REMOTELY SENSED HEIGHT PROFILES OF THE TROPOSPHERIC REFRACTIVE INDEX

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1. Interpretation of the remote sensing (RS) data concerning atmospheric parameters (specifically, the height profile $N(z)$ and structure constant of the refractive index $C_n^2(z)$) requires solution of an ill-posed inverse problem. The incorrectness arises from the fact that small perturbations in the input RS data may give rise to considerable errors in the parameters restored. The input data may be represented by measured variations in the propagation time or Doppler frequency shift, due to refraction, of electromagnetic waves arriving from different elevation angles. The possibility of practical implementation of the regularization approaches to the treatment of RS data concerning height profiles of the atmospheric refractive index was shown in papers [1, 2, 4]. However, neither of the cited papers treating RS data presented a due analysis of the effect upon the quality of inversion of the inadequate mathematical formalization of the equations relating the input and the output data. This is the reason why we suggest in the present paper an analysis of the interpretation inaccuracies of RS data in the presence of distortions in functional relations and of a degrading background like additive noise.

2. The operator form of the formal RS equation can be written as

$$g = Af + n \quad , \tag{1}$$

where g is the input data vector, f - the refractive index height profile which is sought for (in other cases, the structure constant profile), A - generally is an integral operator, and n - the additive noise distorting the input data. In the Bayes approach, the estimate for the unknown value is [4]

$$\hat{f} = (C_f^{-1} + A^+ C_n^{-1} A)^{-1} A^+ C_n^{-1} g \ , \tag{2}$$

where C_f and C_n are correlation operators of the sought function f and noise n , respectively, and the "plus" superscript denotes conjugation. The accuracy of this estimate depends primarily on the adequacy of the model adopted for the RS data, i.e. on the amount of distortion introduced in the operator A, and on the quality of the a priori information (i.e. errors in the correlation operators C_f and C_n). To analyse the effect of these factors on the accuracy of $\hat{f}$, it is convenient to represent (2) in the spectral form, namely

$$\hat{f} = \sum_k \lambda_k^{-1} (A^+ C_n^{-1} g, \varphi_k)\, \varphi_k \quad , \tag{3}$$

where λ_k and φ_k denote eigenvalues and eigenfunctions, respectively, of the operator

$$M = C_f^{-1} + A^{+} C_n^{-1} A \ .$$

The transformation operator A of RS problems may be distorted either in the additive or multiplicative mode, i.e.

$$A = A_0 + \delta A \qquad \text{or} \qquad A = A_0 (1 + \delta A) \ .$$

To evaluate the limitations to the accuracy of $\hat{f}$ owing to these factors, we shall appeal to the perturbation theory for self-adjoint operators.
In terms of the values introduced and with the use of the notation, the general solution to equation (1) takes the form

$$\hat{f}_\Delta = \sum_k \lambda_k^{-1} \left(1 - \frac{\Delta \lambda_k}{\lambda_k}\right) \Big\{ [A^{+} C_n^{-1} g, \varphi_k] \varphi_k + \\ + [\delta A^{+} C_n^{-1} g, \varphi_k] \varphi_k + [A^{+} C_n^{-1} g, \Delta \varphi_k] \varphi_k + [A^{+} C_n^{-1} g, \varphi_k] \Delta \varphi_k \Big\} \ . \tag{4}$$

The difference between this estimate and the true value of the parameter restored can be easily determined with the aid of (3) and (4).

The total error δf of equation has been minimized by the proper choice of the filter transfer function, namely

$$\tilde{W}(y_k) = [B(y_k) + \tilde{K}(y_k)]^2 / \{ [B(y_k) + \tilde{K}(y_k)]^2 + \Omega_{\Delta M}(y_k) \} \ . \tag{5}$$

By analysing statistical characteristics of the errors in the operator A we have obtained

$$\Omega_{\Delta M}(y_k) = \frac{4}{N_0} \tilde{\Psi}(y_k) \, \Omega_{\delta A}(y_k) \quad ,$$

where we have used that $C_n = 0.5 \cdot \mathrm{No} \cdot I$ (No is the power density of white noise and I denotes the unit matrix).
The gain resulting from application of the optimum filtration can be evaluated as a ratio of the mean square variations $\langle \delta f_\Sigma^2 \rangle$ and $\langle \delta f_{opt}^2 \rangle$. As shown by the quality analysis of the extra filtration procedure, additive distortions of the operator A at such frequencies that $\mu^{-1} = \mathrm{No}/2N_f \ll 1$ are equivalent to the presence of extra noise, along with the white additive noise of spectral density $2 N_f \Omega_{\delta A}(y_k)$. Meanwhile, multiplicative distortions are not taken into account in the inversion of the operator A being equivalent to an additional error $2 N_f \Omega_{\delta A}(y_k)$ in each spectral component

of the field $\hat{f}(y_k)$.

3. The basic equation for restoring the tropospheric refractive index profile from the radio data $u(\vartheta)$ is [1, 4]

$$u(\vartheta) = \int_{z_1}^{z_2} K(z,\vartheta)\, N(z)\, dz \quad . \tag{6}$$

This equation is an approximate equation as it does not allow either for curvature of the wave trajectory or effect of the turbulent component $\Delta N(z)$ of the refractive index. Therefore, the formalized inverse problem of RS should be formulated with an account of distortions in the operator and input data, $u = A_u N + \varepsilon$.

A similar equation relates the height-dependent structure constant $C_n^2(z)$ to statistical field characteristics in the turbulent troposphere, viz. $B = A_B C_n^2 + \varepsilon$, where $B(\vec{r}, t)$ is the space and time covariance function of amplitude or phase fluctuations. The operator A_B is distorted if the initial equation is not exact, e.g. the equation governing the complex phase in the first Rytov approximation. The quality of restoration of the refractive index and the structure constant of its fluctuations in the technique suggested, in particular the expected improvement of the $\hat{N}$ and $\hat{C}_n^2$ estimates as a result of the extra filtering have been tested in numerical experiments. The test profiles of the refractive index $N(z)$ and C_n^2 were taken to be exponential. The measure of the error in the operator specification was

$$\delta_A = \left[\left(\sum_{i=1}^{m} \sum_{j=1}^{n} \eta_{ij}^2 \right) \frac{h_m}{h_n} \right]^{1/2} \quad , \tag{7}$$

where η_{ij} was a uniformly distributed random value representing distortions in A_u and A_B , and h_m and h_n were steps of the uniform grids.

The a priori information was represented by the correlation functions

$$C_{f ij} = \frac{\gamma}{\rho\sqrt{2\pi}} \exp\left(-\frac{|z_i - z_j|}{2\rho}\right)$$

and

$$C_{\varepsilon ij} = \sigma_\varepsilon^2\, \delta_{ij} \quad .$$

Parameters γ, ρ and σ_ε^2 were chosen to match the known literature data on the refractive index and structure characteristic, respectively [3-5]. The estimates of $\hat{N}$ were obtained from the observational input data corresponding to elevation angles ϑ = 83 to 88°. The maximum space diversity between the reception sites for composing the correlation function (necessary to estimate C_n^2) was several Fresnel zone radii.

Thus, a theoretical analysis based on the Bayes approach has yielded practical algorithms for forming parameter estimates and evaluating the estimate accuracy in the presence of uncertainties in the functional operators, input data and a priori information. An extra filtering procedure has been suggested to improve the estimates. The maximum criterion applied to the total error of the estimate has allowed the frequency characteristic of the filter to be derived. Numerical experiments confirm the efficiency of the suggested approach.

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