

# Surface Modes in a Two-Dimensionally Electromagnetic Band-Gap Structure with Termination Condition

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## Abstract

As was well known, the excitation of surface modes in a 2D EBG (two-dimensionally electromagnetic band-gap structure) is a key factor affecting the superlensing phenomenon. In the paper, we employed the rigorous mode-matching method incorporated with the Floquet's solutions to systematically calculate the dispersion characteristics of the bound modes supported by a 2D EBG structure with termination condition. It is interesting to observe that the surface mode is caused by the perturbation from the termination condition, enabling the movement of the dispersion curve from the pass-band region to the stop-band region. The perturbed dispersion curves and electric field pattern over the structure are drawn together with the unperturbed ones (without termination condition) to identify the type of waveguide mode.

## 1. Introduction

Periodic structures again attract considerable attentions in recent year since the anomalous phenomenon- negative refraction was found. The term "negative refraction" means that the medium refracts the electromagnetic wave in the direction opposite to that obeying the Snell's law. This phenomenon was predicted by Veselago [1] in 1968 by introducing the negative permittivity and permeability into the Maxwell's equations. He knew that in such a homogeneous medium the  $\mathbf{k}$ ,  $\mathbf{E}$  and  $\mathbf{H}$  form a left-handed set of vectors. A striking observation of superlensing using a uniform medium having negative permittivity and permeability was found by Pendry [2]. He argued that the superlensing is due to the excitation of surface modes. Only the surface is carefully terminated, the excitation of surface modes and an improvement of image is achieved. Moreover, he claimed that the superlensing effect is not because of negative permittivity and permeability, or inward directed equifrequency contours. In this paper, the purpose is to investigate the physical insight of the surface modes supported by a 2D EBG. Unlike the paper [3] dealing with the structure with incomplete unit cell, here, we imposed a termination condition on a complete 2D EBG (each unit cell is complete). The structure could be terminated by an open-circuit (perfect magnetic conductor) or short-circuit (perfect electric conductor) conditions. From the literature, we know that the surface mode is the wave decaying along both directions from the interface between 2D EBG and uniform surrounding medium. To understand the guiding characteristic of such an interface wave, the dispersion relation of all the bound modes supported by such a 2D EBG structure has to be calculated first. In this paper, the rigorous mode-matching method combined with the Floquet's solutions [4, 5] is employed to carry out the computation for both scattering and guiding characteristics. Besides, the band-structure of the corresponding infinite 2D EBG medium was calculated for facilitating the understanding of wave propagating within the structure. From the numerical results given later on, we could understand that the surface mode is evolved from the waveguide mode in the 1D periodic layers. Due to the perturbation caused by the termination condition, the waveguide mode resident in the pass-band region transforms into the surface mode stayed in the stop-band region.

## 2. Method of Analysis

As shown in the figure 1, a two-dimensionally electromagnetic band-gap structure consists of rectangular dielectric rods array immersed in a uniform medium. The structure is infinitely in extent along the  $x$ - and  $y$ - direction, while it has finite thickness along the  $z$  direction. Below the

periodic structure at a distance  $h$ , there exists an open-circuit or short-circuit termination condition. Since the electric and magnetic fields are assumed to be invariant along the  $y$  direction, the problem can be individually treated as  $E_y$ - and  $H_y$ - mode, respectively. Notice that the OC (open-circuit) or SC (short-circuit) termination condition is used to simplify the mathematical formulation for a symmetric structure. If the  $2h$  is not equal to the width of the uniform separator sandwiched by two adjacent 1D periodic layers, the EBG structure with termination condition can be regarded as an EBG structure with defect in its central layer. As far as such a 2D periodic structure is concerned, the scattering characteristic and dispersion relation of waveguide modes supported by the structure are the two important topics needed to be studied.

Returning to figure 1, the 2D EBG structure could be considered as a finite stack of 1D periodic layer (along the  $z$  direction). The scattering characteristic of plane wave by a 1D periodic structure can be considered as a basic building block for constructing the scattering characteristic of the overall structure. This boundary-value problem of the 1D periodic structure can be rigorously resolved by first solving the electric and magnetic fields in an infinite 1D periodic medium using the plane wave expansion method [5]. After matching the boundary condition at the two interfaces between the 1D periodic medium and uniform surrounding medium, we could obtain the input-output relation of the 1D periodic layer. By cascading the input-output relation for each 1D periodic layer, the scattering characteristic including the reflection and transmission efficiency of each space harmonic can be determined. Moreover, the dispersion relation of the waveguide (eigen-solutions for source-free problem) can be easily achieved by imposing the transverse resonance condition on the input interface. Since the detail mathematical procedure is well-known and could be found in literature [4, 5], the complicated mathematical formulations is neglected here.

### 3. Numerical Results

In the following numerical examples, the structure parameters are given as follows. The relative dielectric constants of the dielectric cylinder and uniform surrounding media are 5.0 and 1.0, respectively. The width of the square cylinder, periods along the  $x$  and  $z$  directions are 0.5, 1.0 and 1.0, respectively. The number of 1D periodic layer is 5, whereas the number of periods along the  $x$  direction for is infinity.

Figure 2(a) depicts the dispersion relation of guided waves in the 2D EBG structure. The vertical and horizontal axes represent the normalized frequency ( $d_x/\lambda$ ) and normalized propagation constant along the  $x$  direction ( $\beta_x d_x/2\pi$ ), respectively. The straight line with unity slope is the light line ( $\beta_x = k_o$ ); the region with slope greater than unity represents the slow-wave (or bound-wave) region where the electromagnetic waves are confined, whereas the region with slope less than unity is the fast-wave region where the electromagnetic waves confined in the defect region or radiating into the surrounding medium. Since the wave guiding characteristics in defect region and leaky-wave phenomena are well known in literature [5]; however, the bound waves are rarely being studied, only the dispersion roots in the bound wave region are displayed for investigating their physical consequences. In this figure, the *projected band-structure* of the corresponding infinite 2D EBG was drawn; the region drawn in light yellow colour is the pass-band region, while the region in white colour is the stop-band region. The two different termination conditions are considered in this numerical example, which are open-circuit and without termination (infinite in extent of the uniform medium). The curves in black and red colours represent the dispersion roots of open-circuit and without termination, respectively. These dispersion curves are very similar to those of the bound waves in multiple uniform dielectric layers, excluding the coupling between the space harmonics due to the periodicity along the  $x$  direction, which are shown in the upper pass-band region. Since the structure contains five 1D periodic layers, there exist five dispersion curves of the fundamental mode shown in the figure. For the case without termination, all the dispersion curves are inside the pass-band region. It is interesting to note as the open-circuit termination ( $h=0.01$ ) was imposed the two curves will leave the pass-band region and go into the stop-band region. Obviously, the open-circuit termination condition will enable the dispersion curves of un-terminated case to move downward. Thus, the lowest one will go away the pass-band region. The dispersion curve in the stop-band region represents that the electromagnetic field experiences a strong reflection in the 2D EBG structure. Moreover, since its effective dielectric constant is greater than that of the surrounding medium, the wave is decaying in the surrounding medium. Therefore, this

wave is decaying along the both directions from the interface between 2D EBG and uniform medium at  $z = 0$ . To demonstrate it, we plot the distribution of the electric field strength ( $E_y$  component) over the structure at the normalized frequency 0.25. We can observe the field is decaying along the 2D EBG region; however, the decaying below the  $z = 0$  region is not easy to observe because that the termination plane is very close to the interface ( $h=0.01$ ).

In the next example, we changed the termination distance  $h$  to observe the variation on the dispersion curves. We calculate the dispersion curves for OC termination condition ( $h = 0.245$ ) and redraw the figure 2. It is apparently to see that in Figure 3(a) the two groups of dispersion curves are close to each other. To explain this phenomenon, we should recall the mathematical procedure for obtaining the dispersion roots. The dispersion roots are determined by solving the non-trivial solution of the transverse resonance equation. The input impedance matrix looking upward at the interface between air and 2D EBG remains the same for the two different cases, while they are totally different for the input impedance looking downward. However, since the  $x$ -direction effective dielectric constant is greater than unity, the wave is decaying along the  $z$  direction in the uniform medium. As the termination distance is increasing, the input impedance will approach the case of without termination. In mathematical representation, the input impedance of an open-circuit termination is given as  $-jZ_o \cot \kappa h$ , where  $Z_o$  and  $\kappa$  are the characteristic impedance and the propagation constant along the  $z$  direction in the uniform medium, respectively. Because the propagation constant  $\kappa$  along the  $z$  direction is a pure imaginary number (we assumed that the dielectric material and surrounding medium are lossless), the cotangent function will become a hyper cotangent function. Moreover, if the argument  $\kappa h$  is increasing, the  $-jZ_o \cot \kappa h$  will approach  $Z_o$ ; i.e. the characteristic impedance of wave in uniform medium. This explains the convergence of the two groups of dispersion curves shown in figure 3(a).

In addition to the dispersion curves, we also demonstrate the distribution of the electric field strength on the overall structure. The field patterns were plotted at the normalized frequency chosen at 0.25, where the effective refractive index are 1.1972 and 1.1542 for the open-circuit and un-terminated cases, respectively. Figure 3(b) and 3(c) show the field strength distribution of the two cases. Since the dispersion root is in the pass-band region, the field can penetrate into the 2D EBG structure. Moreover, it is apparently to see that the two patterns are similar to each other. We have also calculated the field patterns for the other modes; however, due to the limited pages, these figures will be presented in the conference.

## References

- [1] V. G. Veselago, "The electrodynamics of substance with simultaneously negative values of  $\epsilon$  and  $\mu$ ", *Sov. Phy. Usp.* 10, 509-514, 1968
- [2] J. B. Pendry, "Negative refraction makes a perfect lens", *Phys. Rev. Lett.* 85, 3966-3969, 2000
- [3] R. Moussa, Th. Koschny and C. M. Soukoulis, "Excitation of surface waves in a photonic crystal with negative refraction: The role of surface termination", *Physical Review B* 74, 115111, 2006
- [4] Tamir, T., and Zhang, S., "Modal Transmission-Line Theory of Multilayered Grating Structures," *J. Lightwave Technol.*, vol. 14, pp.914-927, 1996
- [5] Ruey Bing Hwang; Cherng Chyi Hsiao, "Frequency-selective transmission by a leaky parallel-plate-like waveguide", *IEEE Trans. on Antennas and Propagation*, Volume: 54 Issue: 1 Jan. 2006, Page(s): 121- 129

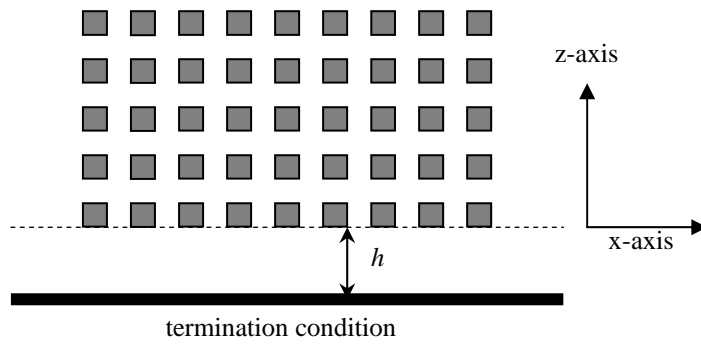


Figure 1: structure configuration

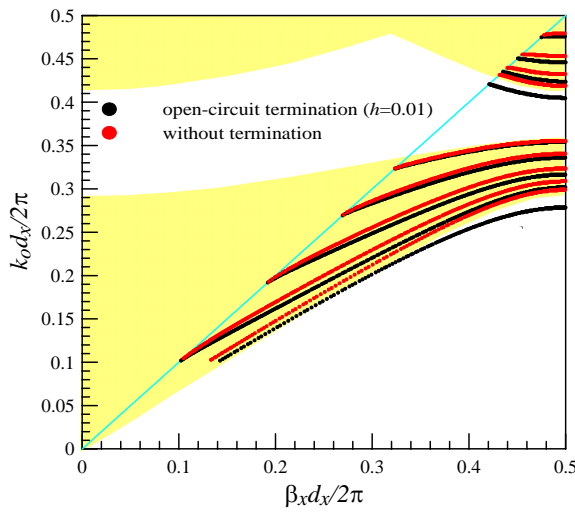


Figure 2(a)

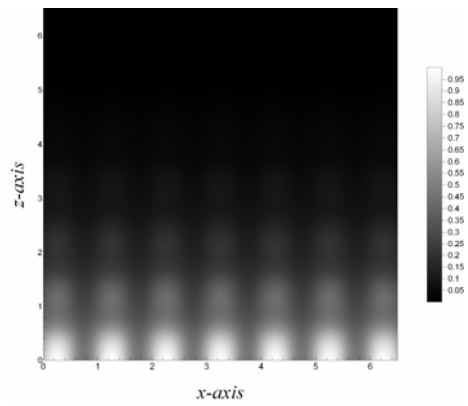


Figure 2(b)

Figure 2: (a) dispersion relation of the bound waves, and (b) distribution of the electric field strength  $E_y$  at the normalized frequency 0.25

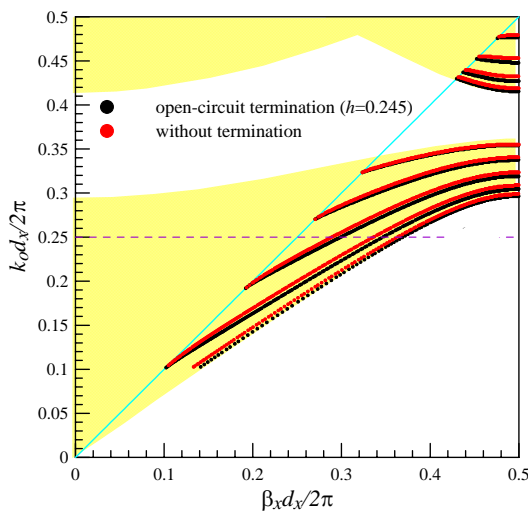


Figure 3(a)

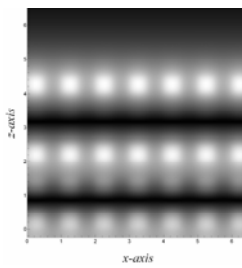


Figure 3(b)

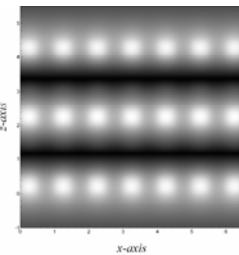


Figure 3(c)

Figure 3: (a) dispersion relation of the bound waves, and the distribution of the electric field strength  $E_y$  at the normalized frequency 0.25, (3b) for OC termination and (3c) for un-terminated case