PROPAGATION OF ELECTROMAGNETIC WAVES IN ICE DERIVED FROM ITS DIELECTRIC PROPERTIES 1. WAVE VELOCITY AND BIREFRINGENCE

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ABSTRACT

Based on the measured dielectric properties of ice, the propagation velocity and birefringence of electromagnetic waves in ice at MHz and GHz frequencies are discussed. The velocity of electromagnetic waves in solid ice is around 169 m/μsec and it decreases with increasing temperature at a rate of 0.023 m/μsec per 1 °C. Dielectric anisotropy of ice single crystal indicates that wave velocity can be varied within 1 m/μsec depending on the angle between electric field vector and crystal orientations of ice. Dielectric anisotropy also indicates that the polarization state of the electromagnetic waves are determined by the dielectric permittivity tensor in the polycrystalline ice.

1. INTRODUCTION

The relative complex dielectric permittivity of ice at frequencies from HF to microwave is of importance for the analysis of remote sensing data on the cryosphere because it determines the propagation of electromagnetic waves with such frequencies in large ice masses. The electromagnetic waves at frequencies between ten and a few hundred MHz are commonly used for the radio echo sounding (RES) of the ice sheets and Glaciers to explore the internal structure of them [1 and 2]. Satellite borne synthetic aperture radars (SAR) operating at 1.275 GHz on J-ERS1 and at 5.3 GHz on E-ERS1 and were recently launched.

The crystal structure of ice has a uniaxial symmetry and dielectric permittivity indicates the crystal orientation dependence from DC to optical frequencies [3, 4, 5 and 6]. Natural ice in the cryosphere is a polycrystalline aggregate and shows various fabric patterns which represent the distribution of the crystal orientation of grains in it. At the same time, natural ice is a dielectric mixture composed of ice, air bubbles, and impurities. Major impurities usually found in natural ice is classified as acid, salt and solid particles.

The real part of the relative complex dielectric permittivity of ice, ε', obtained experimentally is shown in Fig.1 schematically. The dispersion at the lower frequencies is caused by the Debye relaxation mechanism. Another dispersion is observed in the infrared region. ε' at frequencies between the two dispersion is usually denoted as ε_∞ as shown in Fig. 1 and it is a constant. Therefore, ε' at frequencies from HF to microwave becomes ε_∞ in the temperature range of the cryosphere.

In this paper we discuss the propagation velocity and birefringence of the electromagnetic waves from the measured real part of the dielectric permittivity. Attenuation is discussed from the measured dielectric loss in our another paper in this volume.

2. WAVE VELOCITY

The velocity of electromagnetic wave, v, is related to the permittivity, ε', as a following equation;

$$v = \frac{c}{\sqrt{\varepsilon'}} \qquad (1)$$

where c is the velocity of the electromagnetic waves in vacuum. ε' depends on crystal anisotropy, density and impurity in ice. Evans [7] reviewed that ε's obtained in the earlier studies could be, 3.17 ± 0.07, over the frequency range from 10 to 10^5 MHz. The studies carried out after Evans' review (e.g. [5, 8 and 9]) reported the values of ε' also within the same range. Thus, ε' is approximately constant around 3.17 with a very slight temperature dependence [5 and 8].

Fujita et al. [5] measured the dielectric permittivity of ice parallel to the c-axis ($\varepsilon'_{//c}$) and perpendicular to the c-axis ($\varepsilon'_{\perp c}$) at 9.7 GHz at temperatures between -32.5 °C and -2.5 °C. $\varepsilon'_{//c}$ and $\varepsilon'_{\perp c}$ are given by following equations;

$$\varepsilon'_{//c} = 3.189\ (\pm 0.006) + 0.00092\ (\pm 0.00007)\ T_c \tag{2}$$

$$\varepsilon'_{\perp c} = 3.152\ (\pm 0.003) + 0.00086\ (\pm 0.00005)\ T_c \tag{3}$$

where T_c is the temperature expressed in °C. The dielectric anisotropy $\Delta\varepsilon'$, $\varepsilon'_{//c}-\varepsilon'_{\perp c}$ is

$$\Delta\varepsilon' = 0.037\ (\pm 0.007) + 0.00006\ (\pm 0.00009)\ T_c \tag{4}$$

The temperature dependence of $\Delta\varepsilon'$ is negligibly small in the temperature range of the cryosphere. Fujita et al. [5] indicates that in the frequency range of HF, VHF, and microwave frequencies $\Delta\varepsilon'$ is 0.037. When ice is considered as an isotropic medium, ε' is expressed as,

$$\varepsilon' = \frac{2}{3}\varepsilon'_{\perp c} + \frac{1}{3}\varepsilon'_{//c} \tag{5}$$

Based on Evans' review, Robin et al. [1] concluded that wave velocity in ice is

$$v = 169 \pm 2\ [\mathrm{m/\mu sec}] \tag{6}$$

Substituting eqs. 2, 3 and 5 into eq. 1, the wave velocity becomes as in Fig. 2. Figure 2 indicates that wave velocity decreases with increasing temperature at a rate of 0.023 m/μsec per 1 °C and is also included in the range expressed in eq. 6. However, v depends on angle between the electric field vector and crystal orientation of ice and can be varied within about 1 [m/μsec].

Fujita et al. [10] showed that ε' depends on the density of ice, ρ. This was obtained based on the measurement at 9.7 GHz with ice samples whose ρ was larger than 600 [kg/m^3]. ε' increases linearly with ρ with a gradient

$$\left(\frac{d\varepsilon'}{d\rho}\right) = 3.08 \times 10^{-3}\ [\mathrm{m^3 kg^{-1}}] \tag{7}$$

Thus one can express v of ice medium with density ρ as

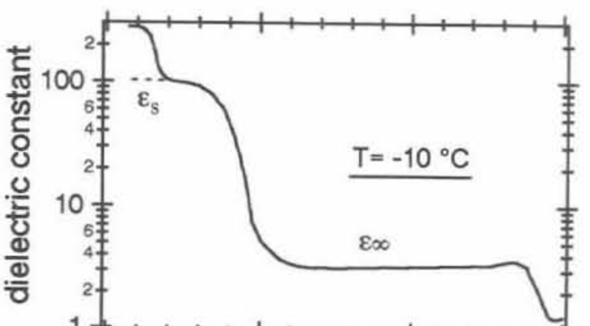

Figure 1. The schematic dispersion spectrum of ice at -10 °C.

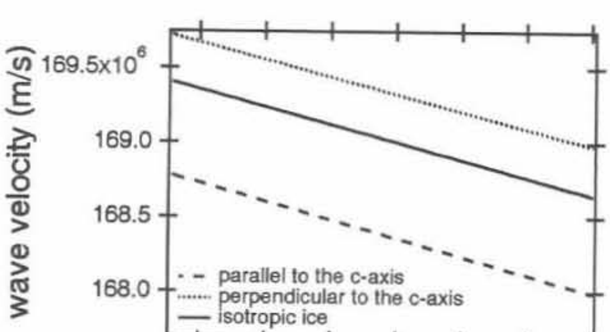

Figure 2.The propagation velocity of electromagnetic waves in ice as a function of temperature from HF to microwave frequency. The velocity depend on the angle between the electric field vector and the crystal orientation of ice.

$$v = \frac{c}{\sqrt{\varepsilon'_{pure} - \left(\frac{d\varepsilon'}{d\rho}\right)\rho}} \quad (8)$$

where ε'_{pure} means the dielectric permittivity of air bubble free ice.

Fujita et al.[11] showed that when acid concentration in ice is lower than 1×10^{-3} [mol/L], ε' of polycrystalline is within the range between $\varepsilon'_{//c}$ and $\varepsilon'_{\perp c}$. Thus influence of acid is negligibly small when PH value is larger than 3. The PH value usually found in the cryosphere is well above 3 even when it is so called "acid snow". Thus we need not consider the influence of acid impurity to the wave velocity in ice in the cryosphere.

3. BIREFRINGENCE OF THE POLYCRYSTALLINE ICE

Hargreaves [12] obtained the macroscopic dielectric permittivity tensor of the polycrystalline ice from ice fabric data. Applying the dielectric mixture theory [13] to the mixture of anisotropic dielectrics, he shows that the dielectric permittivity tensor of the polycrystalline is expressed by;

$$\varepsilon' = \sum_{j=1} f_j \, \varepsilon^{(j)} \quad (9)$$

where f_j is the volume fraction of the j-th crystal grain and $\varepsilon^{(j)}$ is the dielectric permittivity tensor of the j-th crystal grain. Equation 9 is derived on an assumption that the volume of the considering ice mass is small compared with the wavelength but sufficiently large compared with the volume of individual crystal grains. The dielectric permittivity tensor ε_p of a crystal grain is given by,

$$\varepsilon_p = \begin{pmatrix} \varepsilon'_{\perp c} & 0 & 0 \\ 0 & \varepsilon'_{\perp c} & 0 \\ 0 & 0 & \varepsilon'_{//c} \end{pmatrix} \quad (10)$$

when we take the c-axis as one of the principal axes.

Using eq. 9 and 10, one can express the dielectric permittivity tensor component parallel to the electric field. If we take the angle between the electric field vector and the c-axis of j-th grain as θ_j, it is written as,

$$\varepsilon' = \varepsilon'_{\perp c} + \sum_{j=1}^{N} f_j \, \Delta\varepsilon' \cos\theta_j \quad (11)$$

where N is the total number of the crystal grains in the ice medium. For simplicity, we approximate that the volume of grains are the same, then eq. 11 is rewritten by

$$\varepsilon' = \varepsilon'_{\perp c} + \Delta\varepsilon' D_a \quad (12)$$

$$D_a = \frac{1}{N} \sum_{j=1}^{N} \cos\theta_j \quad (13)$$

Here D_a is a factor which expresses the degree of contribution of $\Delta\varepsilon'$ to the component of dielectric permittivity tensor [14].

Hargreaves [15] pointed out that when plane-wave propagates in the uniaxially birefringent medium, only two directions of the electric field vector are allowed. When a lineally polarized wave in air is incident normally on the medium, the incident electric field vector becomes resolved into two components along the directions of allowed electric field vectors in the medium. These two directions are uniquely specified by the direction of propagation and the symmetry axis of dielectric permittivity tensor. Initially the two components are in phase, but after the passage of the medium, since they have different magnitudes of propagation vector, they are out of phase. When the wave return from the medium to air, it is elliptically polarized.

The Da calculated from actual crystal orientation of the Antarctic Ice Sheet is given in [14] and shown in Fig. 3 as an example. This example shows actual ice sheet is almost uniaxially birefringent. The detailed theory about propagation of electromagnetic wave in the uniaxial birefringent medium is given in Hargreaves [15].

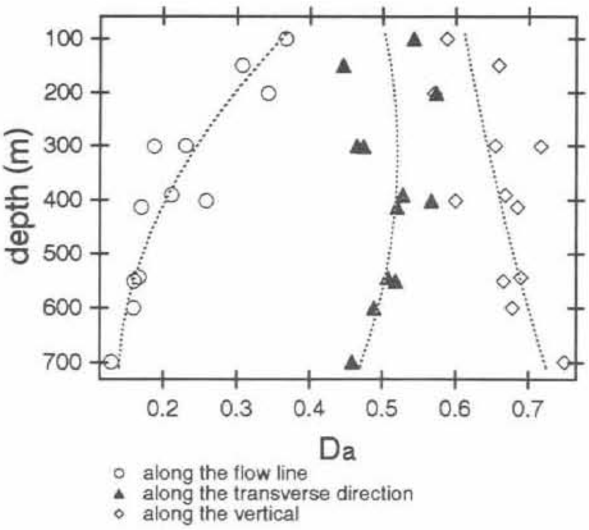

Figure 3. Contribution of the dielectric anisotropy to each component of the dielectric permittivity tensor in the ice sheet at Mizuho Station in East Antarctica [14]. Da is defined in eqs. 12 and 13. Calculation of Da was carried out along the following three axes; the axis along the flow line, the axis along the transverse direction and the axis along the vertical.

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