PROPAGATION OF VLF ELECTROMAGNETIC WAVES IN AN IDEALISED EARTH-CRUST WAVEGUIDE WITH OVER BURDEN

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1. SUMMARY

The present paper is concerned with propagation characteristics and simulation studies of electric field distributions of VLF waves and derivation of range equation for a satisfactory communication link between a land-based transmitter and a receiver deeply submerged in ocean via the earth-crust.

2. FORMULATION OF THE PROBLEM

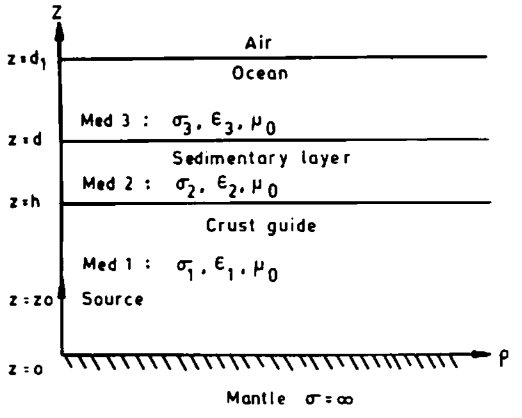
The problem is formulated [1, 2] in terms of Hertz function π_z for the three media (Fig. 1) i = 1, 2, 3 as follows :-

$$\pi_z^{(1)} = \pi_z^{(p)} + \int_\Gamma \left[A(c_1)\exp(-ik_1c_1z) + B(c_1)\exp(ik_1c_1z)\right] H_0^{(2)}(k_1s_1\rho)\,dc_1.$$

$$\pi_z^{(2)} = \int_\Gamma D(c_1)\exp(-ik_2c_2z) + E(c_1)\exp(ik_2c_2z)] H_0^{(2)}(k_1s_1\rho)\,dc_1.$$

$$\pi_z^{(3)} = \int_\Gamma F(c_1)\exp(-ik_3c_3z)\, H_0^{(2)}(k_1s_1\rho)\,dc_1.$$

Where $\pi_z^{(p)}$ denotes primary excitation function 3 ; $S_i = (1-C_i^2)^{1/2}$ is the sine of the angle of incidence. $A(C_1)$, $B(C_1)$, etc are unknown functions. $k_i^2 = -i\omega\mu_0(\sigma_i + i\omega\epsilon_i)$ $\pi_z^{(1)}(s_1)$ is transformed to λ - plane by letting $\lambda_j = k_1 s_j$.



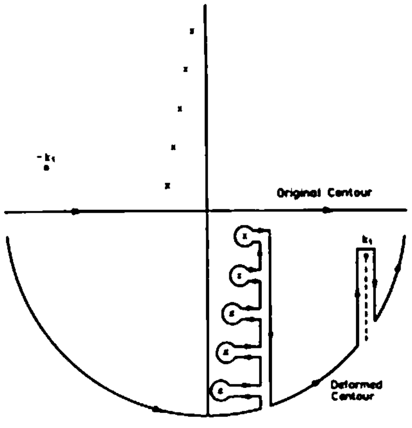


Fig. 2 Integration Contour for $\pi_z^{(1)}$ in λ-plane
x x : Poles ; • : Branch point

Fig.1 - Earth-Crust Profile

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Solution of $\int_{\Gamma} = \int_{\text{poles}} = 2\pi i \sum_{j} \text{Res. at } \lambda_j) + \int_{B.C.}$ yields $\pi_z^{(1)}$ which consists of pole and branch-cut (B.C) waves corresponding to guided and lateral waves respectively. The contour of integration is shown in Fig. 2.

3. SIMULATION STUDIES OF FIELD DISTRIBUTIONS

The field components E_ρ, E_z, H_ϕ for guided and lateral waves are obtained from the usual relations between $\vec{E}$, $\vec{H}$ and $\vec{\pi}$. Fig. 3 shows a laboratory model tank filled with artificially made sea-water (4) and the insulated field-launching and field-sampling probes connected to a VHF generator and line-tuned detector respectively. The tank is designed by using the relations

$$\frac{\sigma_m}{\sigma_c} = \frac{H_e}{H_m}\sqrt{\frac{K_m}{K_e}} \quad (2) \quad \text{and} \quad \frac{f_m}{f_e} = \frac{H_e}{H_m}\sqrt{\frac{K_e}{K_m}} \quad (3)$$

Where H, K, σ and f denote respectively range, dielectric constant, conductivity and frequency respectively. The subsecripts m and c refer to the model and earth-crust respectively. Experimental data for 400 MHz are transformed to VLF data for the real earth and compared with theoretical ΣE_{zG} corresponding to different values of S_j for ρ = 5 km to ρ = 500 km (see Fig. 4).

Fig. 3 - Experimental Set Up

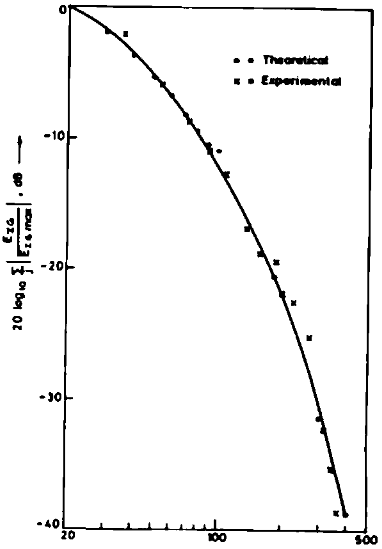

Fig. 4 - Theoretical and experimental $20 \log_{10} \sum_j |E_{zG}/E_{zG\,max}|$ vs ρ

4. POWER CARRIED BY GUIDED AND LATERAL WAVES

The total average guided ($P_{\rho G}$) and lateral (P_{ZL}) waves power are given by

$$P_{\rho G} = \frac{1}{2} \text{Re} \int\int^{2\pi\;\; h} \vec{z}\; E_{zG} \times \hat{\phi}\; \overset{*}{H}_{\phi G}\, d\phi\, dz, \;(4) \quad P_{zL} = \frac{1}{2} \text{Re} \int_0^{2\pi}\int_{\rho=0}^{\infty} \hat{\rho}\; E_{\rho L} \times \hat{\phi}\, \overset{*}{H}_{\phi L}\, \rho\, d\rho\, d\phi, \;(5)$$

Fig. 5 and 6 show constant power level contour and effective lateral wave power flowing into the over burden at different radial distances(ρ)respectively.

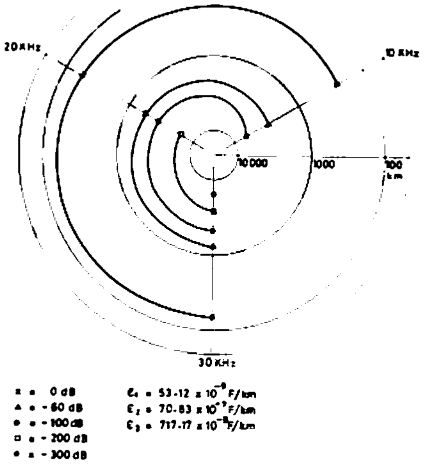

Fig. - 5 - Constant power level contour

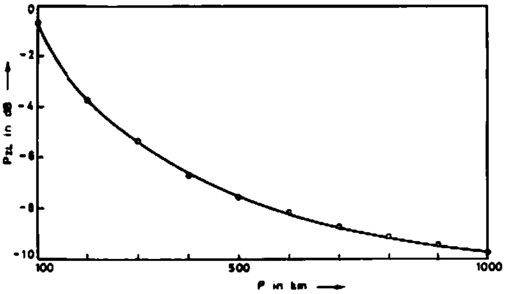

Fig. 6 - Effective lateral wave power P_{ZL} vs ρ

5. ATTENUATION CONSTANTS

The attenuation constant in media 1 due to conduction (P_{lc}) and dielectric (P_{ld}) losses is given by eq. (6). α_1 is also calculated using eq. (7)[2]

$$\alpha_1 = \frac{P_{Lc}(z=0)+P_{Ld}(z=h)+P_{ZL}}{2(P_{PG}+P_{ZL})} \quad (6) \qquad \alpha_1 = -\operatorname{Im} k_1 S_j \quad (7)$$

The attenuation constants in media 2 and 3 are given [5] by

$$\alpha_{2,3} = w\sqrt{\epsilon_{2,3}\,\mu_0}\left[\frac{\left\{(\sigma_{2,3}/w\epsilon_{2,3})^2+1\right\}^{1/2}-1}{2}\right]^{1/2} \quad (8)$$

6. RANGE EQUATION

The power (P) received at a terminal ρ, z = R_3 inside the ocean due to a source at ρ = 0, z = z_0 is given by eq. (9), where R_2 = d-h = 1 km.

$$P = \left(\frac{3\lambda}{8\pi}\right)^2 (1-|R_{23}|^2)\exp(-2\alpha_2 h_2)\ \exp(-2\alpha_3 R_3)\operatorname{antilog}(P_1-\alpha_1 P)(1-|R_{12}|^2) \quad (9)$$

where R_3 is the distance from z = d and P_1 = P_{ZL} for lossless boundary at z = h. Fig. 7 shows power received at the ocean floor (R_3 = 0) for 1 watt power input to the crust-guide at ρ = 0, z = z_0.

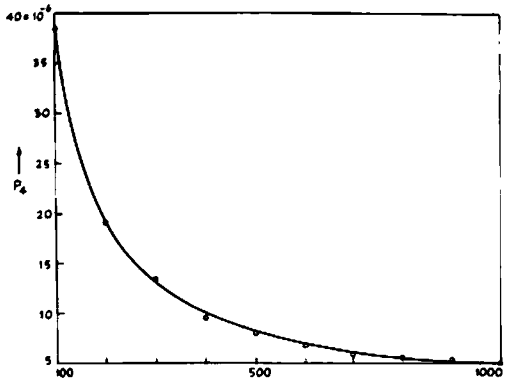

FIG 7 - POWER RECEIVED AT THE OCEAN-FLOOR AT DIFFERENT RADIAL DISTANCES FOR A POWER-INPUT OF ONE WATT.

7. CONCLUSIONS

For computer-aided numerical calculations, we have used $f_e = 5\,\text{KHz}$ to $30\,\text{KHz}$, $f_m = 400\,\text{mHz}$, $\epsilon_1 = 35.42 \times 10^{-9}$ F/Km. to 53.12×10^{-9} F/Km, $\epsilon_2 = 70.83 \times 10^{-9}$ to 88.54×10^{-9} F/Km, $\epsilon_3 = 717.17 \times 10^{-9}$ F/Km. $\sigma_1 = 10^{-4}$ to 10^{-6} V/Km, $\sigma_2 = 10^{-1}$ to 1 V/Km. $\sigma_3 = 4 \times 10^{3}$ V/Km, $h = 20$ Km, $d = 21$ Km, $\mu_0 = 4\pi \times 10^{-4}$ h/Km.
we find that $(\alpha_1 + \alpha_2) \ll \alpha_3$ and $\alpha(av)$ = 0.0062 nepers/km (eq. 6, 8), but $\alpha(av) = 0.00879$ nepers/km (eq. 7), whereas for direct transimission through the ocean, even for a distance 0.1 km, α = exp(-79.4) at f = 10 KHz and exp (-137.6) at f = 30 KHz which are prohibitively high.

Hence we conclude that a communication link between land and deeply submerged terminals inside the ocean via the earth-crust is advantageous to a direct link between land and ocean, provided the existence of a low-loss earth-crust is a reality and location of a transmitting terminal inside the earth-crust can be implemented. The proposed method is invulnerable to electromagnetic interference due to lightening discharge, jamming, EMNP, etc.

8. REFERENCES

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