

THE IONOSPHERIC IRREGULARITIES AND  
THE REFLECTION COEFFICIENT OF RADIO WAVE

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INTRODUCTION

The reflection coefficient or absorption index of radio wave in the ionosphere has been paid great attention by scientists in the society of radio wave propagation owing to the inhomogeneous structure and random variation of the propagation medium, which results in radio fading with a wide frequency extent and various time scales and causes some difficulties in the accurate calculation of the field intensity and reflection of radio wave. Ionosonde generally uses the method of pulse reflection which means that both receiver and transmitter are on the same side of propagation screen (the ionosphere). Therefore, the structure, especially the fine structure of the reflection screen is very important for radio echo. It is because of this, the present paper devotes to the research of relationship between the ionospheric irregularities and the reflection coefficient of short radio wave.

THEORETICAL CONSIDERATION

The ionosphere contains a great deal of irregularities having various scales. The small-scale irregularities may be divided into two types — the characteristic dimension  $2\pi a$  of one type is greater than wavelength  $\lambda$ ; the characteristic dimension  $2\pi a$  of the other is less than wavelength  $\lambda$ . These two types of the small-scale irregularities possess different effects on radio wave. The first small-scale irregularities (with the characteristic dimension  $>$  wavelength) cause radio wave forward scattering and have sharp reradiation pattern; and the second small-scale irregularities (with less characteristic dimension) result in radio wave backscattering and have obtuse reradiation. Thus the first can be called the sharp scatterer and sharp scattering; and the second the diffuse scatterer and diffuse scattering. Obviously, the diffuse scatterers in the reflection screen of the ionosphere are much more than the sharp scatterers. Therefore, the composite field intensity received at the ground ionosonde near transmitter for the first echo should be (Ataev, 1957)

$$E_1 = C\rho \left[ \alpha \frac{e^{i(\omega t - k2h)}}{2h} + \sum_1 \alpha_1 \frac{e^{i(\omega t + \Omega_1 t - k2h_1)}}{2h_1} + \sum_p \alpha_p \frac{e^{i(\omega t + \Omega_p t - k2h_p)}}{h_p \cdot h_p} \right], (1)$$

where C is a transmitting constant,  
 $\rho$  is amplitude reflection coefficient,  
 $\alpha$  is the assignment coefficient of effective reflection by regular layer,  $\alpha_1$  is that by first type of irregularities, and  $\alpha_p$  that by second type of irregularities,

$\omega$  is probing frequency multiplied by  $2\pi$ ,  
 $t$  is the time,  
 $\Omega_1$  is Doppler shift by the first type of small-scale irregularities, and  $\Omega_p$  that by the second,  
 $k$  is wave number,  
 $h$  is the height of reflection,  $h_1$  is the distance (or height) from the first type of irregularities to the ionosphere, and  $h_p$  that from the second.

Hereafter subscript 1, m, and n are used for the first type of irregularities, and p, q and r for the second. Expression (1) represents a sum of three kinds of reflection and scattering from the ionosphere. Owing to very low values of Doppler shift, we can omit  $i\Omega_1 t$  and  $i\Omega_p t$  which will not effect the statistic characteristics of the results. The harmonic factor  $i\omega t$  can also be omitted. Because of instantaneous invariability of the ionosphere between the first reflection and the second reflection of radio wave, the second echo can be derived as

$$\begin{aligned}
 E_2 = C\rho^2 & \left[ \alpha^2 \frac{e^{-ik4h}}{4h} + \sum_1 \alpha_1 \alpha \frac{e^{-ik(2h_1+2h)}}{2h_1+2h} + \sum_p \alpha_p \alpha \frac{e^{-ik(2h_p+2h)}}{h_p(h_p+2h)} + \right. \\
 & + \sum_m \alpha \alpha_m \frac{e^{-ik(2h+2h_m)}}{2h+2h_m} + \sum_m \sum_1 \alpha_1 \alpha_m \frac{e^{-ik(2h_1+2h_m)}}{2h_1+2h_m} + \\
 & + \sum_m \sum_p \alpha_p \alpha_m \frac{e^{-ik(2h_p+2h_m)}}{h_p(h_p+2h_m)} + \sum_q \alpha \alpha_q \frac{e^{-ik(2h+2h_q)}}{(2h+h_q)h_q} + \\
 & \left. + \sum_q \sum_1 \alpha_1 \alpha_q \frac{e^{-ik(2h_1+2h_q)}}{(2h_1+h_q)h_q} + \sum_q \sum_p \alpha_p \alpha_q \frac{e^{-ik(2h_p+2h_q)}}{h_p(h_p+h_q)h_q} \right]. \quad (2)
 \end{aligned}$$

It must be pointed out that some terms should have slightly different form; for example, more correct writing in the exponential of the fifth term is  $-ik(h_1+h_1'+h_m'+h_m)$ , because the elementary wave from the first scattering on the ionospheric irregularities is not definitely reaching the origin point (the transmitting and receiving antennas). Thus  $h_1'$  and  $h_m'$  have, generally speaking, very little differences from  $h_1$  and  $h_m$ . However, this will neither influence the statistic nature of homogeneous distribution in the elementary wave phases, nor the calculation concerning the even moments of the different order echo amplitudes, i. e. the mean even square(s) of the amplitudes.

Similarly, we can arrive at a field expression for the third echo.

If one uses  $x$  for  $\alpha e^{-ik2h}$ ,  $y$  for  $\sum_1 \alpha_1 e^{-ik2h_1}$  etc., and  $z$  for  $\sum_q \alpha_q e^{-ik2h_p}$  etc., where  $\alpha_p = \alpha_p/h_p \doteq \alpha_p/h$ , if the symbol  $A_0$  is used for  $C/h$  which means the amplitude of radio wave when it first reaches the ionospheric screen, as a result, the expressions (1) and (2) and field of the third reflection can be simplified as

$$E_1 = \frac{1}{2} A_0 \rho (x+y+2z) \quad (3a)$$

$$E_2 = \frac{1}{4}A_0\rho^2(x^2+y^2+2z^2+2xy+\frac{8}{3}xz+\frac{8}{3}yz),$$

$$\text{and } E_3 = \frac{1}{6}A_0\rho^3(x^3+y^3+\frac{3}{2}z^3+3x^2y+\frac{46}{15}x^2z+3xy^2+\frac{7}{2}xz^2+\frac{92}{15}xyz+\frac{46}{15}y^2z+\frac{7}{2}yz^2) \quad (3c)$$

respectively. Using principles of statistic mathematics and introducing parameters

$$\xi = \frac{\sum_1 \alpha_1^2 / \alpha^2}{\beta_1^2} = \frac{1}{\beta_1^2} \quad \text{and} \quad \eta = \frac{4 \sum_p \gamma_p^2 / \alpha^2}{\beta_2^2} = \frac{1}{\beta_2^2}, \quad (4)$$

we can obtain that

$$\overline{A_1^2} = \frac{1}{4}A_0^2\rho^2\alpha^2(1+\xi+\eta), \quad (5a)$$

$$\overline{A_1^4} = \frac{1}{16}A_0^4\rho^4\alpha^4[2(1+\xi+\eta)^2-1], \quad (5b)$$

$$\overline{A_1^6} = \frac{1}{64}A_0^6\rho^6\alpha^6[6(1+\xi+\eta)^3-(1+\xi+\eta)+4], \quad (5c)$$

$$\overline{A_2^2} = \frac{1}{16}A_0^2\rho^4\alpha^4(1+4\xi+2\xi^2+\frac{16}{9}\eta+\frac{1}{2}\eta^2+\frac{16}{9}\xi\eta), \quad (5d)$$

etc.

$\beta_1$  and  $\beta_2$  in expression (4) are the ratios of signal to noise caused by the sharp scatterers and the diffuse scatterers respectively. The composite signal-to-noise ratio contributed by two types of irregularities,  $\beta$ , is

$$1/\beta^2 = \xi + \eta = 1/\beta_1^2 + 1/\beta_2^2. \quad (6)$$

From expressions (5) including expression about  $\overline{A_2^4}$ , we arrive at formulae

$$\overline{A_1^4}/\overline{A_1^2}^2 = 2 - \frac{1}{(1+\xi+\eta)^2} = \phi_1(\beta_1, \beta_2) \quad (7a)$$

$$\text{and} \quad \overline{A_2^4}/\overline{A_2^2}^2 = f_1/f_2 = \phi_2(\beta_1, \beta_2), \quad (7b)$$

where

$$f_1 = 1 + 16\xi + 72\xi^2 + 96\xi^3 + 24\xi^4 + 7.11\eta + 15.43\eta^2 + 10.67\eta^3 + 1.5\eta^4 + 64\xi\eta + 61.73\xi\eta^2 + 10.67\xi\eta^3 + 128\xi^2\eta + 30.86\xi^2\eta^2 + 42.67\xi^3\eta \quad (3a)$$

and

$$f_2 = 1 + 8\xi + 20\xi^2 + 16\xi^3 + 4\xi^4 + 3.56\eta + 4.16\eta^2 + 1.78\eta^3 + 0.25\eta^4 + 17.78\xi\eta + 10.32\xi\eta^2 + 1.78\xi\eta^3 + 21.33\xi^2\eta + 5.16\xi^2\eta^2 + 7.11\xi^3\eta. \quad (3b)$$

Expressions (7) are irrelative to the reflection coefficient but are influenced by the irregularities as shown in Fig. 1.

Finally, from (5a) and (5b) we obtain a formula of the reflection coefficient as

$$\rho = 2(\overline{A_2^2}/\overline{A_1^2})^{\frac{1}{2}} \cdot 1/\psi^{\frac{1}{2}}, \quad (9)$$

where

$$\psi = (1 + 4\xi + 2\xi^2 + \frac{16}{9}\eta + \frac{1}{2}\eta^2 + \frac{16}{9}\xi\eta) / (1 + \xi + \eta)^2 \quad (10)$$

which shows the relationship with the ionospheric irregularities.

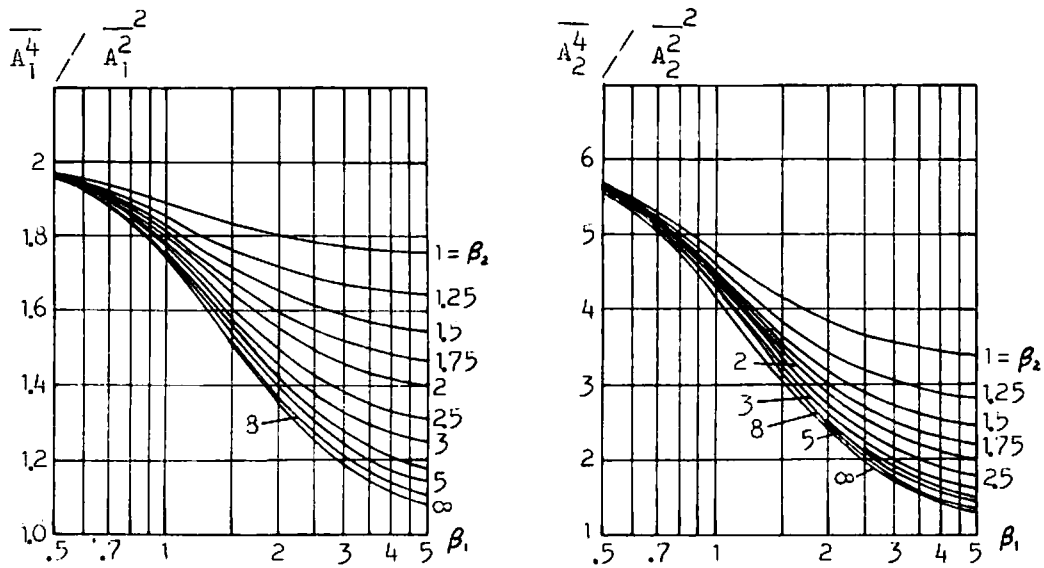


Fig. 1. The dependence of  $\frac{A_1^4}{A_1^2}$  and  $\frac{A_2^4}{A_2^2}$  on  $\beta_1$  and  $\beta_2$ .

Expressions (7a, 7b, 8a, 8b, 9 and 10) show that the first echo itself, the second echo itself, the reflection coefficient of radio wave on the ionosphere and its correction factor are all influenced by both the sharp-direction scatterers and the low-direction scatterers. One has to consider two types of the ionospheric irregularities to obtain an accurate expression of the reflection coefficient characterizing the true absorption of radio wave in the irregular ionosphere. As for the first echo expression (7a) and the second echo expressions (7b, 8a and 8b), it can be seen from Fig. 1 that when there is no the diffuse scattering of the low-direction scatterers, i. e.  $\beta_2 \rightarrow \infty$ ; when  $\beta_1$  increases, curves on the left side and on the right side approach unit in ordinates which mean that the effect of the sharp-direction scattering of the first irregularities weakens or disappears. When  $\beta_2$  has other values, the curves on both sides approach

$$\frac{A_1^4}{A_1^2} = 2 - \frac{1}{(1+\eta)^2} \quad (11)$$

and

$$\frac{A_2^4}{A_2^2} = \frac{1+7.11\eta+15.43\eta^2+10.67\eta^3+1.5\eta^4}{1+3.56\eta+4.16\eta^2+1.78\eta^3+0.25\eta^4} \quad (12)$$

When  $\beta_1$  decreases, two groups of curves gathers gradually. At last, whatever  $\beta_2$  is, two groups tend to 2 and 6 respectively, which indicates more obvious influence of the ionospheric irregularities (Al'pert, 1973). Similarly,  $\rho$  is also influenced by  $\beta_1$  and  $\beta_2$  (Huang and Lung, 1985).

#### REFERENCES

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