## Shape Estimation of Space Debris Using Single-Range Doppler Interferometry

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#### 1 Introduction

Space debris, or fragments of satellites or rocket bodies left on the earth's orbit, is recognized as a serious environmental problem in the space [1]. Shape estimation is an important issue in the oservation of space debris, since it is directly related to the impact of collision. Statistical study has shown that their size estimated from the radar cross section (RCS) assuming a spherical shape do not agree with the size estimated from the atmospheric drag[2]. Analysis of RCS variations associated with the spin motion of space debris observed by a VHF radar suggests prolate shapes rather than spherical ones[3].

A commonly used technique for imaging a rotating object with a radar is the Range-Doppler Interferometry (RDI), or the Inverse Synthetic Aparture Radar (ISAR) technique, in a different notation. The application of this technique is, however, limited to a case where the size of the target is much larger than the radar wavelength. The most important target of the observation of space debris is those in the size region of 1–10 cm, for which shielding is impractical. Here we propose an imaging technique which is applicable to the imaging of objects whose size is comparable to the radar wavelength.

#### 2 RDI and SRDI

RDI makes use of the fact that echoes from different parts of a rotating object have different Doppler shifts. It has already been applied to the imaging of orbital objects, such as the case of Salyut-7/Kosmos-1686 comlex body observed by X-band FGAN radar[4].

While the cross-range resolution is obtained by the Doppler effect, the range resolution is determined by the system bandwidth, which is limited by the signal-to-noise ratio. Since the detection of objects in the size region of 1-10 cm at a distance of 100-1,000 km is a difficult task even with powerful radars, it is not realistic to obtain a range resolution of less than about 1 m.

However, it is possible to obtain a two-dimensional image using only the cross-range information if the same target can be observed for more than one spin period. We hereafter refer this method as Single Range Doppler Interferometry (SRDI) in contrast to RDI. Figure 1 schematically compares the two methods.

The cross-range resolution of both RDI and SRDI is given by

$$\Delta a \simeq \lambda/2\Delta\theta \quad . \tag{1}$$

where  $\Delta\theta$  is the range of rotational angle used in the imaging of a point of the target. Although the cross-range resolution becomes better by choosing large  $\Delta\theta$ , the image deteriorates because different parts of the target are used in imaging a point. Here we set the upper limit of  $\Delta\theta$  to 1 radian aiming to image the rough shape of the entire body.

It should be noted that  $\Delta\theta$  itself cannot be directly measured, which means that we cannot distinguish a small body with a large angular velocity from a large body spinning slowly. The

rotation period needs to be determined by continuously tracking the target for more than one period, and by taking auto-correlation of the received signal.

### 3 Signal Processing of SRDI

In SRDI, we need to represent the Doppler spectrum of the target a a function of rotation angle  $\theta$ . The Doppler shift of the scattered electric field from a point source at the angular coordinate  $(r, \theta_0)$  relative to the center of rotation is given by

$$\omega_{\mathbf{d}}(\theta) = -\mathrm{j}\frac{4\pi\omega_{0}r}{\lambda}\cos(\theta + \theta_{0}) , \qquad (2)$$

where  $\omega_0$  is the angular frequency of rotation. The determination of the instantaneous frequency is not straight forward in the case  $r \sim \lambda$ , where  $\omega_{\rm d}(\theta)/\omega_0$  is only on the order of 10. Here we use the short-time Fourier transform with a raised cosine window of the half angular width of 30°. Figure 2 shows an example of the Doppler spectrum thus computed for a point target at a distance of  $2\lambda$  from the center of rotation. As expected from Eq. 2, the signal from a point target makes a circular trace on this figure.

In order to reconstruct the original target image, a migration technique is applied. It is to integrate all points along the circular path which corresponds to each point target. An example of this procedure for two point targets at radius  $2\lambda$  and  $1\lambda$ , respectively, is shown in Figure 3. This figure demonstrates that SRDI method has a resolution of about  $1/2\lambda$ , which is the same as the cross-range resolution of RDI shown in Eq. 1.

### 4 Scattering from a Continuous Body

While the above simulation assumed isotropic point targets, the actual target usually consists of a continuous body, which shows a strong aspect sensitivity due to the current induced on its surface. In order to reduce its effect, the scattered power of the spectrum at each  $\theta$  is normalized by the total reflection power before applying the migration operation. Figure 4 shows the result of imaging of a perfectly conducting ellipsoid of  $4\lambda \times 0.5\lambda$ . The scattered wave is computed using a wire grid model which consists of wires of the radius of  $0.005\lambda$ . The ellipsoid is divided

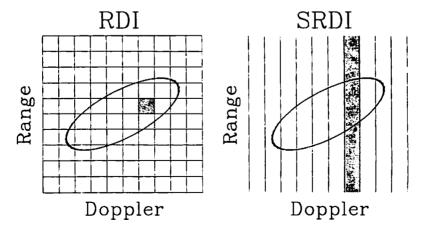


Figure 1: DRI(ISAR) method and SRDI method.

# Image before migration Point target (2.0λ from center)

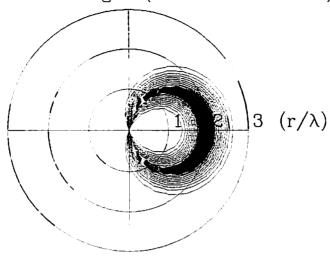


Figure 2: Doppler spectrum of a point target located at  $2\lambda$  from the center versus the rotation angle.

# Image after migration 2 point targets

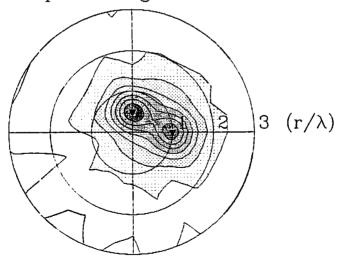


Figure 3: Image of two point targets after the migration. Triangles denotes the location of given point targets.

## Image after migration Ellipsoid $(4.0\lambda \times 0.5\lambda)$

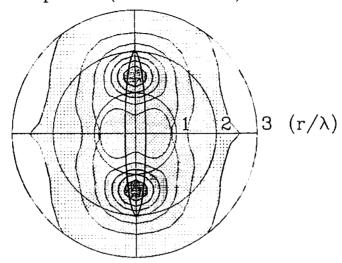


Figure 4: SRDI image of conducting ellipsoid. Thick curve shows the given shape.

into 32 and 2 sectors in the angular and radial directions, respectively. Although the prolate shape of the body and its size can be roughly estimated. The two prominent points are still emphasized in the reconstructed image.

## 5 Summary

A method of imaging a target whose size is smaller than the range resolution is proposed. Its characteristics are examined by means of numerical simulation. Although the method reconstructs the target clearly for the case of isotropic point scatterers, further study is needed to properly interpret the image of a continuous body.

#### References

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