic waves in pair-correlated random distributions of aligned scatterers, J. Math. Phys., 19, 215-230, 1978.

2. V. Twerksy, Multiple scattering of waves by periodic and by random distributions, in Electromagnetic Scattering, edited by P.L.E. Uslenghi, Academic Press, New York, 221, 251, 1978.

3. V.K. Varadan and V.V. Varadan (Eds.) Acoustic, Electromagnetic and Elastic Wave Scattering-Focus on the T-matrix Approach, Pergamon Press, New York, 1980.

4. V.K. Varadan, V.N. Bringi, V.V. Varadan and A. Ishimaru, Multiple Scattering theory for waves in discrete random media and comparison with experiments, Radio Sci., 18, 321-327, 1983.

5. A. Killey and G.H. Meeten, Optical extinction and refraction of concentrated latex dispersions, J. Chem. Soc., Faraday Trans. 2, 587-599, 1981.

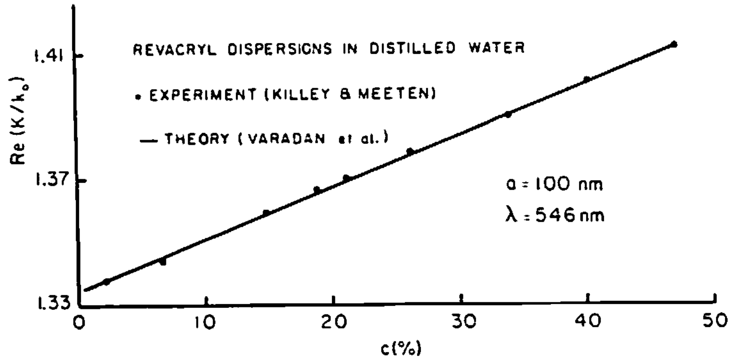


Fig. 1 Phase velocity vs concentration c for Revacryl dispersions in distilled water at λ = 546 nm.

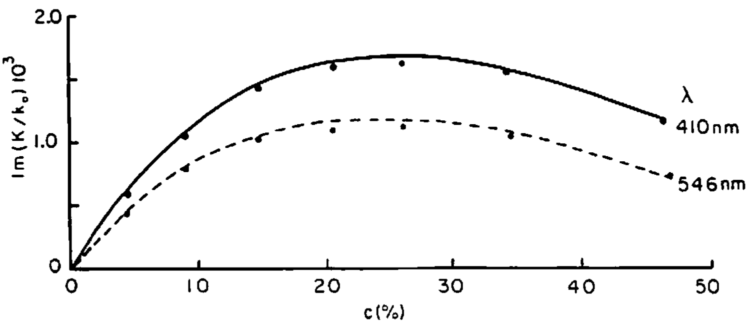


Fig. 2 Coherent attenuation vs concentration c for Revacryl dispersions in distilled water at λ = 410 nm and 546 nm.

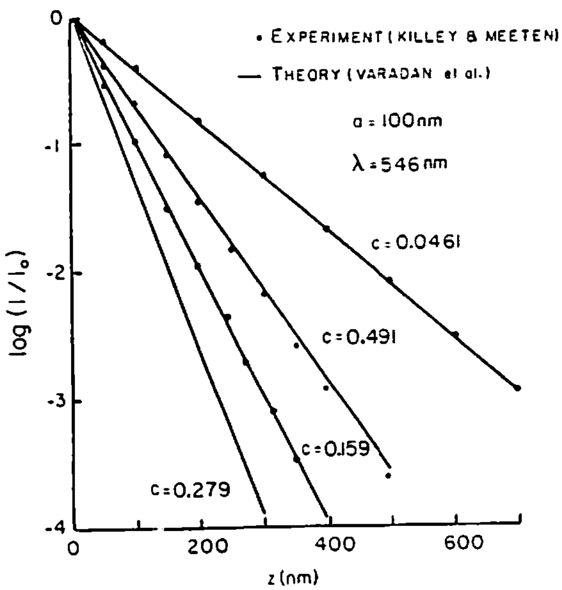


Fig. 3 Coherent intensity as a function of propagation depth z for various values of c at λ = 546 nm.