Numerical Analysis of Radar Cross-Section of Partially Convex Targets with Large Sizes in Random Media

Hosam El-Ocla

Dept. of Comp. Science, Lakehead University, Ontario, Canada

Dept. of Microwaves, Electronics Research Institute, Giza, Egypt

Mitsuo Tateiba

Dept. of Comp. Science and Commun. Eng., Kyushu University, Fukuoka, Japan

## 1 Introduction

Many researchers have investigated problems of wave propagation and scattering in random media for a long time. Scattered waves propagating in continuous random media are calculated efficiently, by a method that uses a current generator to clarify the medium effects on radar detection [1]. One of the random medium effects is the enhancement in radar cross-section (RCS) of targets and can be explained by the coherent addition of doubly scattered waves, which add in phase only in the backward direction [2]. In earlier study [3], numerical results for radar cross-section (RCS) of conducting convex bodies such as elliptic cylinders have elucidated that the spatial coherence length (SCL) of incident waves around target is one of the key parameters for the clarification of random media effects on the RCS. Later, we considered the targets as conducting cylinders with partially convex cross-sections and limited sizes less than one wavelength of waves in free space [4, 5]. The RCS is calculated on the assumption that twice the backscattering enhancement occurs for point target. It was pointed out that there are some anomalies in the backscattering enhancement of RCS of the targets in random media. These anomalies occur when H-wave incident on convex portion of the target. However, these anomalies are absent in case of E-wave incidence. Recently, we extended our study and considered large size targets in different random media [6]-[8]. Most recently, a new approach was proposed to estimate the RCS of targets in random media [9]. In these studies it was found that the target configuration parameters together with the SCL of the incident wave around the target play a leading role in determination of the RCS of partially convex targets. In this work, we analyze the RCS behavior of concave-convex targets with different linear polarizations. We assume the linear polarization including E and H-wave incidences with different values of SCLs and target complexity. In the previous work, it has clarified the obvious difference in the behavior of the RCS between both of concave and convex illumination portions of concave-convex targets. Each of these regions deserves a separate study; we, here, concentrate on the wave backscattering from convex illumination portion only. The time factor  $\exp(-iwt)$  is assumed and suppressed in the following sections.

#### 2 Formulation

Geometry of the problem is shown in Figure 1. A random medium is assumed as a sphere of radius L around a target of the mean size  $a \ll L$ , and also to be described by the dielectric constant  $\varepsilon(\mathbf{r})$ , the magnetic permeability  $\mu$ , and the electric conductivity  $\sigma$ . For simplicity  $\varepsilon(\mathbf{r})$  is expressed as

$$\varepsilon(\mathbf{r}) = \varepsilon_0 [1 + \delta \varepsilon(\mathbf{r})] \tag{1}$$

where  $\varepsilon_0$  is assumed to be constant and equal to free space permitivity and  $\delta \varepsilon(\mathbf{r})$  is a random function with



Figure 1: Geometry of the problem of wave scattering from a conducting cylinder in random media.

$$\langle \delta \varepsilon(\mathbf{r}) \rangle = 0, \qquad \langle \delta \varepsilon(\mathbf{r}) \, \delta \varepsilon(\mathbf{r}') \rangle = B(\mathbf{r}, \mathbf{r}'), \quad \text{and}$$
(2)

$$B(\mathbf{r}, \mathbf{r}) \ll 1, \qquad kl(\mathbf{r}) \gg 1$$
 (3)

Here, the angular brackets denote the ensemble average and  $B(\mathbf{r}, \mathbf{r})$  is the local intensity of random medium. Also,  $\mu$  and  $\sigma$  are assumed to be constant;  $\mu = \mu_0$ ,  $\sigma = 0$ .

For practical turbulence, the condition (3) may often be satisfied and therefore we can assume the forward scattering approximation and the scalar approximation. Consider the case where a directly incident wave is produced by a line source distributed uniformly along the y axis. Then the incident wave is cylindrical and becomes plane approximately around the target because the line source is very far from the target. Here, let us designate the incident wave by  $u_{in}(\mathbf{r})$ , the scattered wave by  $u_s(\mathbf{r})$ , and the total wave by  $u(\mathbf{r}) = u_{in}(\mathbf{r}) + u_s(\mathbf{r})$ .

The target is assumed as a conducting cylinder. The cross-section of the cylinder is expressed by

$$r = a[1 - \delta \cos 3(\theta - \phi)] \tag{4}$$

where  $\delta$  is the concavity index and  $\phi$  is the rotation index. We can deal with this scattering problem two-dimensionally under the condition (3); therefore, we represent  $\mathbf{r}$  as  $\mathbf{r} = (x, z)$ . According as polarization of incident waves:  $E_y$  or  $H_y$ , where  $E_y$ ,  $H_y$  are the y components of electric and magnetic fields, respectively, we can impose two types of boundary condition on wave fields on the cylinder surface S. That is, the Dirichlet condition (DC) for E-wave incidence and the Neumann condition (NC) for H-wave incidence, respectively, are assumed as

$$u(\mathbf{r}) = \mathbf{0}, \qquad \text{for DC} \tag{5}$$

$$\frac{\partial}{\partial n}u(\mathbf{r}) = \mathbf{0}, \qquad \text{for NC}$$
(6)

where  $\partial/\partial n$  denotes the outward normal derivative at **r** on *S*.

Accordingly, using the current generators  $Y_{\rm E}$ ,  $Y_{\rm H}$  and Green's function in the random medium  $G(\mathbf{r} \mid \mathbf{r}')$ , we can express the scattered wave as

$$u_{s}(\mathbf{r}) = \int_{S} d\mathbf{r}_{1} \int_{S} d\mathbf{r}_{2} \left[ G(\mathbf{r} \mid \mathbf{r}_{2}) Y_{E}(\mathbf{r}_{2} \mid \mathbf{r}_{1}) G(\mathbf{r}_{1} \mid \mathbf{r}_{t}) \right]$$
  
for E-wave incidence  
$$= -\int_{S} d\mathbf{r}_{1} \int_{S} d\mathbf{r}_{2} \left[ \left( \frac{\partial}{\partial n_{2}} G(\mathbf{r} \mid \mathbf{r}_{2}) \right) Y_{H}(\mathbf{r}_{2} \mid \mathbf{r}_{1}) G(\mathbf{r}_{1} \mid \mathbf{r}_{t}) \right]$$
  
for H-wave incidence (7)

Here  $Y_{\rm E}$  and  $Y_{\rm H}$  are the operator that transforms incident waves into surface currents on S and depends only on the scattering body. The current generator is expressed in terms of wavefunctions which satisfy Helmholtz equation and the radiation condition, and is well defined in [3]-[9]. We can obtain the RCS by calculating  $\langle |u_{\rm s}|^2 \rangle$  from (7).

$$\sigma = \langle |u_s(\mathbf{r})|^2 \rangle \cdot k(4\pi z)^2 \tag{8}$$

#### 3 Numerical Results

We conduct and analyze numerically the RCS  $\sigma$  by changing the target in shape and size, angle of incidence, and the wave incidence polarization including E and H-wave incidences. We restrict the shape and size to  $\delta = 0, 0.1, 0.2$  and  $0.1 \leq ka \leq 21$ , respectively, where k is the wave number in free space, (see figures 2, 3, and 4).

From the numerical results, we can notice that there are two factors that affect obviously the behavior of RCS: the first is the effect of SCL; as the SCL increases, the behavior of RCS with target size ka in random media becomes closer to its behavior in free space except for the magnitude of RCS. The second is the effect of target curvature and can be seen clearly with changing the target complexity  $\delta$ . As  $\delta$  increases and/or SCL decreases, as the RCS decreases due to the decrease of the effective illumination region, and vice versa. In the case of H-wave incidence, RCS suffers from oscillated behavior in free space. This oscillated behavior that is absent in case of E-wave incidence is due, as a matter of fact, to the effect of creeping waves.

At the point of tangency, each wave creeps around the surface at a velocity less than that in free space and that is attenuated by tangential radiation. For low ka values, wave can continue to creep around the target many times and, therefore, the interference between the specularly reflected and creeping waves is obvious enough to affect the RCS resulting in that oscillated behavior. However, with larger ka, the creeping waves travel along the cylinder and they become weaker and weaker the farther they have to travel due to radiation. Therefore the creeping waves attenuation reduces its effectiveness rapidly resulting in diminishing the interference effect gradually with ka. For random medium case, RCS has some oscillations that are attributed to the effect of creeping waves propagating in random medium that in turn has another oscillating effect on the RCS with both polarizations as was shown in [6, 7].

## 4 Conclusion

In this work we have analyzed numerically the parameters that have a clear effect on the behavior of RCS of conducting targets in continuous random media. We assume partially convex targets of large sizes of about three wavelengths with putting into consideration the polarization of incident waves. Also, we postulate that the incident waves around the target are incoherent enough and the SCL keeps a finite value. Here, we considered the case where the incident wave illuminates the convex portion of a concave-convex target.

Both of the target configuration and the SCL are the two factors that affect the RCS irrespective of the incident wave polarization. The effect of the creeping waves, produced in case of H-wave incidence, on the RCS was discussed. We have explained that the creeping waves effect diminish gradually with ka.

As a result, this analysis demonstrates clearly the effects of target configuration parameters together with the effects of SCL and double passage of waves in random media on the RCS.

# References

- M. Tateiba, and E. Tomita, IEICE Trans. Electron., vol.E75-C, no.1, pp.101–106, Jan. 1992.
- [2] E. Jakeman, J. Opt. Soc. Am., Vol. 5, No. 10, pp. 1638–1648, 1988.
- [3] Z. Q. Meng, and M. Tateiba, Waves in Random Media, vol.6, pp.335–345, 1996.
- [4] H. El-Ocla and M. Tateiba, Waves in Random Media, Vol. 11, No. 1, pp. 21-32, 2001.
- [5] H. El-Ocla and M. Tateiba, IEICE Trans. Commun., Vol. E84-B, No. 9, pp. 2583-88, 2001.
- [6] H. El-Ocla and M. Tateiba, Waves in Random Media, Vol. 12, No. 3, pp. 387–397, 2002.
- [7] H. El-Ocla and M. Tateiba, Waves in Random Media, Vol. 13, No. 2, pp. 125–136, 2003.
- [8] H. El-Ocla and M. Tateiba, Progress in Electromagnetics Research PIER 42, pp. 107–130, 2003.
- H. El-Ocla and M. Tateiba, IEEE Antennas and Wireless Propagation Letters, Vol. 2, pp. 173–176, 2003.

#### Acknowledgment

This work was supported in part by the National Science and Engineering Research Council of Canada under Grant 250299-02.



Figure 2: RCS vs. target size in free space and at two different SCLs and  $\delta = 0$  where (a) E-wave incidence, (b) H-wave incidence.



Figure 3: As figure 2, but for  $\delta = 0.1$ .



Figure 4: As figure 2, but for  $\delta = 0.2$ .