A SIMULATION OF ESTIMATING THE ELECTRON DENSITY PROFILE OF THE LOWER IONOSPHERE BY VLF WAVES (IN THE CASE OF AN UNKNOWN INCIDENT ANGLE AND RELATIVE PHASE DATA)

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1. Introduction

The methods of remote sensing of the lower ionospheric electron density profile are classified into three groups as follows; (1) the method by Rocket measurement[1,2,3], (2) ground observation by short wave[4,5] or VHF wave[6], (3) ground observation of VLF wave[7]. The method (1) is restricted in time and does not provide continuous profile, (2) has a problem as high power transmission equipment is needed. Deek's method[7] of (3) requires experience and is not systematic. We reported one method[8] and its improved version[9] of estimating the electron density profile of the lower ionosphere from the knowledge of the absolute phase data of VLF reflection coefficients with a given incident angle. In this paper, we treated the case of the relative phase data of the receiving antenna output in the case of an unknown incident angle. An example of this simulation shows a good result. The method described here is applicable not only to the electron density profile of the lower ionosphere but also to the plasma diagnosis.

2. A method of estimating the electron density profile by using the receiving antenna output

In the previous paper[8,9], we reported a method of estimating the electron density profile from the given incident angle and the amplitude and the absolute phase values of the ionospheric reflection coefficient data. At first, the technique to obtain the incident angle for the propagation condition of the given transmitting and receiving sites and the given electron density profile is described, followed by a method of estimating the electron density profile by using the relative phase data of the receiving antenna output. In order to simplify the discussion, we have taken into consideration the VLF wave propagation without the earth, the wave of which is reflected only once from the ionosphere and is received with both loop antennas parallel and perpendicular to the incident plane.

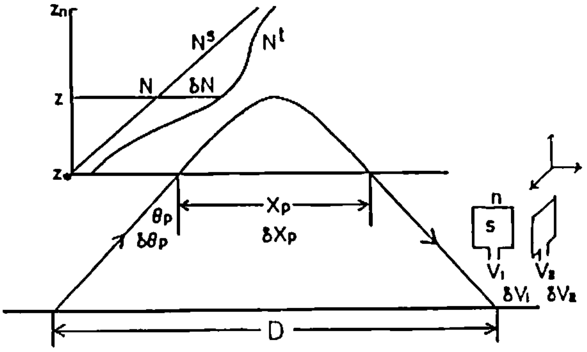

Fig.1 The geometry of the ionospheric propagation model. V_p ($p=1,2$) is the output of the loop antenna and δV_p, $\delta\theta_p$ and δX_p are the perturbation of V_p, θ_p and X_p respectively by $\delta N(z)$ over whole layer.

2.1 The incident angle for the ionosphere and the receiving antenna output

The incident angle of the plane wave for the given transmitting and receiving sites, electron density profile N(z) and collision frequency profile $\nu(z)$ is obtained by solving eq.(1),

$$X_p = -\frac{1}{k\cos\theta}\frac{d\phi_p}{d\theta}\Big|_{\theta_p}^{[10]} = D - 2Z_o\tan\theta_p \tag{1}$$

where Xp is the distance from the incident to the exit points at the bottom (altitude z_0) of the ionosphere. (suffix p=1 and p=2 indicate the electric field strength polarizations parallel and perpendicular to the incident plane of the reflected wave respectively.) K and Φp are the wave number in free space and the phase value of the reflection coefficient Rp respectively. By using this incident angle θp, a receiving loop antenna output Vp is expressed by the following equation for the normalized unit incident electric field strength,

$$\begin{aligned} V_p &= -jknS(\cos\theta)^{p-1}\cdot R_p\cdot\exp[-jk(D\sin\theta+2z_o\cos\theta)]\Big|_{\theta_p} \\ &= A_p R_p(\theta)\Big|_{\theta_p} \end{aligned} \tag{2}$$

where n and S are the turn number and the area of the loop antenna respectively.

2.2 A method of estimating the electron density profile by using the relative phase data of the receiving antenna output

When N(z) at an arbitrary altitude z in the ionosphere is increased by $\delta N(z)$, the values of Vp,Ap and Rp have the perturbations δVp,δAp and δRp respectively by $\delta N(z)$ over the whole layer. And δVp is expressed as follows;

$$\delta V_p = \delta A_p R_p + A_p \delta R_p \tag{3}$$

Now, let Vp^s be the antenna output calculated from the initially postulated profile $N^s(z)$ and let Vp^t be the antenna output measured in practice with the phase shift value $e^{j\psi}$. Then δVp is expressed as follows;

$$\delta V_p = V_p^s - V_p^t e^{-j\psi} \tag{4}$$

δRp satisfies the following integral equation[8,9] with respect to $\delta N(z)$,

$$\delta R_p = \int_{z_0}^{z_n} K_p(z)\,\delta N(z)\,dz \tag{5}$$

where z_n is the top altitude of the ionosphere. Substituting eqs.(4),(5) into eq.(3), we have the following equations for p=1, p=2 respectively,

$$V_1^s - V_1^t e^{-j\psi} - \delta A_1 R_1 = \int_{z_0}^{z_n} A_1 K_1\,\delta N(z)\,dz \tag{6}$$

$$V_2^s - V_2^t e^{-j\psi} - \delta A_2 R_2 = \int_{z_0}^{z_n} A_2 K_2\,\delta N(z)\,dz \tag{7}$$

We eliminate $e^{-j\psi}$ from eqs.(6),(7) and obtain the following equation;

$$V_1^sV_2^t - V_2^sV_1^t - V_2^t\delta A_1R_1 + V_1^t\delta A_2R_2 = \int_{z_0}^{z_n}(A_1V_2^tK_1 - A_2V_1^tK_2)\delta N(z)\,dz \qquad (8)$$

Let us evaluate the magnitude of the 1st and 2nd term of the right hand side of eq.(3). When the distribution change of $\delta N(z)$ is relatively small, $\delta ApRp$ is much smaller than $Ap\delta Rp$ in the numerical calculation . In order to solve eq.(8), we set $\delta Ap=0$. That is, eq.(3) becomes $\delta Vp=Ap\delta Rp$. According to the technique used in solving eq.(5), we obtain $\delta N(z)$ from eq.(8). And we can obtain $N^{s\prime}(z)$ by using

$$N^{s\prime}(z) \leftarrow N^s(z) - \delta N(z) \qquad (9)$$

Next, δAp can be obtained by the following equation from the incident angle $\theta p'$ and θp that are calculated by $N^{s\prime}(z)$ and $N^s(z)$ respectively.

$$\delta A_p = \frac{\partial A_p}{\partial \theta_p}\cdot\delta\theta_p \ , \quad \delta\theta_p = \theta_p - \theta_p' \qquad (10)$$

where Ap is given by eq.(2). Applying this value of δAp of eq.(10) into eq.(8), we can solve $\delta N(z)$ in eq.(8) with the initial profile $N^s(z)$ and improve $N^s(z)$ by using

$$N^s(z) \leftarrow N^s(z) - \delta N(z) \qquad (11)$$

For the other frequency data, the profile is improved further by the above iterative calculation procedure. When the left-hand side of eq.(8) converges near zero by continuing iterative calculation, the derived electron density profile approaches the electron density profile that corresponds to the measured antenna output Vp^t.

3. Simulation and consideration

In order to examine the method described above, we have made a simulation. The simulation is made by assuming the VLF transmission path from Omega station at Tsushima (located at 34°36'42''N and 129°27'21''E) to Kanazawa (located at 36°32'42''N and 136°41'0''E). The propagation conditions are as follows; frequencies are 10.2,11.33 and 13.6kHz, the distance is 683.7km, the azimuthal angle is 80.2° with the earth magnetic meridian plane, the geomagnetic dip angle is 49.4° , the gyro frequency is 1.336MHz. By using Vp^s and Vp^t calculated from $N^s(z)$, $N^t(z)$ and $\nu(z)$ shown in Fig.2, we obtain the thick calculated curve in Fig.2 which converges well to the altitude range between 62km and 77km of N^t profile. In the case of using the relative phase data, the information of the data is decreased in order to eliminate the phase shift value, compared with the use of the absolute phase data. The result of simulation by using absolute phase data is shown in Fig.3, and that converges better than the result in Fig.2. The condition of the simulation in Fig.3 is the same as in Fig.2 except the absolute phase condition.

4. Conclusion

In this paper, a method of estimating the electron density profile by using the relative phase data of the receiving antenna output is described. To examine the method, we made the simulation by assuming the Omega wave transmission path from Tsushima to Kanazawa and obtained a good result that converges well, although not as good as that obtained by using the absolute phase data, since the relative phase data is incomplete. The future problem to be solved must take into consideration the ground wave and multi-reflection of the ionosphere.

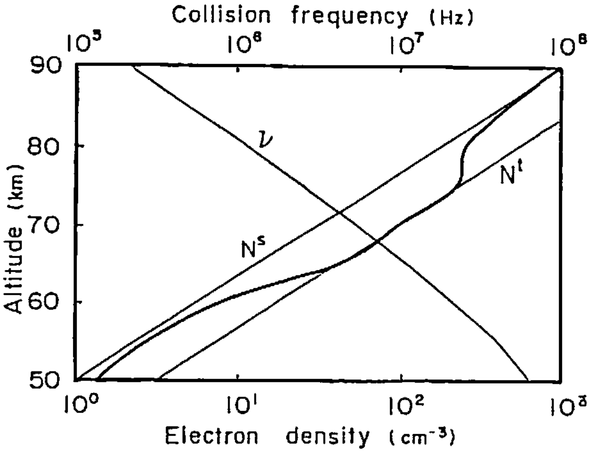

Fig.2 The example of the simulation of the estimation by using the relative phase technique. The thick line is derived by the iterative numerical calculation with intention to obtain the profile N^t from the profile N^s. The propagation path is from Tsushima to Kanazawa in Japan with Omega waves.

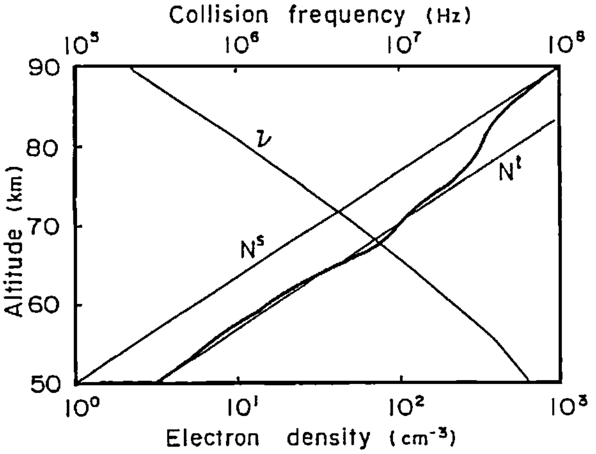

Fig.3 The example of the simulation of the estimation by using absolute phase technique. The simulation conditions are the same as in Fig.2 except for the absolute phase data.

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