# Computer Aided Design of Dielectric Waveguide Bends : Guided-Mode Extracted Integral Equations

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**INTRODUCTION** New integral equations which can be called guided-mode extracted integral equations are applied to the computer-aided design (CAD) of dielectric slab waveguide bends. This method is exact, have wide applicability, does not employ mode-expansion techniques and can treat problems by the conventional moment method. So, it is suitable for the basic theory of CAD software of arbitrary-shaped dielectric waveguide bends. In order to show the validity of this method, numerical results of transmission characteristics in various dielectric waveguide bend-shapes are illustrated.

**THEORY** The geometry of the problem is shown in Fig.1. The dielectric slab waveguide 1 of width $d_1$ and waveguide 2 of width $d_2$ which satisfy the single-mode condition and whose indices of refraction are given by $n_2$, are joined together to form a given bend angle through the arbitrary-shaped bend-section as shown in Fig. 1. The indices of refraction in the upper surrounding space and in the lower surrounding space are given by $n_1$ and $n_3$, respectively. We denote the boundaries between the surrounding space and waveguides 1,2+(bend-section) by $C_j$ $(j=1\text{-}6)$ as shown in Fig. 1. A dominant TE(or TM)-mode is assumed to be incident from waveguide 2 to the arbitrary-shaped bend. The incident wave is reflected, transmitted and scattered by the arbitrary-shaped bend. The z-component of incident wave and the reflected wave in the waveguide 2 are denoted by $E^{-(2)}(\mathbf{x})$ and $RE^{+(2)}(\mathbf{x})$, respectively, and that of the transmitted wave in the waveguide 1 is denoted by $TE^{-(1)}(\mathbf{x})$ in Fig.1, where $R$ and $T$ are reflection and transmission coefficients, respectively. The total field $E_z(\mathbf{x})$ created by the bend will be very complicated in the vicinity of the bend. Only reflected+incident wave, however, can survive at points far away from the bend in the waveguide 2 and only transmitted wave can survive at points far away in the waveguide 1. We decompose the total field $E_z(\mathbf{x})$ in the dielectric waveguides into field components as

$$E_z(\mathbf{x})=E^C(\mathbf{x}) + TE^{+(1)}(\mathbf{x}), \quad \text{on } C_1 \text{ and } C_2, \tag{1}$$
$$E_z(\mathbf{x})=E^C(\mathbf{x}) + E^{-(2)}(\mathbf{x}) + RE^{+(2)}(\mathbf{x}), \quad \text{on } C_3 \text{ and } C_4, \tag{2}$$
$$E_z(\mathbf{x})=E^C(\mathbf{x}), \quad \text{on } C_5 \text{ and } C_6, \tag{3}$$

respectively, for the case of TE-mode. The field denoted by $E^C(\mathbf{x})$ represents the difference between the total field and the reflected+incident waves in the waveguide 2, the difference between the total field and the transmitted wave in the waveguide 1 and the total field in the bend-section. The field denoted by $E^C(\mathbf{x})$ in (1) and (2) is called the disturbed field [1]-[4]. If we adopt the condition that the radiation filed cannot exist at points far away from the bend in the both dielectric waveguides, we can find that the reflection coefficient $R$ and transmission coefficient $T$ can be expressed in terms of the field $E^C(\mathbf{x})$ and $\partial E^C(\mathbf{x})/\partial n$ on the boundaries, where $\partial/\partial n$ denotes derivative with respect to the outward normal vector $\mathbf{n}$ to $C_j$ $(j=1\text{-}6)$ as shown in Fig.1. By using these expressions, we can obtain new boundary integral equations for the unknown functions $E^C(\mathbf{x})$ and $\partial E^C(\mathbf{x})/\partial n$ as follows [3],[4]:

$$E^C(\mathbf{x})/2=\int_{C_1+C_2+C_3+C_4+C_5+C_6}[P_2(\mathbf{x}|\mathbf{x}')\partial E^C(\mathbf{x}')/\partial n'-E^C(\mathbf{x})\partial P_2(\mathbf{x}|\mathbf{x}')/\partial n']dl'+S_2(\mathbf{x}) \tag{4}$$

for the case where the observation point $\mathbf{x}$ approaches to the boundaries $C_j$ $(j=1\text{-}6)$ from the inner space of the waveguides+(bend-section) and

$$\begin{aligned}E^C(\mathbf{x})/2=&-\int_{C_1+C_3+C_5}[P_1(\mathbf{x}|\mathbf{x}')\partial E^C(\mathbf{x}')/\partial n'-E^C(\mathbf{x})\partial P_1(\mathbf{x}|\mathbf{x}')/\partial n']dl'\\&-\int_{C_2+C_4+C_6}[P_3(\mathbf{x}|\mathbf{x}')\partial E^C(\mathbf{x}')/\partial n'-E^C(\mathbf{x})\partial P_3(\mathbf{x}|\mathbf{x}')/\partial n']dl'+S_1(\mathbf{x})\end{aligned} \tag{5}$$

for the case where $\mathbf{x}$ approaches boundaries from the surrounding space of the waveguides+(bend-section). In integral equations *(4)* and *(5)*, kernel functions $P_j(\mathbf{x}|\mathbf{x}')$ $(j=1\text{-}3)$, are composed of free space Green's functions of index of refraction $n_j$ $(j=1\text{-}3)$ and terms which represent the effects of guided modes and $S_j(\mathbf{x})(j=1,2)$ are impressed terms. If fields $E^C(\mathbf{x})$ and $\partial E^C(\mathbf{x})/\partial n$ on the boundaries $C_j(j=1\text{-}6)$ are obtained, the scattered field can be also calculated. Since we can consider that the fields of guided-modes are extracted from the total field in the unknown functions of integral equations *(4)* and *(5)*, we can call these equations guided-mode extracted integral equations(GMEIEs).

**Numerical Examples:** Since the disturbed field $E^C(\mathbf{x})$ on the boundaries $C_j$ $(j=1\text{-}4)$ of the both waveguides become zero sufficiently far away from the bend-section, we can treat the problem like the scattering problem by the isolated object of finite size by the integral equations *(4)* and *(5)*. If boundary conditions are introduced, integral equations *(4)* and *(5)* can be solved by the conventional moment-method (boundary-element method). We applied integral equations *(4)* and *(5)* to the analysis of three types of slab waveguide bends which have been proposed in order to reduce the radiation bending loss. In these numerical examples, parameters of the dielectric slab are given by $n_1=n_3=1.0$, $n_2=1.5$, $k_0 d_1=k_0 d_2=2.68$ and bending angle is *20* degrees. The bend-shape of type (a) as shown in Fig.2 is based on the idea of using total reflection [5],[6]. The bend-shape of type (b) as shown in Fig.3 is based on the idea of reducing the wavefront-velocity of the inner side of the bend [7]. The two waveguide are connected by the straight waveguide which has same width as that of both waveguides in the bend-shape of type (c)[8]. Numerical examples of the dependences of the power transmission coefficient $\Gamma_T$ and normalized scattering power $\Gamma_S$ on bend-shapes are shown with normalized total energy $\Gamma_{TOTAL}=\Gamma_T+\Gamma_R+\Gamma_S$ for TE-mode(solid circles) and TM-mode (vacant circles) in Figs. 2-4. $\Gamma_{TOTAL}$ must be unity from energy conservation law. The abscissa represents the parameter of bend-shape as shown in these figures. Numerical error of $\Gamma_{TOTAL}$ of these numerical examples were smaller than 0.01. The power reflection coefficients $\Gamma_R$ were smaller than the numerical error in these numerical examples. The typical scattering patterns of TE-mode for each bend-shape are also shown in these figures. The pattern of broken curve shows the result of the sharp corner bend whose shape is shown by the broken line in each figure. These example show the basic transmission characteristics of each waveguide bend-shape. The transmission characteristics of bend-shape type (a) and type (c) is depend on the polarization of the wave. However, the transmission characteristics of bend-shape type (b) does not so much depend on the polarization of the wave. We can determine the optimum bend-shape in each waveguide bend from these examples. These numerical examples show the validity of the proposed GMEIEs.

**CONCLUSION:** Integral equations called GMEIEs have been applied to the CAD of dielectric slab waveguide bends. The numerical results of transmission characteristics and scattering pattern confirm the validity of the present method. The basic idea of GMEIE is very general. Hence, the theory is applicable to the more complicated waveguide discontinuity problems such as dielectric waveguide branches and these problems are under consideration. The theory is also applicable to problems of other fields such as acoustic and elastic waves.

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## References

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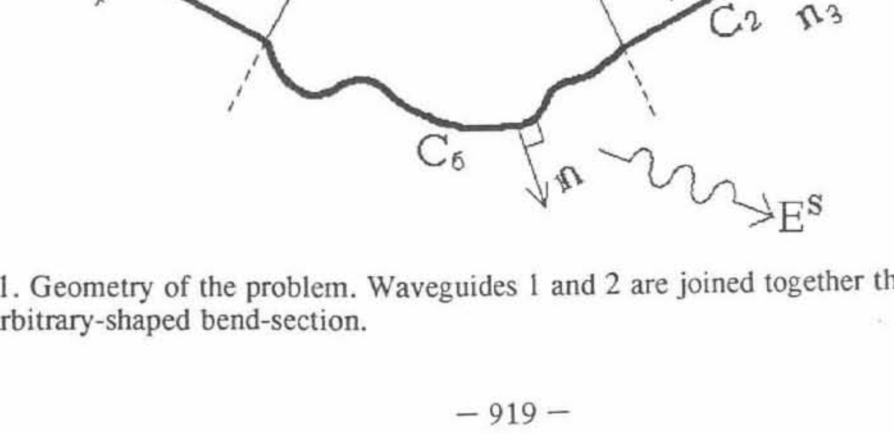
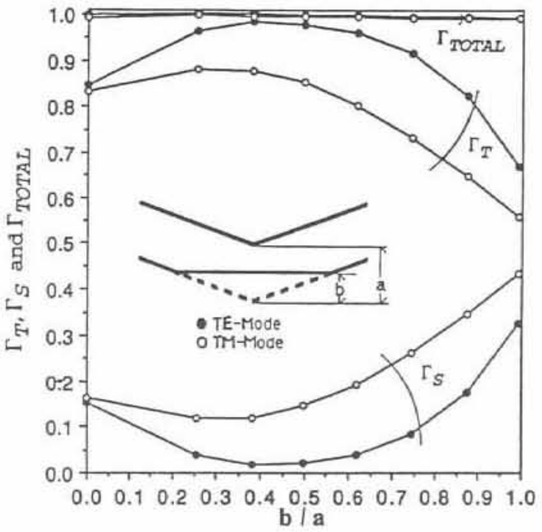



Fig. 1. Geometry of the problem. Waveguides 1 and 2 are joined together through the arbitrary-shaped bend-section.



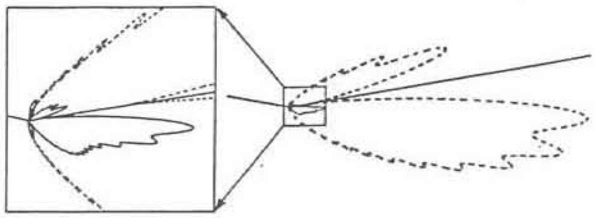
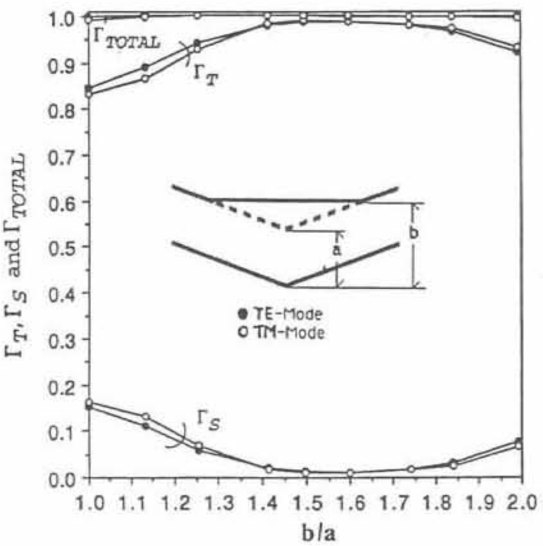



Fig. 2. Left figure shows power transmission coefficient $\Gamma_T$, normalized scattered power $\Gamma_S$ and their total $\Gamma_{TOTAL}$ of bend-shape type (a) whose shape is given in the left figure. The abscissa represents the parameter of bend-shape b/a as shown in the left figure. In the upper figure, solid curve shows scattering pattern of TE-mode for the case of b/a=0.38 of this bend-shape and broken curve shows that of sharp corner bend (b/a=0).



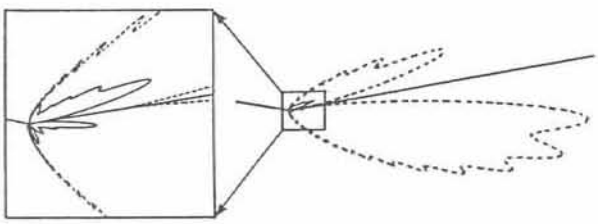
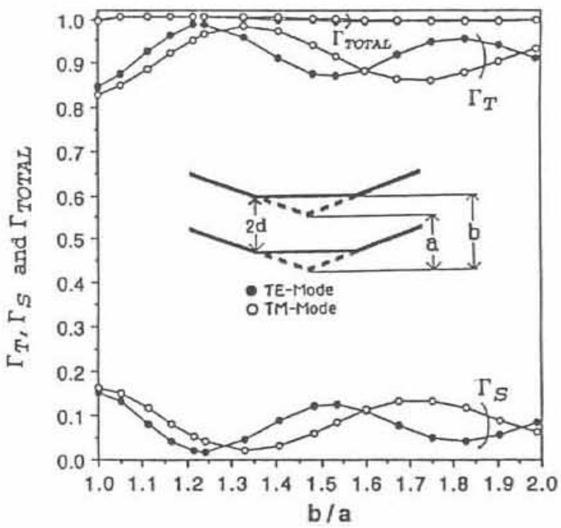



Fig. 3. Left figure shows power transmission coefficient $\Gamma_T$, normalized scattered power $\Gamma_S$ and their total $\Gamma_{TOTAL}$ of bend-shape type (b) whose shape is given in the left figure. The abscissa represents the parameter of bend-shape b/a as shown in the left figure. In the upper figure, solid curve shows scattering pattern of TE-mode for the case of b/a=1.60 of this bend-shape and broken curve shows that of sharp corner bend (b/a=1.0).



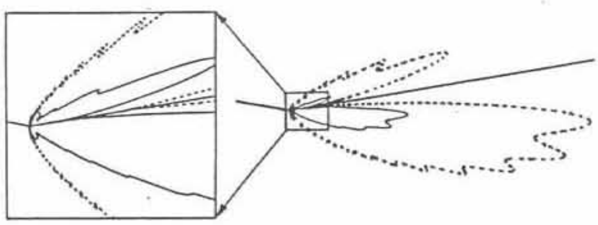



Fig. 4. Left figure shows power transmission coefficient $\Gamma_T$, normalized scattered power $\Gamma_S$ and their total $\Gamma_{TOTAL}$ of bend-shape type (c) whose shape is given in the left figure. The abscissa represents the parameter of bend-shape b/a as shown in the left figure. In the upper figure, solid curve shows scattering pattern of TE-mode for the case of b/a=1.24 of this bend-shape and broken curve shows that of sharp corner bend (b/a=1.0).