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ELECTROMAGNETIC WAVES ON PARTIALLY FINITE PERIODIC ARRAYS OF SMALL LOSSLESS PENETRABLE SPHERES

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1. Introduction

The subject of this paper is the field of a three-dimensional (3D) partially finite periodic array of lossless magnetodielectric spheres illuminated by a plane wave propagating in a direction parallel to the array axis; that is, with the propagation vector of the plane wave normal to the interface between the array and free space. By "partially finite" we mean that the array is finite in the direction of the array axis and is of infinite extent in the directions transverse to the array axis. The work is based on our earlier investigations of traveling waves on linear [one-dimensional (1D)] periodic arrays of acoustic monopoles [1], electric dipoles [2], and magnetodielectric spheres [3]-[5], using a spherical-wave source scattering-matrix formulation, and the extension of the linear array work to two-dimensional (2D) and three-dimensional (3D) arrays of these scatterers given in [6]. In these investigations we considered periodic infinite arrays of identical small scatterers each characterized by a scattering coefficient that relates the field scattered from the element to the field incident on the element, and assumed that only the fields of the lowest order spherical multipoles (acoustic monopoles, electromagnetic dipoles) are significant in analyzing scattering from the array elements. The focus of our investigations has been the k–β equation (or diagram) — in our work more properly referred to as the kd–βd equation (or diagram) — that relates the traveling wave electrical separation distance βd of the array elements in the direction parallel to the array axis, to the corresponding free-space electrical (or acoustical) separation distance kd, where $k = \omega/c$ is the free-space wavenumber with $\omega > 0$ the angular frequency and c the free-space speed of light. Our investigation of periodic arrays of lossless scatterers has been motivated in part by the theoretical demonstration by Holloway *et al.* that a doubly negative (DNG) material can be formed by embedding an array of spherical particles in a background matrix [7]. Our work in the above references has confirmed the conclusion of Holloway *et al.*.

In this paper we apply the analyses we have performed to obtain the kd–βd equation for an infinite periodic 3D array of magnetodielectric spheres, to obtain exact computable expressions for the field of a partially finite periodic array of these elements when the array is illuminated by a plane wave propagating in a direction parallel to the array axis. By treating partially finite arrays of spherical scatterers we move considerably closer to being able to analyze actual models of DNG materials constructed from periodic arrays of spherical particles than is possible with our analyses of 3D infinite arrays. Details and derivations that space does not permit us to give here are given in full in [6].

2. Analysis

We investigate the field excited by a plane wave incident from free space on a 3D periodic partially finite array of lossless magnetodielectric spheres, using a method due to Foldy [8]. The array is finite in the direction of the array axis and infinite in the directions transverse to the array axis. The direction of incidence of the illuminating plane wave is parallel to the array axis, normal to the interface between the array and free space. It is assumed that the spheres can be modeled by pairs of crossed electric and magnetic dipoles, each of the dipoles perpendicular to the array axis. The z axis of a Cartesian coordinate system is taken to be the array axis and N+1 equispaced planes parallel to the xy plane of magnetodielectric spheres are located at $x = nd$, $n = 0,\ 1,\ 2,\ \cdots,\ N$. In each plane the spheres are centered at $(x, y) = (mh, lh)$, $l, m = 0,\ \pm1,\ \pm2,\ \cdots$ with the electric and magnetic dipoles oriented in the x and y direction, respectively. The electric and magnetic field vectors of the incident plane wave illuminating the array from the left are

$$E_{x,\mathrm{inc}}(z) = \mathrm{e}^{\mathrm{i}kz}, \quad H_{y,\mathrm{inc}}(z)/Y_0 = \mathrm{e}^{\mathrm{i}kz} \tag{1}$$

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so that all spheres in any plane of the array are excited identically. We let $E_x^n(z_n)$ and $H_y^n(z_n)$ be the external electric and magnetic fields, respectively, incident on a sphere in the nth plane, with $z_n = nd, n = 0, 1, 2, \cdots, N$, and make use of the scattering equations

$$b_{-n} = S_- E_x^n(z_n), \quad b_{+n} = S_+ H_y^n(z_n)/Y_0 \tag{2}$$

where b_- and b_+ are the coefficients of the scattered electric and magnetic dipole waves, respectively, and S_- and S_+ are the normalized magnetodielectric sphere electric and magnetic dipole scattering coefficients, respectively, given by

$$S_- = -\mathrm{i}\frac{3}{2}\, b_1^{sc}, \quad S_+ = -\mathrm{i}\frac{3}{2}\, a_1^{sc} \tag{3}$$

with b_1^{sc} and a_1^{sc} the Mie electric and magnetic dipole scattering coefficients, respectively, given by [9, sec.9.25, eqs.(11),(10)]. We then obtain expressions for $E_x^n(z_n)$ and $H_y^n(z_n)/Y_0$ by summing the contribution of the incident plane wave $\mathrm{e}^{\mathrm{i}kz_n}$ and the contribution of the fields scattered from all the array elements other than the $(0, 0, z_n)$ sphere

$$E_x^n(z_n) = \mathrm{e}^{\mathrm{i}kz_n} + \frac{1}{(kh)^3}\left\{\sum_{\substack{j=0\\j\neq n}}^{N} S_- E_x^j(z_j)\,\sigma_{11}(|j-n|d) - \sum_{\substack{j=0\\j\neq n}}^{N} S_+ \frac{H_y^j(z_j)}{Y_0}\,\sigma_{12}[(j-n)d] + S_- E_x^n(z_n)\,\sigma_2\right\} \tag{4a}$$

and

$$\frac{H_y^n(z_n)}{Y_0} = \mathrm{e}^{\mathrm{i}kz_n} + \frac{1}{(kh)^3}\left\{\sum_{\substack{j=0\\j\neq n}}^{N} S_+ \frac{H_y^j(z_j)}{Y_0}\,\sigma_{11}(|j-n|d) - \sum_{\substack{j=0\\j\neq n}}^{N} S_- E_x^j(z_j)\sigma_{12}[(j-n)d] + S_+ \frac{H_y^n(z_n)}{Y_0}\sigma_2\right\} \tag{4b}$$

where

$$\sigma_{11}(|j-n|d) = \sum_{m=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} \frac{\mathrm{e}^{\mathrm{i}kh\rho_{mljn}}}{\rho_{mljn}}\left[\frac{-2\mathrm{i}}{\rho_{mljn}}\left(kh + \frac{\mathrm{i}}{\rho_{mljn}}\right)\frac{m^2}{\rho_{mljn}^2} + \left((kh)^2 + \frac{\mathrm{i}kh}{\rho_{mljn}} - \frac{1}{\rho_{mljn}^2}\right)\frac{l^2 + [(j-n)d/h]^2}{\rho_{mljn}^2}\right] \tag{5a}$$

$$\sigma_{12}[(j-n)d] = \sum_{m=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} \frac{\mathrm{e}^{\mathrm{i}kh\rho_{mljn}}}{\rho_{mljn}}\left((kh)^2 + \frac{\mathrm{i}kh}{\rho_{mljn}}\right)\frac{(j-n)d/h}{\rho_{mljn}} \tag{5b}$$

and

$$\sigma_2 = \sum_{\substack{m=-\infty\\(m,l)}}^{\infty} \;\neq\; \sum_{\substack{l=-\infty\\(0,0)}}^{\infty} \frac{\mathrm{e}^{\mathrm{i}kh\rho_{ml}}}{\rho_{ml}}\left[\frac{-2\mathrm{i}}{\rho_{ml}}\left(kh + \frac{\mathrm{i}}{\rho_{ml}}\right)\frac{l^2}{\rho_{ml}^2} + \left((kh)^2 + \frac{\mathrm{i}kh}{\rho_{ml}} - \frac{1}{\rho_{ml}^2}\right)\frac{m^2}{\rho_{ml}^2}\right] \tag{6}$$

where

$$\rho_{mljn} = \rho_{ml|j-n|} = \sqrt{m^2 + l^2 + [(j-n)d/h]^2},\ \rho_{ml} = \sqrt{m^2 + l^2} \tag{7}$$

and thus we have a system of $2(N+1)$ equations for the $2(N+1)$ unknowns $E_x^n(z_n), H_y^n(z_n)/Y_0$, $n = 0, 1, 2, \cdots, N$

$$\left[1 - S_-\sigma_2/(kh)^3\right] E_x^n(z_n) - \frac{S_-}{(kh)^3}\sum_{\substack{j=0\\j\neq n}}^{N}\sigma_{11}(|j-n|d)E_x^j(z_j) + \frac{S_+}{(kh)^3}\sum_{\substack{j=0\\j\neq n}}^{N}\sigma_{12}[(j-n)d]\frac{H_y^j(z_j)}{Y_0} = \mathrm{e}^{\mathrm{i}kz_n} \tag{8a}$$

$$\left[1 - S_+\sigma_2/(kh)^3\right]\frac{H_y^n(z_n)}{Y_0} - \frac{S_+}{(kh)^3}\sum_{\substack{j=0\\j\neq n}}^{N}\sigma_{11}(|j-n|d)\frac{H_y^j(z_j)}{Y_0} + \frac{S_-}{(kh)^3}\sum_{\substack{j=0\\j\neq n}}^{N}\sigma_{12}[(j-n)d]E_x^j(z_j) = \mathrm{e}^{\mathrm{i}kz_n}. \tag{8b}$$

Rapidly convergent expressions for σ_{11}, σ_{12}, and σ_2 are obtained in [6, sec.11.3]. It is then straightforward to solve the system of equations (8) for the values of $E_z^n(z_n)$ and $H_y^n(z_n)/Y_0$. The field at any point on the array axis (other than at elements of the array) can then be calculated using (4) with $E_z^n(z_n)$ and $H_y^n(z_n)$ replaced by $E_z(z)$ and $H_z(z)$, and by replacing j and n by n and z/d, respectively, in the expressions for σ_{11} and σ_{12}. Given values of $E_x(z)$ we can calculate the reflection coefficient of the wave scattered back in the negative z direction for $z < 0$ as well as the transmission coefficient of the wave traveling in the positive z direction for $z > Nd$. Since the amplitude of the plane wave incident on the partially finite array is 1, the reflection coefficient, R, is the complex coefficient of the wave $\mathrm{e}^{-\mathrm{i}kz}$ for $z < 0$ with R obtained from the equation

$$R = E_x(z)\,\mathrm{e}^{\mathrm{i}kz}, \quad z < 0\,. \tag{9}$$

The complex transmission coefficient can be obtained from the equation

$$T = [E_x(z) + \mathrm{e}^{\mathrm{i}kz}]\,\mathrm{e}^{-\mathrm{i}kz}, \quad z > Nd\,. \tag{10}$$

3. NUMERICAL RESULTS

In this section as an example of the reflection coefficient curves for partially finite arrays of magnetodielectric spheres we show plots of the reflection coefficient for a partially finite array of diamond spheres with $\epsilon_\mathrm{r} = 5.84, \mu_\mathrm{r} = 1$. The value of N is 100 (that is, there are 101 equispaced infinite planes of spheres normal to the array axis), and the ratio of the radius of the spheres, a, to the separation of adjacent sphere centers, d, is 0.45. We show plots for two cases, one where there is no loss, and one with loss inserted into the propagation constant of the incident plane wave when calculating the values of $E_x^n(z_n)$ and $H_y^n(z_n)/Y_0, n = 0, 1, \cdots, N$ from (8). The value of the loss constant, ε, is chosen via the equation $\mathrm{e}^{-Nkd\varepsilon} = 10^{-P}$ with $P = 1$. Given the value of ε, the values of the incident plane wave, $\mathrm{e}^{\mathrm{i}kz}$, at the locations $z = z_n = nd,\ n = 0, 1, \cdots, N$ in the RHS of (8) are then multiplied by the respective factors $\mathrm{e}^{-\varepsilon nkd},\ n = 0, 1, \cdots, N$. The purpose of inserting loss is to reduce the multiple interactions between the leading and trailing interfaces of the partially finite array, thereby producing a reflection coefficient that is nearly equal to that of the leading interface alone.

In Fig. 1 we show a plot of the magnitude of the reflection coefficient for the partially finite lossless diamond array. The pronounced oscillations of the pattern are the result of reflections between the two ends of the array. Thus, the array behaves somewhat like a Fabry-Perot resonator. The intervals of the plot where the magnitude of the reflection coefficient equals one correspond exactly to the bandgaps in the kd–βd diagram of an infinite periodic array of diamond spheres (see [6, fig. 21]), that is, the intervals of kd where no traveling wave exists to convey power from one end of the array to the other.

In Fig. 2 we show the plot of the magnitude of the reflection coefficient for the partially finite diamond array with loss inserted into the incident plane wave, together with a plot for $kd < 1$ of the magnitude of the reflection coefficient obtained from the Clausius-Mossotti bulk permittivity and permeability for the infinite 3D diamond array [10, sec.8-1, eq.(3-35)],[6, sec.9.1]. The oscillations of Fig. 1 have been considerably reduced. However, this loss decreases the magnitude of the reflection coefficient in the stopbands to a value slightly less than one. Note that for values of kd less than one there is excellent agreement between the lossy partially finite array reflection coefficient and the Clausius-Mossotti reflection coefficient apart from a small interval of kd between zero and about 0.1. This is to be expected since the derivation of the Clausius-Mossotti bulk parameter expressions assumes a separation of the array elements sufficiently small so that the array can be regarded as a homogeneous medium. As kd becomes smaller than 0.1, the total thickness of the slab with decaying incident field (simulating a loss) becomes smaller than a free-space wavelength and its scattering becomes weaker.

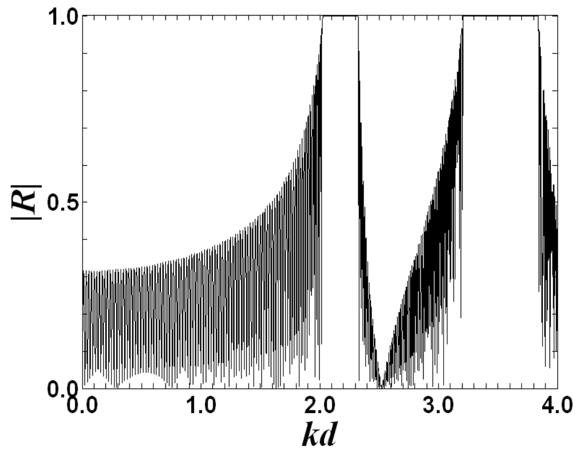

Figure 1: Reflection coefficient of a lossless partially finite 3D array of diamond spheres with $\epsilon_r = 5.84,\ \mu_r = 1,\ a/d = .45$.

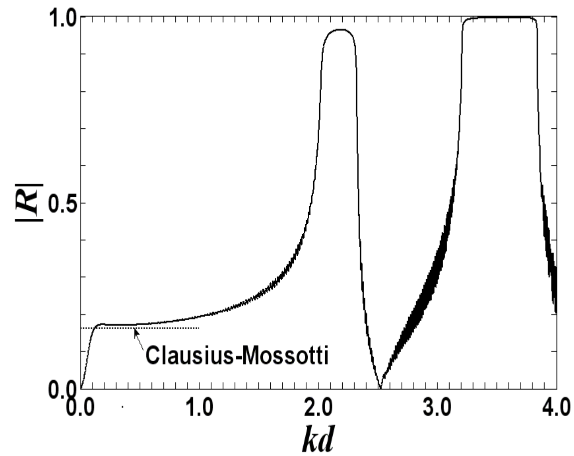

Figure 2: Reflection coefficient of a lossy partially finite 3D array of diamond spheres with $\epsilon_r = 5.84,\ \mu_r = 1,\ a/d = .45$.

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