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Application of Tai's Trial Function in an Improved Circuit Theory Two-Term Representation

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Abstract

A new trial function is used to derive the elements of the impedance matrix in the Improved Circuit Theory (ICT) for its application to multielement antennas. Results of the new ICT impedance are comparable in accuracy with the general method of moments. All the new functions are expressed in closed-form, and so still presents an ICT algorithm which is superior in terms of computer running time compared to method of moments.

1. Introduction

The Improved Circuit Theory (ICT) for Multielement Antennas uses a King-Middleton two-term trial function for the current distribution in an extended variational principle to derive the formula for the generalized impedance for multielement antennas of typical configuration as shown in Fig. 1 [1]. This represents a great improvement in the EMF theory evaluation of center-fed dipole antenna arrays. This is because the ICT method is able to take into adequate consideration the effects of mutual coupling without necesarily using an excessive number of current expansion functions as it is case with other methods like method of moments (MoM) [2]. The ICT method has also been implemented at much reduced CPU time by completely eliminating all time consuming numerical integration functions in its algorithm [3-4].

However, as we would show later, the ICT method is inadequate for much longer antenna lengths. Therefore for accurate evaluation of such antenna systems we are forced to use more conventional methods like MoM which whose application is not restricted to the antenna length but which is well known to be inherently time intensive and requires considerable amount of computer storage space.

In this paper, we present the results of the impedance of the ICT method as a result of a new choice in a trial function shown by C. T. Tai [5-6] to be far more superior in quality when used to derive the input impedance of center-fed dipole arrays. It is shown that the new impedance formula leads to the possibility of much extended application of the ICT method and also establishes the valid region of the conventional ICT method.

2. Tai's Trial Function in ICT Two-term Representation

In the classical King-Middleton ICT two-term representation, the generalized impedance matrix is derived with the with the two-term current functions

$$f_i^1(z_i) = \frac{\sin k(h-|z_i|)}{\sin kh} \quad (1)$$

$$f_i^2(z_j) = \frac{1-\cos k(h-|z_j|)}{\cos kh}. \quad (2)$$

But C. T. Tai has shown the these two trial functions are not sufficiently accurate for antenna lengths greater than one wavelength for the case of single elements [5-6]. To extend Tai's suggestiong to multielement antennas we have also replaced second component of the trial function as follows

$$f_i^2(z_j) = \frac{k(h-|z_j|)\cos(h-|z_j|)}{kh\cos kh}. \quad (3)$$

Eqs. 1 and 3 have therefore been used in a new two-term representation to derive the elements of the impedance matrix all expressible in closed-form has shown in the appendix.

3. Results and Discussion

We have validated the new ICT formula by comparing the input impedances computed with it to other conventional methods. Fig. 2 compares the admittances computed for different antenna lengths using C. T. Tai's trial function , King-Middleton ICT, C. T. Tai's single element variational [5] and MoM [2]. We can see the considerable agreement between our new formula and MoM in particular. The results also extablishes the region of validity of the conventional ICT formula because as can be deduced from Figs. 2, it is inadequate for element lengths above 1.8λ.

The attractiveness of the ICT method is captured in Table 1 which compares a typical CPU time on an NEC PC-9801VX for the various methods.

4. Conclusion

We have shown that the use of C. T. Tai's trial function in a two-term ICT representation leads to an impedance formula which has the possibility of expanding the region of validity of the ICT method in the analysis of linear wire antennas. Since all components of the new formula are expressible in closed-form. ICT is still computationally efficient in terms of CPU time. We have also established the valid region of the conventional ICT method.

The results reported are applicable to only equal length center-fed dipole arrays, but the same procedure can be used to derive a more general formula for an array of center-fed dipoles of arbitrary length.

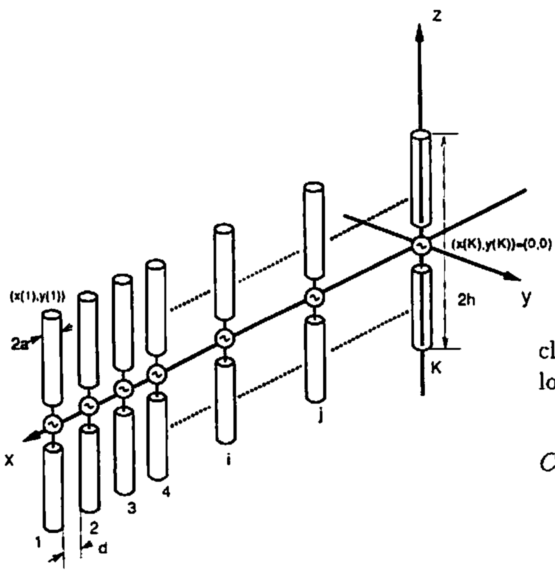

Figure 1: Array of Cylindrical Dipoles.

5. Appendix

Using the current functions in Eqs. 3 and 5 in a two-termrepresentation, it can be shown that the elements of the impedance matrix in ICT can be expressed as

$$\begin{aligned} Z_{ij}^{11} &= j30csc^2kh\,\{(2+\cos 2kh)S_{kd}(kh) \\ &+ \sin 2kh\,[C_{kd}(2kh) - 2C_{kd}(kh)] \\ &- \cos 2kh\,[S_{kd}(2kh) - S_{kd}(kh)]\} \quad (4) \end{aligned}$$

$$\begin{aligned} Z_{12}^{12} &= \frac{j60\csc 2kh}{kh}[2kh\cos 2khC_{kd}(2kh) \\ &+ 2kh\sin 2khS_{kd}(2kh) - \cos 2khCU1_{kd}(2kh) \\ &- \sin 2khSU1_{kd}(2kh) + 2(1+\cos 2kh)CU1_{kd}(kh) \\ &+ 2\sin 2khSU1_{kd}(kh) - kh(1+3\cos 2kh)C_{kd}(kh) \\ &- 3kh\sin 2khS_{kd}(kh)] \quad (5) \end{aligned}$$

$$\begin{aligned} Z_{12}^{22} &= j\frac{30}{k^2h^2(1+\cos 2kh)}\{[-\sin 2khCU2_{kd}(2kh) \\ &+ \cos 2khSU2_{kd}(2kh) + [4kh\sin 2kh \\ &- \cos 2kh]CU1_{kd}(2kh) + [4kh\sin 2kh \\ &- \cos 2kh]CU1_{kd}(2kh) - [4kh\cos 2kh \\ &+ \sin 2kh]SU1_{kd}(2kh) + (1-4(kh)^2)\sin 2kh \\ &+ 2kh\cos 2kh]C_{kd}(2kh) + [2kh\sin 2kh \\ &- (1-4(kh)^2)\cos 2kh]S_{kd}(2kh) \\ &+ 2\sin 2khCU2_{kd}(kh) - 2\cos 2khSU2_{kd}(kh) \\ &+ 2[1+\cos 2kh - 4kh\sin 2kh]CU1_{kd}(kh) \\ &+ 2[-kh+\sin 2kh + 4kh\cos 2kh]SU1_{kd}(kh) \\ &- 2\left[kh + kh\cos 2kh - (3(kh)^2-1)\sin 2k\right] \\ &\cdot C_{kd}(kh) + 2[(kh)^2 + 1 - kh\sin 2kh \\ &- (3(kh)^2-1)\cos 2kh]S_{kd}(kh)\Big\} \quad (6) \end{aligned}$$

$C_D(x)$ and $S_D(x$ are well documented closed-form functions defined in while the following are new functions [7]:

$$\begin{aligned} CU1_D(x) &= \frac{1}{2}[-2(D+j)e^{-jD} + j(e^{-jv}+e^{-ju}) \\ &+ D^2[(\frac{e^{-jv}}{v} + \frac{e^{-ju}}{u}) + j][E_i[-jv] \\ &+ E_i[-ju] - 2E_i[-jD]]] \quad (7) \end{aligned}$$

$$\begin{aligned} SU1_D(x) &= \frac{1}{2}[jD^2(\frac{e^{-jv}}{v} - \frac{e^{-ju}}{u}) - (e^{-jv} - e^{-ju}) \\ &+ D^2[E_i[-jv] - E_i[-ju]]] \quad (8) \end{aligned}$$

$$\begin{aligned} CU2_D(x) &= \frac{1}{2}[(\frac{1}{2} + j\frac{v}{2} - (\frac{D^2}{2v})^2 + j\frac{D^4}{4v})e^{-jv} \\ &- (\frac{1}{2} + j\frac{u}{2} - (\frac{D^2}{2u})^2 + j\frac{D^4}{4u})e^{-ju} \\ &- D^2[1 + (\frac{D}{2})^2] \\ &\cdot [E_i[-jv] - E_i[-ju]]] \quad (9) \end{aligned}$$

$$\begin{aligned} SU2_D(x) &= \frac{j}{2}[(-1 - jD + \frac{D^2}{2} - j\frac{D^3}{2})e^{-jD} \\ &+ (\frac{1}{2} + j\frac{v}{2} - (\frac{D^2}{2v})^2 + j\frac{D^4}{4v})e^{-jv} \\ &+ (\frac{1}{2} + j\frac{u}{2} - (\frac{D^2}{2u})^2 + j\frac{D^4}{4u})e^{-ju} \\ &- D^2[1 + (\frac{D}{2})^2][E_i[-jv] \\ &+ E_i[-ju] - 2E_i[-jD]]] \quad (10) \end{aligned}$$

$$\begin{aligned} u &= \sqrt{D^2+x^2} - x \\ v &= \sqrt{D^2+x^2} + x \quad (11) \end{aligned}$$

$$E_i[-jz] = C_i(z) - C_i(0) - jS_i(z) \quad (12)$$

$C_i(z)$ and $S_i(z)$ are the well known cosine and sine integral functions respectively.

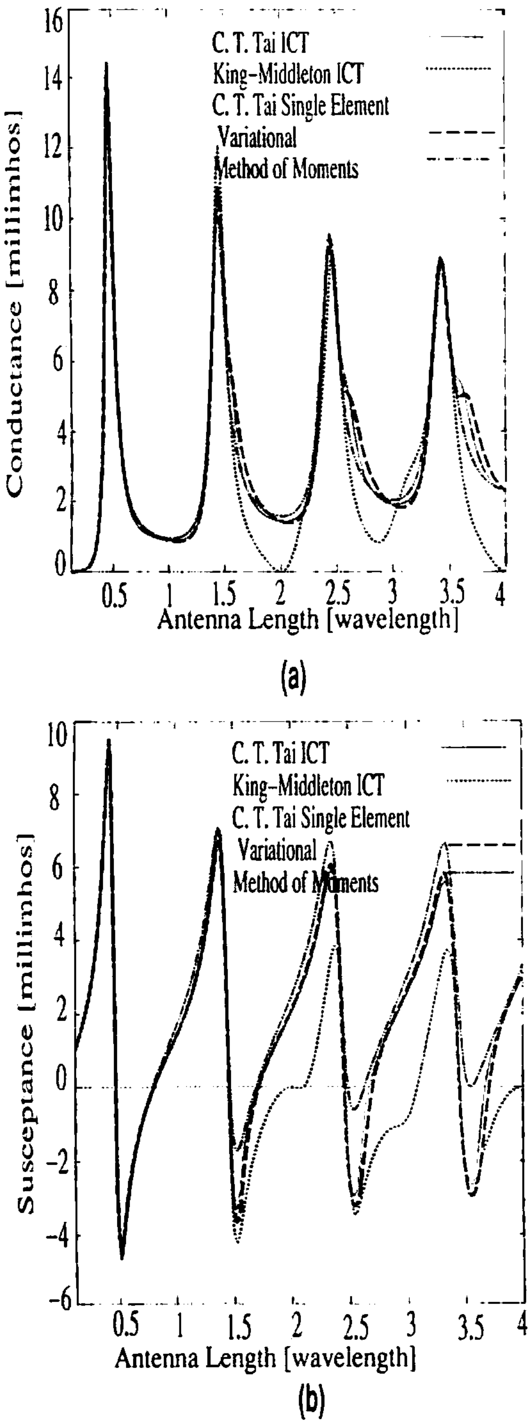

Figure 2: Input Admittance for $\Omega = 10$ (a) Conductance, (b) Susceptance.

Table 1: CPU Time on NEC PC-9801 VX.

Method	Time [seconds]
Tai ICT	0.11
King-Middleton ICT	0.27
Faster ICT [3]	0.07
Tai Variational EMF	0.03
Method of Moments	96

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