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DEFORMATION OF THE SPATIAL SPECTRUM OF SCATTERED ELECTROMAGNETIC WAVES BY COLLISION MAGNETIZED TYRBULENT PLASMA

Akira ISHIMARU
Washington University, Department of Electrical Engineering
Seattle, WA 98195, USA
E–mail: ishimaru@ee.washington.edu
George JANDIERI, George ABURJANIA, Madlena DAVITADZE
Georgian Technical University, Department of Physics
Kostava str., 77, 380075 Tbilisi, Georgia
E–mail: **jandieri@access.sanet.ge**
Vakhtang JANDIERI
Kyushu University, Department of Computer Science and Communication Engineering
Fukuoka 812-8581, Japan
jandieri_vacho@hotmail.com

1. Introduction

It is known that statistical characteristics of scattered radiation in randomly inhomogeneous media depend on both scattered features and absorption of medium [1-3]. The influence of absorption is ambiguous: in "classical" case absorption "eat" the spatial harmonics scattered under the big angles and by that weakens its fluctuations, in other case it leads to faster growth of power of scattered radiation and anomalous (in comparison with the similar transparent medium) increase of fluctuations. It was shown that asymmetric feature of the task is necessary condition of anomalous growth of fluctuations in medium with large–scale inhomogeneities. One reason is oblique illumination of boundary of the layer and the scattered waves become inhomogeneous. Absorption in other case can lead to anomalous increase of fluctuations – non-isotropic component of a complex refractive index leads to asymmetric dumping of scattered waves. With the practical point of view the most interesting case is inclined incidence of small amplitude plane electromagnetic wave on semi infinite turbulent collisional plasma under the influence of external magnetic field. In small angle approximation, using geometrical optic method, the deformation of the angular power spectrum (APS) of multiply scattered radiation has been investigated in this paper.

2. Formulation of the task

Let us assume that a plane semi–infinite layer of turbulent collisional magnetized plasma be irradiated by a monochromatic electromagnetic wave with frequency ω from vacuum. Plane XZ coincides with the plane formed by a vector of a static external magnetic field $\mathbf{B}_0$ and the wavevector of refracted wave $\mathbf{k}$, θ_0 is an angle between the imposed magnetic field and Z axis. The electron concentration in plasma layer is $\mathrm{N}(\mathbf{r}) = \mathrm{N}_0 + \mathrm{N}_1(\mathbf{r})$ ($\mathrm{N}_1 << \mathrm{N}_0$), where N_0 is background electron density, $\mathrm{N}_1(\mathbf{r})$ represents the random concentration of irregularities in the plasma and is a random function of position. The eikonal equation $\tilde{\mathbf{k}}^2 = (\omega^2 / c^2)\tilde{n}^2$ in the ray–(optics) approximation [4,5] allows to calculate phase fluctuation of the wave outside the plasma layer with smooth inhomogeneities, where $\tilde{\mathbf{k}}(\mathbf{r}) = -\nabla\tilde{\varphi}$ is the complex wavevector, $\tilde{\varphi}$ is the complex phase of a wave, and $\tilde{n}^2 = \tilde{n}^2(\mathrm{N}(\mathbf{r}), \omega, \tilde{\mathbf{k}})$ is the complex refractive index. Scattered refracted wave is extraordinary. Statistical characteristics of scattered field are determined by fluctuations of a complex phase of the extraordinary plane wave [1-3]. Expanding the wave phase characteristics in series: $\tilde{\mathbf{k}} = \tilde{\mathbf{k}}_0 + \tilde{\mathbf{k}}_1(\mathbf{r}) + \ldots$, $\tilde{\varphi} = \tilde{\varphi}_0 + \tilde{\varphi}_1 + \ldots$ and solving differential equation, taking into account the boundary conditions, fluctuation component of the phase may be written as:

$$\tilde{\varphi}_1 = \tilde{\alpha} \int_{-\infty}^{+\infty} d\kappa_x \int_{-\infty}^{+\infty} d\kappa_y \, \exp(i\kappa_x x + i\kappa_y y) \int_0^z d\xi \, \tilde{N}_1(\kappa_x, \kappa_y, \xi) \exp\left[-i\kappa_x \frac{\partial \tilde{k}_z}{\partial k_x} (z - \xi) \right], \qquad (1)$$

where $\tilde{\alpha} = -\omega \left(\frac{\partial(\tilde{n}\omega)}{\partial \omega} \frac{\partial \omega}{\partial \tilde{k}_z} \right)^{-1} \frac{\partial \tilde{n}}{\partial N_0}$, $\partial \tilde{k}_z / \partial k_x = \beta + i\gamma$ (hereinafter index "0" at component of wavevector $\tilde{\mathbf{k}}_0$ is omitted for brevity). The transverse correlation function of the phase

$$R_{\tilde{\varphi}}(\rho_x, \rho_y, z) = 2\pi\tilde{\alpha}^2 \int_{-\infty}^{\infty} d\kappa_x \int_{-\infty}^{\infty} d\kappa_y \frac{\exp(i\kappa_x \rho_x + i\kappa_y \rho_y + 2\kappa_x \gamma z)}{2\kappa_x \gamma} \left[1 - \exp(2\kappa_x \gamma z) \right] \Phi(\kappa_x, \kappa_y, -\beta\kappa_x)$$

is expressed through a three-dimensional spatial power spectrum $\Phi(\kappa_x, \kappa_y, \kappa_z)$ of statistically homogeneous electron concentration fluctuations. In the most interesting case of strong fluctuations of the phase $< \tilde{\varphi}_1 \tilde{\varphi}_1^* > >> 1$ APS (Fourier transformation of the correlation function of scattered field) has a Gaussian form

$$S(\kappa_x, \kappa_y, z) = S_0 \exp\left[-\frac{(\kappa_x - k_x - \Delta\kappa_x)^2}{2 < \kappa_x^2 >} - \frac{\kappa_y^2}{2 < \kappa_y^2 >} \right], \qquad (2)$$

where S_0 is maximum of the spectrum, $\Delta\kappa_x$ determines the displacement of the APS maximum of the received radiation caused by random inhomogeneities, while $< \kappa_x^2 >$ and $< \kappa_y^2 >$ determine the widths of this spectrum in XZ and YZ planes, respectively. The spectrum Φ satisfies the normalization condition $4\pi \int_0^{\infty} d\kappa \, \kappa^2 \Phi(\kappa_x, \kappa_y, 0) = < N_1^2 >$. The APS is similar to ray intensity (brightness) in transform equation. The solutions of radiation transfer equation for the APS in small angle approximation and complex geometrical optics approximation are the same. Analyses show that anomalous broadening of the APS and displacement of its center of gravity are caused by both medium anisotropy and oblique incident of a wave on the boundary of a layer and is represented by the expression $\partial \tilde{k}_z / \partial k_x \neq 0$.

Components of wavevector $\tilde{\mathbf{k}} = (\omega / c)\{p, \; 0, \; \tilde{q}\}$ of an inclined incident plane wave on the boundary of the magnetoplasma satisfy the equation [4]:

$$a_4 \tilde{q}^4 + a_3 \tilde{q}^3 + a_2 \tilde{q}^2 + a_1 \tilde{q} + a_0 = 0, \qquad (3)$$

where

$$a_0 = \left[(1-p^2)(1-is) - v \right] \left\{ \left[(1-p^2)(1-is) - v \right] (1-v-is) - (1-p^2)u \right\} - (1-p^2) p^2 u_x v$$

$$a_1 = -2(1-p^2) p v \sqrt{u_x u_z}, \quad a_2 = -2(1-is) \left\{ \left[(1-p^2)(1-is) - v \right] (1-v-is) - (1-p^2)u \right\} +$$

$$+ v \left[p^2 u_x - (1-p^2) u_z - u \right], \quad a_3 = 2 p v \sqrt{u_x u_z}, \quad a_4 = (1-is)[(1-is)^2 - u] - v[(1-is)^2 - u_z],$$

$u = \omega_B^2 / \omega^2$, $v = \omega_p^2 / \omega^2$ and $s = v_{eff} / \omega$ are non–dimensional parameters, $u_x = u \sin^2 \theta_0$, $u_z = u \cos^2 \theta_0$, ω_p and ω_B are the plasma frequency and the angular gyrofrequency of electrons, respectively, v_{eff} is the effective collision frequency of electrons with other plasma particles. The problem is to determine the angle of refraction and the angle of inclination of an imposed magnetic field for concrete plasma parameters, at which γ will be equal zero. In this case oblique refraction of a wave and anisotropy of medium completely compensate each other. The effect of compensation has been predicted in [2,3]. If compensation is absent, the increase of parameter γ leads to anomalous broadening of the APS and the effect of influence of absorption on the APS will be more brightly.

3. Numerical calculations

Numerical calculations have been carried out for F-layer of the ionosphere, $\omega_H \approx 8.8 \cdot 10^6\ s^{-1}$, $\omega_p \approx 10^7\ s^{-1}$, $\nu \approx 10^5\ s^{-1}$, $\lambda = 100\,m$, $u = 0{,}22$, $v = 0{,}28$, $s = 0{,}053$ using Monte–Carlo method. "Classical" case of Monte–Carlo method takes into account the absorption as "probability of survival" Λ, which in quantum case corresponds the probability of photon absorption at scattering on medium inhomogeneities. For isotropic case $\Lambda = \sigma_s/(\sigma_s + \sigma_a)$, where σ_s and σ_a are the extinction coefficients due to scattering and absorption, respectively. In this paper we shall use "weight" modification of the Monte–Carlo method. Differential cross section for cold turbulent magnetized plasma of statistically isotropic fluctuations is given by the formula [6]:

$$\frac{d\sigma_s}{d\Omega} = \frac{1}{2\pi} \cdot \frac{e^4}{m_e^2 c^2} \cdot \frac{\omega^4}{\omega_p^4} \cdot \Phi(\Delta k) \cdot \frac{\tilde{n}_s^{\,3} \left| \tilde{\vec{e}}_s^{\,*} (\hat{\varepsilon} - 1) \tilde{\vec{e}}_i \right|^2}{\tilde{n}_i \left(\left|\tilde{\vec{e}}_i\right|^2 - \left|\tilde{\vec{k}} \tilde{\vec{e}}_i\right|^2 / k^2 \right) \left(\tilde{\vec{e}}_s^{\,*} \hat{\varepsilon} \tilde{\vec{e}}_i \right)}, \tag{4}$$

where $\hat{\varepsilon}$ is dielectric tensor of of plasma, $\tilde{\vec{\mathbf{k}}}_s$, $\tilde{\vec{e}}_s$ and $\tilde{n}_s$ are the wavevector, polarization vector of electric field and refractive index of scattered wave; $\tilde{\vec{\mathbf{k}}}_i$, $\tilde{\vec{\mathbf{e}}}_i$ and $\tilde{n}_i$ correspond to the incident wave, Δk is the module of $\tilde{\vec{\mathbf{k}}}_s - \tilde{\vec{\mathbf{k}}}_i$. The last term in (4) describes the dependence of direction of wave propagation and the features of scattered plasma. Anisotropy is taken into account at a stage of generation of propagation distance of radiation between two consecutive acts of scattering on inhomogeneities. Extinction coefficient due to absorption can be found by formula $\sigma_a = 2\omega c^{-1}\,\mathrm{Im}\ \tilde{n}(\theta)$ in collisional magnetized plasma. We shall use the imaginary component of the refractive index [4]:

$$\tilde{n}^2 = 1 - \frac{2v(1 - v - is)}{2(1 - is)(1 - v - is) - u \cdot \sin^2\theta \pm \sqrt{u^2 \sin^4\theta + 4u(1 - v - is)^2 \cos^2\theta}}. \tag{5}$$

For simplicity of simulation of scattering processes at given plasma parameters u, v and s we have assumed that the extinction coefficient due to scattering σ_s' along the direction of static external magnetic field is equal one. Such choice in fact corresponds to measurement of all average free path length L_f of radiation between two acts of scattering along the direction of static magnetic field. L_f is a reverse value of σ_s. Knowledge of σ_s and σ_a allows to determine probability of a survival Λ for magnetized collisional turbulent plasma. The distortions of the APS of scattered radiation have been checking by simulation of radiation propagation in a layer having thickness $\sigma_s|_{\theta=0} \cdot H = 50$ and at L_f/λ equals 100. Chosen plasma parameters u, v and s correspond to ionospheric plasma layer having thickness 500 km at $\sigma_s \approx 0.0001\,m^{-1}$ and $\Lambda \approx 0.75$ along the direction of imposed static magnetic field. Spatial spectrum of electron concentration fluctuation $\Phi(\Delta k)$ is given by the expression

$$\Phi(\Delta k) = C \begin{cases} \left(\dfrac{\sqrt{2}}{90} k_i \right)^{-g}, & \Delta k \in \left[0, \dfrac{\sqrt{2}}{90} k_i \right] \\ \Delta k^{-g}, & \Delta k \in \left[\dfrac{\sqrt{2}}{90} k_i, \sqrt{2} k_i \right] \\ 0, & \Delta k > \sqrt{2} k_i \end{cases}, \tag{6}$$

where *g* is the index of spectrum, k_i is the module of an incident wavevector, C is the normalized coefficient. The results of simulation are illustrated in Figure 1 for normal wave and at $g = 2.6$. APS of scattered radiation has "double–hump" form due to joint action of anisotropy, absorption and single scattered indicatrix. "Double–hump" form of the angular power spectrum is a reason of waves scattered at small angle close to compensation direction and these waves are weekly attenuate with increasing of penetration depth into the layer.

In opinion of authors, the effect of compensation can be observed in quiet ionosphere. The displacement of the centre of gravity of the APS of scattered radiation in the ionosphere due to influence of absorption is observable value too. The centre of gravity of the spatial spectrum of scattered radiation and the line-of-sight between the removed cosmic source and observer in radiofrequency band can be coinciding if condition of compensation is satisfied.

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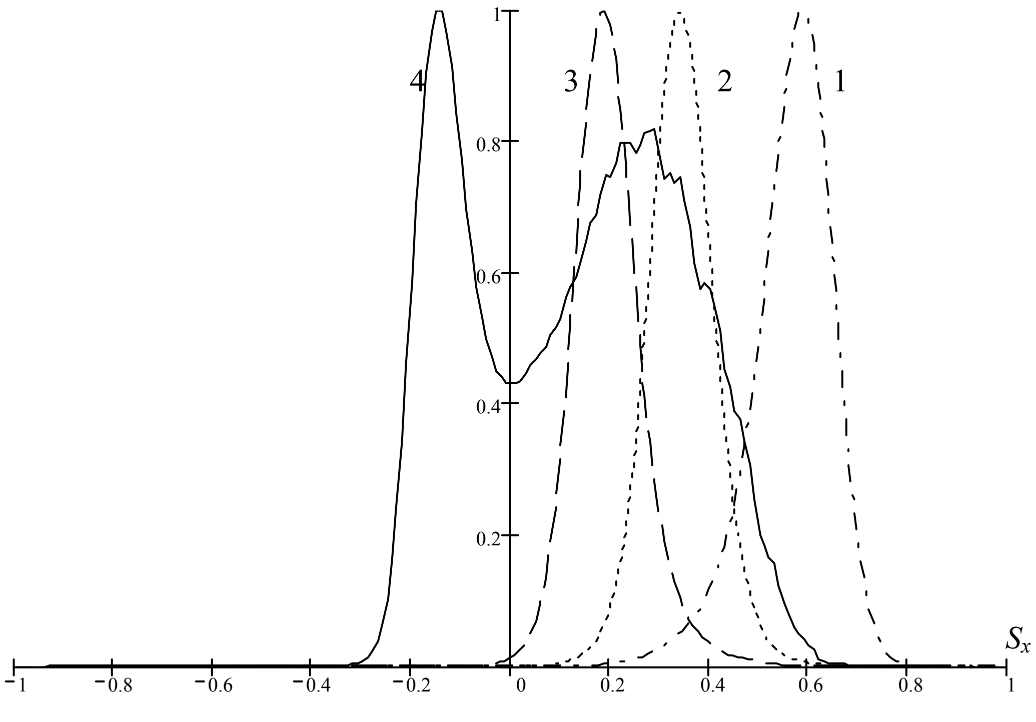

Figure 1. Angular power spectrum of multiple scattered radiation in plasma layer: $u = 0.22$; $\nu = 0.28$; $s = 0.0053$ at a depth of $\sigma_s|_{\theta=0} \cdot H = 50$. The angle of inclination of a static impose magnetic field is $\theta_0 = 40^0$, L_f/λ = 100; $S_x = k_x / k$. All spectra are normalized on their maximal values. The curves 1, 2, 3, 4 are constructed for the refraction angle 40 and 23.5 (the direction of compensation for normal wave), 10 and –10 degrees, respectively.