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Scattering of Electromagnetic Waves by Inhomogeneous Dielectric Gratings Loaded with Perfectly Conducting Strips

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1. Introduction

Recently, the refractive index can easily be controlled to make the periodic structures such as optoelectronic devices, photonic bandgap crystals, frequency selective devices, and other applications by the development of manufacturing technology of optical devices. Thus, the scattering and guiding problems of the inhomogeneous gratings have been considerable interest, and many analytical and numerical methods which are applicable to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials[1]-[4] .

In this paper, we proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings with perfectly conducting strips[10] using the combination of improved Fourier series expansion method[5]-[7] and point matching method[8]-[9].

Numerical results are given for the transmitted scattered characteristics for the case of frequency loaded with three perfectly conducting strips for TE cases. The effects of the inhomogeneous dielectric gratings comparison with that of the homogeneous gratings[11] on the transmitted power are discussed.

2. Method of Analysis

We consider inhomogeneous dielectric gratings loaded with three perfectly conducting strips shown in Fig.1. The grating is uniform in the y-direction and the permittivity $\varepsilon(x,z)$ is an arbitrary periodic function of z with period p. The permeability is assumed to be μ_0. The time dependence is $\exp(-i\omega t)$ and suppressed throughout.

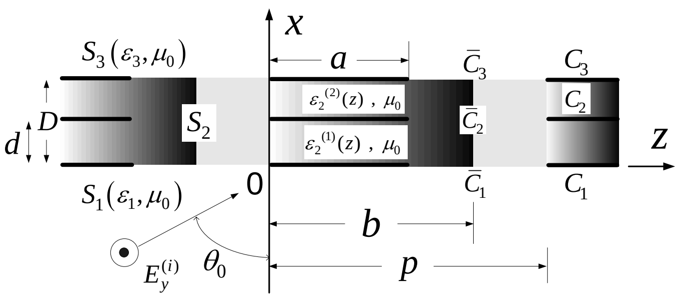

Fig.1 . Structure of inhomogeneous dielectric gratings loaded with three perfectly conducting strips.

In the formulation, the TE wave is discussed. When the TE wave (the electric field has only the y-component) is assumed to be incident from $x>0$ at the angle θ_0,

$$E_y^{(i)} = e^{ik_1(z\sin\theta_0 - x\cos\theta_0)}, \quad k_1 \triangleq \omega\sqrt{\varepsilon_1\mu_0} \tag{1}$$

the electric fields in the regions $S_1\,(x\le 0)$, $S_2\,(0<x<D)$, and $S_3\,(x\ge D)$ are expressed[10] as

$\underline{S_1(x\le 0)\ :}$

$$E_y^{(1)} = E_y^{(i)} + e^{ik_1 z\sin\theta_0}\sum_{n=-N}^{N} b_n^{(1)} e^{i\left(-k_n^{(1)}x + 2\pi nz/p\right)} \tag{2}$$

$\underline{S_2(0<x<D)\ :}$

$$E_y^{(2,1)} = \sum_{\nu=1}^{2N+1}\left[A_\nu^{(1)}e^{ih_\nu^{(1)}x} + B_\nu^{(1)}e^{-ih_\nu^{(1)}(x-d)}\right] f_\nu^{(l)}(z) \ \ ; \ E_y^{(2,2)} = \sum_{\nu=1}^{2N+1}\left[A_\nu^{(2)}e^{ih(2)_\nu(x-d)} + B_\nu^{(2)}e^{-ih_\nu^{(2)}(x-D)}\right] f_\nu^{(1)}(z) \tag{3}$$

$$f_\nu^{(l)}(z) \triangleq e^{ik_1\sin\theta_0 z}\sum_{m=-N}^{N} u_m^{(\nu,l)} e^{i2\pi mz/p} \quad ,l=1,\,2$$

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$\underline{S_3(x \geq D)\ :}$

$$E_y^{(3)} = e^{ik_1 z \sin\theta_0} \sum_{n=-N}^{N} c_n^{(3)} e^{i\left\{k_n^{(3)}(x-D)+2\pi nz/p\right\}} \quad ; H_z^{(j)} = \left\{i\omega\mu_0\right\}^{-1} \partial E_y^{(j)} \Big/ \partial x \ , \ (j=1\sim 3) \tag{4}$$

where λ is the wavelength in free space, he wavelength in free space, $b_n^{(1)}, A_\nu^{(1)}, B_\nu^{(1)}, A_\nu^{(2)}, B_\nu^{(2)}$, and $C_n^{(3)}$ are unknown coefficients to be determined from boundary conditions. $k_n^{(j)}$ $(j=1,3)$ is propagation constants in the x direction, and , $h_\nu^{(k)}$,$u_n^{(\nu,l)}(l=1.2)$,the propagation constant and eigenvectors, are satisfy the following eigenvalue equation in regard to h [5]

$$\mathrm{A}\mathbf{U} = h^2 \mathbf{U} \tag{5}$$

where,

$$\mathbf{U}^{(\nu,l)} \triangleq \left[u_{-N}^{(\nu,l)}, \cdots, u_0^{(\nu,l)}, \cdots u_N^{(\nu,l)}\right]^T, T\ :\ \text{transpose}, \quad \mathbf{A} \triangleq \left[a_{m,n}^{(l)}\right], \quad a_{n,m}^{(l)} \triangleq k_1{}^2 \xi_{n,m}^{(l)} - (2\pi n/p + k_1 \sin\theta_0)^2,$$

$$\xi_{n,m}^{(l)} \triangleq \frac{1}{p}\int_0^p \left\{\frac{\varepsilon_2^{(l)}(z)}{\varepsilon_0}\right\} e^{i2\pi(n-m)z/p} dz \ , m,n = (-N, \cdots, 0, \cdots, N)$$

We obtain the matrix form combination of metallic region C and the dielectric region $\overline{C}$ using boundary condition $Z_j = (j-1)p/[(2N+1)]$; $j = 1 \sim (2N+1)$ at the matching points on $x =$ 0, *and D.* Boundary condition using Point Matching are as follows:

$$Z_j \in C_1 \ ; \left[E_z^{(1)} = 0,\ E_z^{(2,1)} = 0\right]_{x=0} \quad , \quad Z_j \in \overline{C_1} \ ; \left[E_y^{(1)} = E_y^{(2,1)}\right]_{x=0} , \ \left[H_z^{(1)} = H_z^{(2,1)}\right]_{x=0} \tag{6}$$

$$Z_j \in C_3 \ ; \left[E_z^{(2,2)} = 0,\ E_z^{(3)} = 0\right]_{x=D} \quad , \quad Z_j \in \overline{C_3} \ ; \left[E_y^{(2,2)} = E_y^{(3)}\right]_{x=D} , \ \left[H_z^{(2,2)} = H_z^{(3)}\right]_{x=D} \tag{7}$$

In the boundary condition at Eq.(6), and Eq.(7), it is satisfied in all matching points by using the orthogonality properties of $\{e^{i2\pi nz/p}\}$,we get following equation in regard to $A_\nu^{(1)}, B_\nu^{(1)}, A_\nu^{(2)}, and\ B_\nu^{(2)}$

$$\mathbf{Q}_1 \mathbf{A}^{(1)} + \mathbf{Q}_2 \mathbf{B}^{(1)} = \mathbf{F} \quad , \quad \mathbf{Q}_3 \mathbf{A}^{(2)} + \mathbf{Q}_4 \mathbf{B}^{(2)} = \mathbf{0} \tag{8}$$

where, $\mathbf{F} \triangleq \left[0\,(Z_k \in C_1)\ ,2k_0^{(1)}\,(Z_k \in \overline{C_1})\right]^T$

$$\mathbf{A}^{(k)} \triangleq \left[A_1^{(k)},\ A_2^{(k)}, \cdots,\ A_{2N+1}^{(k)}\right]^T, k = 1,2 \qquad \mathbf{B}^{(k)} \triangleq \left[B_1^{(k)},\ B_2^{(k)}, \cdots,\ B_{2N+1}^{(k)}\right]^T, k = 1,2$$

$$\mathbf{Q}_1 \triangleq (\mathbf{H}_2^{(1)} + \mathbf{H}_1^{(1)}\mathbf{D}^{(1)}) \ , \mathbf{Q}_2 \triangleq (\mathbf{H}_2^{(1)} - \mathbf{H}_1^{(1)}\mathbf{D}^{(1)}) \ , \mathbf{Q}_3 \triangleq (\mathbf{H}_2^{(2)}\mathbf{D}^{(2)} - \mathbf{H}_3^{(2)}), \ \mathbf{Q}_4 \triangleq (\mathbf{H}_2^{(2)}\mathbf{D}^{(2)} + \mathbf{H}_3^{(2)})$$

$$n = (-N, \cdots, 0\ , \cdots, N)\ , \nu = 1 \sim (2N_f + 1)$$

$$\mathbf{H}_1^{(l)} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ e^{-iNz_j} & \cdots & e^{i0z_j} & \cdots & e^{iNz_j} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} Z_j \in C_1 \\ \left.\right\} Z_j \in \overline{C_1} \end{matrix} \bullet \mathbf{U}^{(l)} \ , \ \mathbf{H}_2^{(l)} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & & 0 & & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{-N}^{(l)} e^{-iNz_j} & & k_0^{(l)} e^{i0z_j} & & k_N^{(l)} e^{iNz_j} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} Z_j \in C_1 \\ \left.\right\} Z_j \in \overline{C_1} \end{matrix} \bullet \mathbf{U}^{(l)}$$

$$\mathbf{H}_3^{(l)} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e^{-iNz_j} & \cdots & e^{i0z_j} & \cdots & e^{iNz_l} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} z_j \in c \\ \left.\right\} z_j \in \overline{c} \end{matrix} \bullet \mathbf{U}^{(l)} \quad , \ \mathbf{U}^{(l)} \triangleq \left[u_n^{(\nu,l)}\right], \ \mathbf{D}^{(l)} \triangleq \left[\eta_1 e^{i\eta_2 h_\nu^{(2,l)}\eta_3} \cdot \delta_{(n+N+1),\nu}\right], \quad l = 1,2$$

$$\eta_1 \triangleq \begin{cases} 1 & ;\ Z_j \in C \\ h_\nu^{(2,l)}; & Z_j \in \overline{C} \end{cases} \ \eta_2 \triangleq \begin{cases} 0\ ;\ Z_j \in C \\ 1\ ;\ Z_j \in \overline{C} \end{cases} , \eta_3 \triangleq \begin{cases} d\ ;\ l = 1 \\ D - d\ ;\ l = 2 \end{cases}$$

Boundary condition on $x = d$ are as follows:

$$Z_j \in C_2 \ ; \left[E_y^{(2,1)} = 0,\ E_y^{(2,2)} = 0\right]_{x=d} \quad , \quad Z_j \in \overline{C_2} \ ; \left[E_y^{(2,1)} = E_y^{(2,2)}\right]_{x=d} , \ \left[H_z^{(2,1)} = H_z^{(2,2)}\right]_{x=d} \tag{9}$$

In the boundary condition at Eq.(9), it is satisfied in all matching points by using the orthogonality properties of $\{e^{i2\pi nz/p}\}$,we get following equation in $A_\nu^{(1)}, B_\nu^{(1)}, A_\nu^{(2)}, and\ B_\nu^{(2)}$

$$\mathbf{R}_1\mathbf{A}^{(1)} + \mathbf{R}_2\mathbf{B}^{(1)} = \mathbf{R}_3\mathbf{A}^{(2)} + \mathbf{R}_4\mathbf{B}^{(2)} \tag{10}$$

where, $\mathbf{R}_1 \triangleq \mathbf{U}^{(1)}\mathbf{D}'^{(1)}$, $\mathbf{R}_2 \triangleq \mathbf{U}^{(1)}$, $\mathbf{R}_3 \triangleq \mathbf{U}^{(2)}$, $\mathbf{R}_4 \triangleq \mathbf{U}^{(2)}\mathbf{D}'^{(2)}$

We get following matrix form combined for the matching points at $z_i = (i-1)z/(2N+1)$; $i = 1, 2N+1$.

$$\mathbf{S}_1\mathbf{A}^{(1)} + \mathbf{S}_2\mathbf{B}^{(1)} = \mathbf{S}_3\mathbf{A}^{(2)} + \mathbf{S}_4\mathbf{B}^{(2)} \tag{11}$$

where,

$\mathbf{D}'^{(1)} \triangleq [e^{ih_\nu^{(2,1)}d} \bullet \delta_{(n+N+1),\nu}]$, $\mathbf{D}'^{(2)} \triangleq [e^{ih_\nu^{(2,2)}(D-d)} \bullet \delta_{(n+N+1),\nu}]$, $\mathbf{S}_1 \triangleq (\mathbf{H}_1^{(1)}\mathbf{D}'^{(1)} + \mathbf{H}_3^{(1)}\mathbf{D}''^{(1)})$, $\mathbf{S}_2 \triangleq (\mathbf{H}_1^{(1)} - \mathbf{H}_1^{(1)}\mathbf{D}'''^{(2)})$

$\mathbf{S}_3 \triangleq \mathbf{H}_3^{(2)}\mathbf{D}''^{(2)}$, $\mathbf{S}_4 \triangleq -\mathbf{H}_3^{(2)}\mathbf{D}'''^{(2)})$, $\mathbf{D}''^{(1)} \triangleq [h_\nu^{(2,1)} \bullet \delta_{(n+N+1),\nu}]$, $\mathbf{D}'''^{(1)} \triangleq [h_\nu^{(2,1)} e^{ih_\nu^{(2,1)}d} \bullet \delta_{(n+N+1),\nu}]$

$\mathbf{D}''^{(2)} \triangleq [h_\nu^{(2,2)} \bullet \delta_{(n+N+1),\nu}]$, $\mathbf{D}'''^{(2)} \triangleq [h_\nu^{(2,2)} e^{ih_\nu^{(2,2)}(D-d)} \bullet \delta_{(n+N+1),\nu}]$

By using matrix relationship between $\mathbf{A}^{(1)}, \mathbf{B}^{(1)}$ $\mathbf{A}^{(2)}, \mathbf{B}^{(2)}$, we get the following homogeneous matrix equation in regard to $A_\nu^{(2)}$ $(\nu = 1 \sim 2N+1)$.

$$\mathbf{W} \cdot \mathbf{A}^{(2)} = \mathbf{F} \tag{12}$$

where、$\mathbf{W} \triangleq \left[\mathbf{Q}_1\mathbf{P}_1 + \mathbf{Q}_2\mathbf{P}_3 - (\mathbf{Q}_1\mathbf{P}_2 + \mathbf{Q}_2\mathbf{P}_4)\mathbf{Q}_4^{-1}\mathbf{Q}_3\right]$

$\mathbf{P}_1 \triangleq (\mathbf{R}_2^{-1}\mathbf{R}_1 - \mathbf{S}_2^{-1}\mathbf{S}_1)^{-1}(\mathbf{R}_2^{-1}\mathbf{R}_3 - \mathbf{S}_2^{-1}\mathbf{S}_3)$, $\mathbf{P}_2 \triangleq (\mathbf{R}_2^{-1}\mathbf{R}_1 - \mathbf{S}_2^{-1}\mathbf{S}_1)^{-1}(\mathbf{R}_2^{-1}\mathbf{R}_4 - \mathbf{S}_2^{-1}\mathbf{S}_4)$,

$\mathbf{P}_3 \triangleq (\mathbf{R}_1^{-1}\mathbf{R}_2 - \mathbf{S}_1^{-1}\mathbf{S}_2)^{-1}(\mathbf{R}_1^{-1}\mathbf{R}_3 - \mathbf{S}_1^{-1}\mathbf{S}_3)$, $\mathbf{P}_4 \triangleq (\mathbf{R}_1^{-1}\mathbf{R}_2 - \mathbf{S}_1^{-1}\mathbf{S}_2)^{-1}(\mathbf{R}_1^{-1}\mathbf{R}_4 - \mathbf{S}_1^{-1}\mathbf{S}_4)$

The mode power transmission coefficients ρ_t is given by

$$\rho_t \triangleq \sum_{n=-N}^{N} \mathrm{Re}\left[k_n^{(3)}\right] |c_n^{(3)}|^2 ,$$

$$\text{where、} \; C_n \triangleq \sum_{n=-N}^{N} \left[A_\nu^{(2)} e^{ih_\nu^{(1)}(D-d)} + B_\nu^{(2)}\right] U_n^{(\nu)} . \tag{13}$$

3. Numerical Analysis

We consider the following profiles of inhomogeneous dielectric gratings:

$$\varepsilon_2^{(1)}(x,z) = \varepsilon_2^{(2)}(x,z) \triangleq \begin{cases} \varepsilon_1 + (\varepsilon_d - \varepsilon_1)z/b : (0 \le z \le b) \\ \varepsilon_a : (b < z < p) \end{cases} \tag{14}$$

The values of parameters chosen are $\varepsilon_1 = \varepsilon_3 = \varepsilon_0$, $a/p = 0.5, \varepsilon_d/\varepsilon_0 = 2.0$, $\theta_0 = 30^\circ$, $\varepsilon_d/\varepsilon_0 = 2.0$, $\varepsilon_a/\varepsilon_0 = 1.0$ and $D/p = 0.3$ .The relative error are less than about 0.1% and the energy error is less than about 10^{-3} for TE waves when we computed with $N = 15$ at $a/b = 1$ and $p/\lambda = 1.5$.

First, we consider the two strip gratings for the case of $d/p = 0$ .Figures 2 shows ρ_t for various values of normalized frequency (p/λ) for $\varepsilon_1/\varepsilon_0 = 1.0$ and 2.0 at $a/b = 1$. From in Figs.2, the maximum of coupling resonance at $p/\lambda < 1.5$ moves toward smaller (p/λ) as $\varepsilon_1/\varepsilon_0$ increases, and the effect of the d/p is more significant at $1.2 < p/\lambda > 1.7$.

Figures 3 shows ρ_t for various values of normalized frequency (p/λ) with the case of $b/p = 0.75$ for the same parameters as in Fig.2. We note that the characteristic tendencies for the effect of the b/p are approximately same at $p/\lambda < 1.0$, but for about $1.0 < p/\lambda > 2.5$, the effect of the inhomogeneous dielectric gratings is more significant comparing with $\varepsilon_1/\varepsilon_0 = 1$.

Figures 4 shows ρ_t for the various values of normalized frequency (p/λ) with the case of $\varepsilon_1/\varepsilon_0 = \varepsilon_d/\varepsilon_0 = 1.5$ for the same parameters Fig.3. We note that the characteristic tendencies for the effect of the equivalent permittivity are approximately same.

Next, we consider the three strip gratings for the case of $d/p = 0.15$.

Figures 5 shows ρ_t for various values of normalized frequency (p/λ) with the same parameters as in Fig.4. We note that the characteristic tendencies for the effect of the d/p are approximately same, but the effect of the d/p and b/p is more significant for the maximum of coupling resonance at $p/\lambda \approx 1.9$ comparing with the homogeneous case.

4. Conclusion

In this paper, we have proposed a new method for the scattering of electromagnetic waves by inhomogeneous

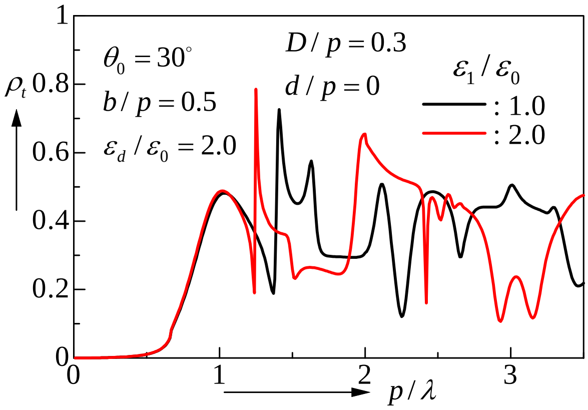

Figure 2. ρ_t vs. p/λ for the case of $b/p = 0.5$

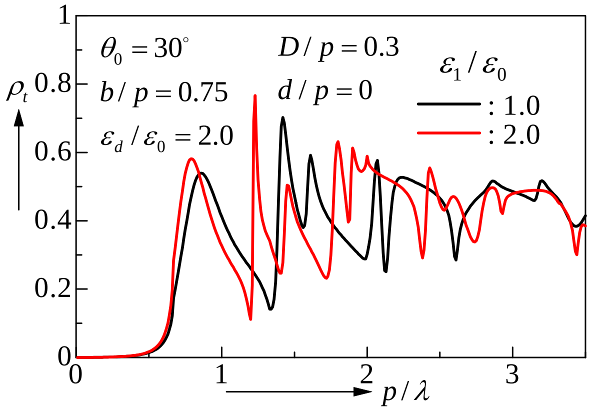

Figure 3. ρ_t vs. p/λ for the case of $b/p = 0.75$

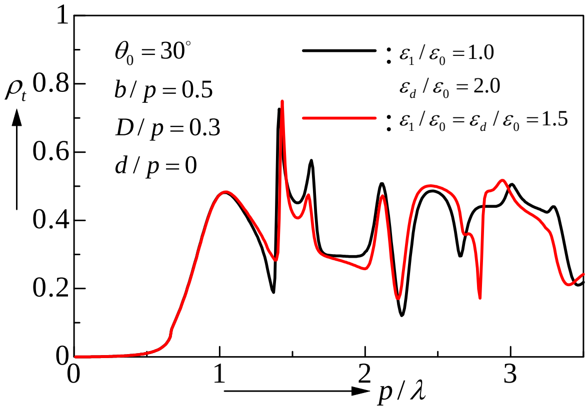

Figure 4. Comparison with homogeneous gratings.

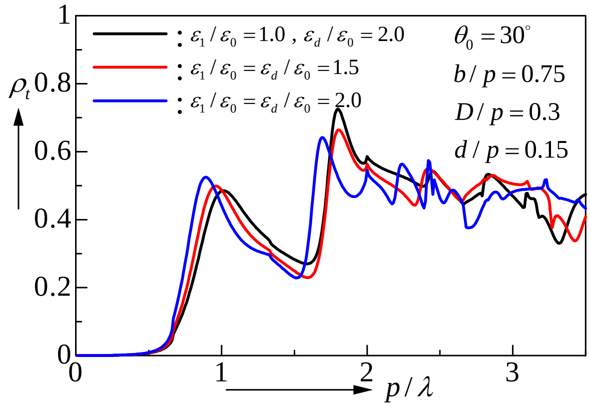

Figure 5. ρ_t vs. p/λ for the case of $d/p = 0.15$

dielectric gratings loaded with perfectly conducting strips using the combination of improved Fourier series expansion method and point matching method.

Numerical results are given for the transmitted scattered characteristics for the case of frequency loaded with the three perfectly conducting strips for TE cases. The effects of the inhomogeneous dielectric gratings comparison with that of the homogeneous case on the transmitted power are discussed.

This method also can be applied to the inhomogeneous dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials.

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