

# Scattering From a Periodic Array of Elliptical Cylinders With a Coated Body of Arbitrary Shape Using Moment Method

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## Abstract

The scattering from a two-dimensional periodic array of elliptical cylinders with a circular shape coated body using Moment Method is examined in this paper. The scattered field is expressed in terms of the integral form by an infinite summation of the surface integral over the cross section of the reference cylinder. The integral form is converted into the matrix equations of  $N$  linear independent unknowns by MoM. The lattice sums technique is used to evaluate the elements of the matrix equation. This matrix equation can be solved by Generalized Minimum Residual (GMRES) method. Numerical results show the effect of changing the relative permittivity of the dielectric coated body on both the total field distribution within each cell and the reflection coefficient for the fundamental Floquet mode of the periodic structure.

## 1. Introduction

In the past, several major research efforts have been undertaken to investigate the scattering properties of a periodic array structures [1]-[3]. Periodic array structures have many applications in the design of antennas and waveguide for bandwidth enhancement or dual-band applications such as wavelength or frequency selection filters [4]. Base on the accuracy needed to analyze these complex structures, the Moment Method (MoM) [5] is an obvious solution approach because MoM is one of the useful numerical methods for solving the problem of scattering by inhomogeneous dielectric bodies. In this research, the dense matrix coefficient expressed in the lattice sums of arbitrary high order can be simplify to an integral form of elementary functions. These elementary functions constitute the lattice sums expression used in [6] as this expression only depend on the geometrical parameter of the structure. Using the lattice sums technique [6], the computational time for the numerical evaluation decreases drastically as compared to [7]. GMRES method [8] is applied as the iterative method to solve the matrix equations since it has been proven to have a good convergence speed [9].

## 2. Formulation

Consider the scattering of a periodic array of dielectric elliptical cylinders with a circular shaped coated dielectric body along the  $x$ -direction with periodicity  $d$  as shown in Fig. 1. The axis dimension, radius and relative permittivity of the cylinder and the coated body are given by  $(a, b, \epsilon_{r2})$  and  $(R_1, \epsilon_{r1})$  respectively.  $d_{12}$  is the distance between the centers of the coated circular body and the cylinder. It is assumed that the incident plane wave  $E_i$  is polarized along the  $z$ -axis. In this case, the scattered field  $E_s$  at any point of the structure differs from that in a selected reference cylinder by only a phase shift. Using Floquet's theorem, the scattered field for an infinite periodic structure can be written in an integral form. The total electric field is the superposition of the incident field and the scattered field as shown as follows:.

$$E(\mathbf{r}) = E_i(\mathbf{r}) - j \frac{k^2}{4} \sum_{l=-\infty}^{\infty} \int_{s_l} H_0^{(2)}(k\rho_l) E(\mathbf{r}_0) \exp(-jk_x l d) (\epsilon_{r0} - 1) ds_l \quad (1)$$

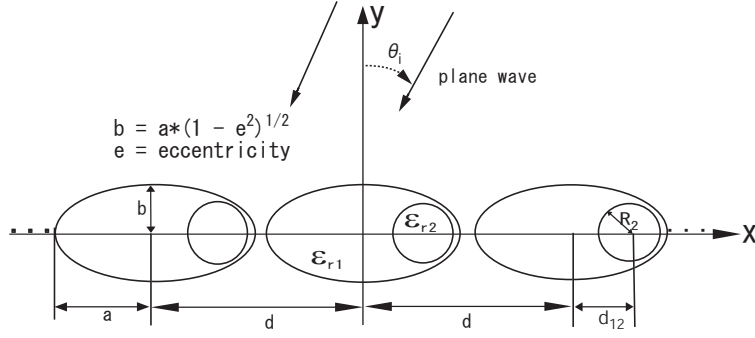


Figure 1: Geometry of a periodic array of elliptical cylinders with coated body.

To obtain the form of the matrix equations in (2) by MoM, the cross section of the reference cylinder is divided in  $N$  number of cells which assume constant  $E_n$  on each cell.

$$\sum_{n=1}^N C_{mn} E_n = E_m^i \quad m = 1, \dots, N \quad (2)$$

The resulting matrix equations can be solved by GMRES method as a suitable iterative solver. The coefficient  $C_{mn}$  is given as follows when  $m \neq n$

$$C_{mn} = j \frac{\pi}{2} [\epsilon_{r0}(n) - 1] k a_n J_1(k a_n) \sum_{l=-\infty}^{\infty} \exp(-j k_x l d) H_0^{(2)}(k \rho_{mn}^l) \quad (3)$$

and when  $m = n$

$$C_{mm} = 1 + j \frac{\pi}{2} [\epsilon_{r0}(n) - 1] \left[ \left\{ k a_n H_1^{(2)}(k a_n) - \frac{2j}{\pi} \right\} + k a_n J_1(k a_n) \sum_{l=-\infty}^{\infty} \exp(-j k_x l d) H_0^{(2)}(k \rho_{mm}^l) \right] \quad (4)$$

Where  $k_x = -k \sin \theta_i$ ,  $k$  is the wavenumber of the free space,  $\theta_i$  the incident angle with respect to the  $y$ -axis,  $H_0^{(2)}(x)$  is the zeroth-order hankel function of the second kind. The distance  $\rho_{mn}^l = \sqrt{(x_m - l d)^2 + (y_m)^2}$  and  $x_m, y_m$  are the centers of the  $m$ -cell with the location of the reference source assume at the origin.  $\sum'$  is the summation except  $l = 0$ . The evaluation of the infinite sum of Hankel functions multiplied by trigonometric angular dependencies in Eqs. (3) and (4) is the most time consuming part in the scattering problem of periodic structures. In order to obtain the summation efficiently with less computational time, the integral forms of the hankel function used in [6] are applied so as to acquire a accurate result for the dense matrix coefficients.

### 3. Numerical Results

We will discuss the power reflection characteristics for the fundamental Floquet mode  $R_0$  whose normalized wavelength ranges from  $(0.75 \leq d/\lambda \leq 1)$ . The cylindrical harmonics of the scatter is truncated to  $|l| = 12$  and this number is closely related to the size of the matrix. We first of all confirm the method presented in this paper by comparing the computational time and the power reflection coefficient for a periodic array of circular cylinders of radius  $0.3d$  and relative permittivity of  $2.0$ . Figure 2 clearly explains the accuracy in using lattice sums method together with MoM to calculate the power reflection coefficient as a function of the normalised wavelength. It is shown that the present result is good agreement with the previous one [2]. MoM can be applied efficiently to the problem of scattering by inhomogeneous dielectric bodies because it conform to various shapes and also takes the advantage of larger matrices size of the scatter. As such, it will be of great significant numerical method when considering scattering from a multilayer structures.

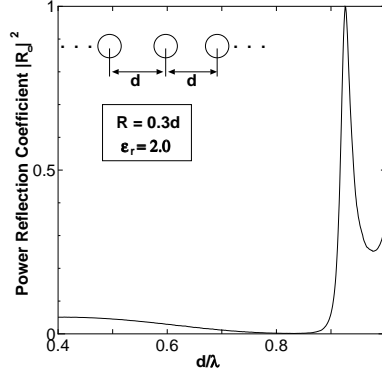


Figure 2: Power reflection coefficient  $|R_0|^2$  vs normalized wavelength  $d/\lambda$  for circular cylinders of radius  $0.3d$  and  $\epsilon_r = 2.0$ .

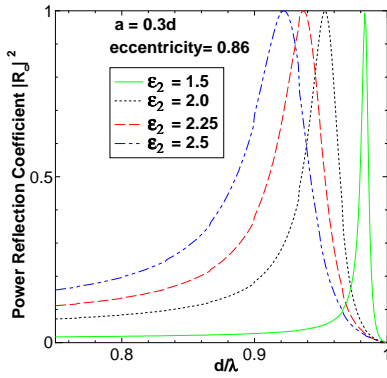


Figure 3: Power reflection coefficient  $|R_0|^2$  vs normalized wavelength  $d/\lambda$  for various  $\epsilon_{r2}$  of the elliptical cylinder of eccentricity 0.86 and major axis of  $0.3d$ .

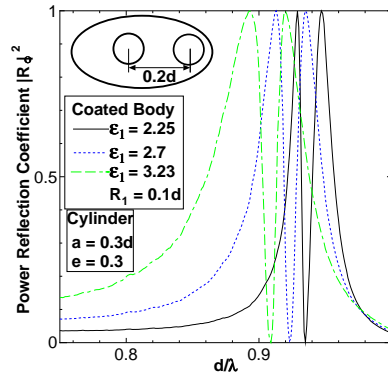


Figure 4: Power reflection coefficient  $|R_0|^2$  vs normalized wavelength  $d/\lambda$  for an elliptical cylinder of eccentricity 0.3 coated with two circular shaped material at the surface.

Figure 3 shows the power reflection coefficients  $|R_0|^2$  as a function of the normalised wavelength  $d/\lambda$  for various  $\epsilon_{r2}$  of the elliptical cylinder that has eccentricity of 0.86 and  $a = 0.3d$ . A single resonance peak is noticeable but the side-band reflectance properties is narrow for the case when  $\epsilon_{r2} = 1.5$  and broaden when  $\epsilon_{r2}$  increases with a depictable lowering of the normalised resonance wavelength. Figure 4 shows the power reflection coefficients  $|R_0|^2$  as a function of the normalised wavelength  $d/\lambda$  where in the elliptical cylinder of eccentricity 0.3 with  $a = 0.3d$  and relative permittivity of 1.5 is coated with two circular dielectric cylinders with radius of  $R_1 = 0.1d$ . One of the body is coated at the center of the cylinder and the other at  $0.2d$  from the center of the elliptical cylinder. We observed two noticeable resonance peaks and there exists the frequency of  $|R_0|^2 = 0$ . The resonance peaks depict the multiple scattering effect between the elliptical cylinder and the coated circular shape bodies. When the  $\epsilon_{r1}$  is increased, the peaks shifted toward a lower  $d/\lambda$  values with broader side-band reflectance properties. Figure 5 shows the total intensity contour map for  $d = 0.902\lambda$ ,  $\epsilon_{r1} = 3.23$ ,  $\epsilon_{r2} = 1.5$ ,  $R_1 = 0.1d$ ,  $d_{12} = 0.2d$  with the eccentricity of the elliptical cylinder of 0.3 and  $a = 0.3d$ . As it is shown in Fig. 5, the total field intensity is very strong between the array on to the point  $y < 0.5\lambda$ . The same intensity is experienced above the array of the near field points between  $0.5\lambda < y > \lambda$  and a propagating plane wave near the point  $y = 2\lambda$  is observed.

#### 4. Conclusion

In this paper, scattering from a periodic array of elliptical cylinders with coated circular shaped dielectric body has been analysed by MoM together with lattice sums technique. The result for a periodic

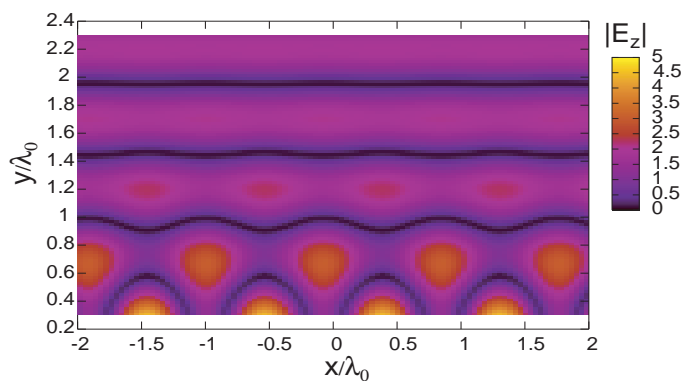


Figure 5: Distribution of the total intensity contour map:  $d = 0.902\lambda$ ,  $\varepsilon_{r1} = 3.23$ ,  $R_1 = 0.1d$  and the eccentricity of the elliptical cylinder is 0.3 with  $a = 0.3d$ .

array of circular shows close agreement with better computational time. This coated material can act as a wavelength selection surface and can be apply in the design of transmitting narrow band filters. We can also conclude that the variation in the relative permittivity of the coated cylinder strongly influence the resonance scattering characteristic, reflection and transmission properties of the periodic structure. Multiple resonance characteristic is noticeable even though the structure is a single layered periodic array of elliptical cylinders. This gives additional degree of freedom for the scattering field. Future work will be the use of Multigrid-Moment Method [9] as an effective iterative solver to the matrix equations which will enhance faster computational time to the numerical solution. Also, we can employ Genetic Algorithm to optimise the periodic structure parameters so as to obtain high or low reflective properties for a given periodic array structure.

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