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Statistical Analysis of Multiple Scattering and Attenuation Due to Many Raindrops Using FDTD

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1. Introduction

Rain measurement system using propagation characteristics of microwave and millimeter wave is very effective for disaster prevention system for local strong rainfalls. Measurement technique of electromagnetic scattering and attenuation characteristics by rain is one of useful evaluation methods of rainfall rate[1,2]. Attenuations of incident wave with 20GHz carrier frequency in random media corresponding to rainfall rate 5 – 50 mm/h are evaluated by three-dimensional FDTD method. In FDTD analysis, incident wave with relatively flat distribution around beam center is transmitted. Firstly, we show measurement system of rain attenuation using microwave Gaussian beam[3]. When the frequency of incident wave is 20GHz, scattering by a raindrop with 1mm diameter can be shown by Rayleigh approximation scattering theory. Next, scattered fields by single and two raindrops are evaluated by FDTD method and Rayleigh scattering theory. Lastly, scattered fields by many raindrops and specific rain attenuation are evaluated using FDTD method[4-6]. Based on these computer simulations, the optimum measurement system of rain using microwave techniques may be accomplished.

2. Rain Measurement System Using Microwave

Rain measurement system using microwave is shown in Fig. 1. A parabolic antenna with diameter $2a$=2.4m is used to transmit and receive microwave of 10-20GHz. Gaussian beam with beam spot r_0= 0.6m-20λ～40λ is transmitted. Reflected wave from a conducting plate is received by the parabolic antenna and specific attenuation A (dB/km) is evaluated.

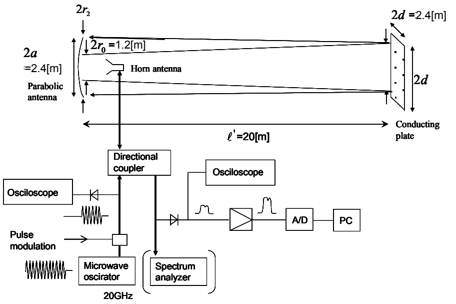


Fig.1 Measurement system of rain attenuation using microwave

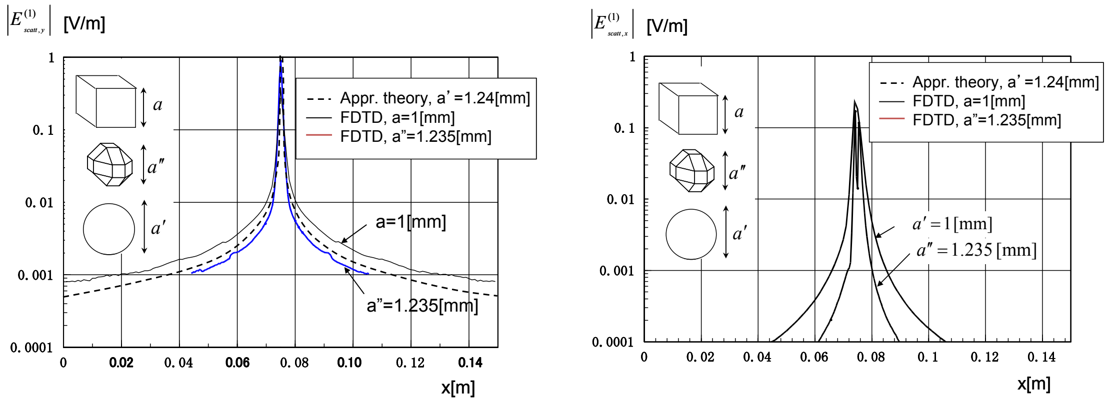
3. Scattering By Single and Two Raindrops

Scattered field of incident beam wave with the relatively large beam waist by a sphere with diameter a' and dielectric constant $\varepsilon = n_r^{*2}\varepsilon_0$ is approximately obtained by Rayleigh scattering when diameter is smaller than 1/10 of wavelength. The amplitudes of scattered fields by theoretical approximate and numerical analysis are compared to test the accuracy of FDTD analysis. Fig.4 shows the comparison of scattered field amplitudes of spherical, cubic and polyhedric raindrop model at $\mathbf{r}_i = (x_i, y_i, z_i) = (0.075, 0.075, 0.075)$. We confirmed that these raindrop models show similar scattering characteristics.

Considering the second order scattering terms, second iterative scattering field by N raindrops are shown by the equivalent dipole of Rayleigh scattering $\mathbf{p}_i$, Green's function $\mathbf{G}$ for the free space, and the first iterative scattering field $\mathbf{E}^{(1)}_{scatt}$ as

$$\mathbf{E}^{(2)}_{scatt} = \mathbf{E}^{(1)}_{scatt} + \sum_{j=1}^{N}\sum_{i=1}^{N}\mathbf{p}_j\left(\mathbf{p}_i\left(\mathbf{E}_{inc}\right)\mathbf{G}\right)\mathbf{G}, \qquad \mathbf{E}^{(1)}_{scatt} = \sum_{i=1}^{N}\mathbf{p}_i\left(\mathbf{E}_{inc}\right)\mathbf{G} \tag{1}$$

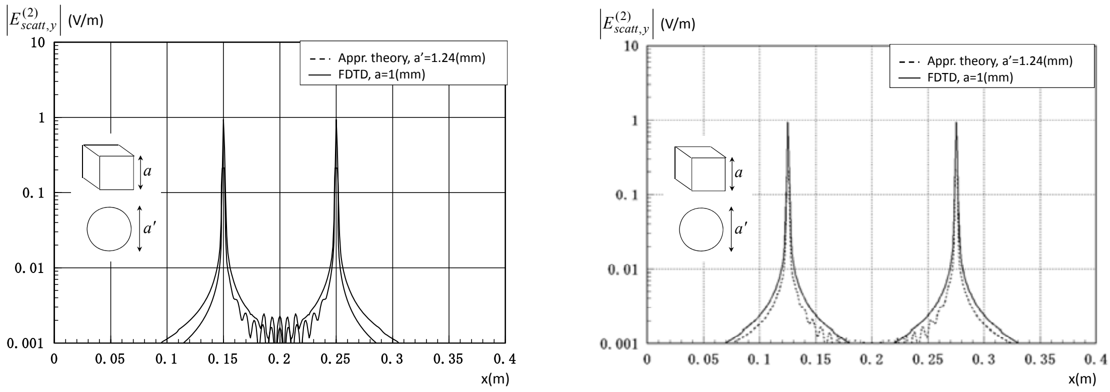
Comparison of FDTD numerical and approximate analytical results for two raindrops in Fig.3 shows interference patterns around raindrop models. When distance of two raindrops is less than 0.1(m), strong interferences of multiple scattering waves are observed between raindrops. Rain region with average distance of 0.1(m) corresponds to rainfall rate R=14mm/h So, FDTD simulation including multiple scattering effects is very effective in accurate evaluation of microwave attenuation by heavy rain greater than 14mm/h.



(a) y-component

(b) x-component

Fig. 2 Comparison of the scattering amplitude on the line y=z=0.075m, y polarization of incident wave. a : Side length of a cube, a' : Diameter of a sphere, a'' :Length of a polyhedron from the bottom to the top



(a) d=6.7λ=0.1(m)

(b) d=10λ=0.15(m)

Fig.3 Comparison of the scattering amplitude on the line y=y$_0$=0.2m, z=0.075m, y polarization of incident wave. Positions of raindrops：(a) x_1 =0.15(m), x_2 =0.25(m), (b) x_1 =0.125(m), x_2 =0.275(m), y_1 = y_2 =0.2(m), z_1 = z_2 =0.075(m)

4. Multiple Scattering and Interference By Many Raindrops

Three-dimensional analysis model of rain attenuation is shown for parallel calculation in s layer domains in Fig.4. Analysis region in one layer is defined as $\ell_x \ell_y \ell_z$. In this paper, fundamental characteristics are shown for one unit layer. For long distance propagation, parallel calculation in multi layers should be performed. Positions of raindrops in rain region depend on homogeneous distribution. The incident wave has flat distribution around beam center as shown in Fig.5. The electric field of the incident wave at z =0(m) is given by

$$E_{inc,y}(x,y,0,t)=E_0\left\{\frac{1}{1+e^{-u(x-x_1)}}+\frac{1}{1+e^{u(x-x_2)}}-1\right\}\left\{\frac{1}{1+e^{-u(y-y_1)}}+\frac{1}{1+e^{u(y-y_2)}}-1\right\}\cdot\left\{\frac{1}{1+e^{-w(t-t_1)}}+\frac{1}{1+e^{w(t-t_2)}}-1\right\}\sin(2\pi ft) \quad (2)$$

Where, $u=10^2$ (m^{-1}), x_1, y_1=0.06(m), x_2, y_2=0.34(m), $w=10^{11}$ (s^{-1}), t_1=0.05(ns), t_2=0.45(ns) are used. Pulse width is t_2-t_1=0.4(ns) and beam width is $x_2-x_1=y_2-y_1$=0.28(m). As shown in Fig.5, beam width of 0.28(m) is appropriate because the amplitudes of the incident wave on the boundary of analysis space, at x=0, ℓ_x, y=0, ℓ_y can be negligibly small and the amplitude distribution in a region of $0.1(\text{m})\le x,y\le 0.2(\text{m})$ is almost uniform constant. When raindrop positions x_i and y_i are restricted as $0.1(\text{m})\le x_i,y_i\le 0.2(\text{m})$, the incident waves at raindrop positions (x_i,y_i,z_i) with same z_i coordinate have almost constant amplitudes. When the sizes of the aperture in x and y directions are $\ell_x=\ell_y=0.4$(m), diffraction angle θ is approximately $\theta\cong\frac{\lambda}{\ell_x}=\frac{\lambda}{\ell_y}=3.75\times10^{-2}$ (rad). In this case, divergences of the beam Δx and Δy in x and y directions after ℓ =0.15(m) propagation are given by $\Delta x=\Delta y=\theta\ell'=5.625\times10^{-3}$ (m) and satisfy $\Delta x,\Delta y<<\ell_x,\ell_y$. Simulation parameters are shown in Table 1. Fig.6 shows a random media model of rainfall rate R=20(mm/h), where N=33 is the number of raindrops, a_i=1(mm) is a side length of raindrops and $n_r^*=6.46-j2.81$ is complex refractive index of raindrops. N=33 is sufficiently number of raindrops for evaluation of statistical scattering characteristics. Fig.7 shows the numerical results of field difference ΔE_y between total field and incident field, as scattered field. This figure shows the effects of scattering and absorption due to randomly distributed raindrops. The amplitude of scattered field around beam center is very fluctuated due to the interference of many scattered waves.

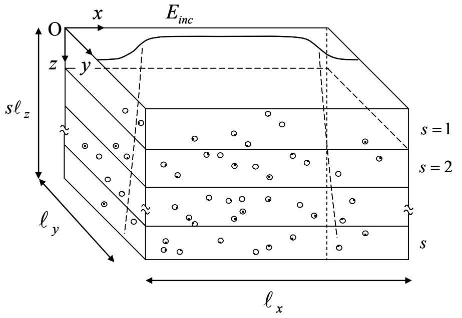


Fig. 4 Analysis model for rain attenuation

Table 1. Parameters for long distance simulation using FDTD computation

Parameters	Values
λ : Wavelength of incident wave ℓ_x,ℓ_y,ℓ_z : Length of subspace in x, y, z direction	0.015m 0.4m (26.7λ), 0.4m (26.7λ), 0.15m (10λ)
f : Frequency of incident wave	20GHz
w : Time parameter for incident pulse t_1,t_2 : Time when pulse gets the half amplitude	10^{11} (1/s) 0.05ns, 0.45ns
u : Spatial parameter for incident pulse x_1,x_2 : Point where pulse gets the half amplitude y_1,y_2 : Point where pulse gets the half amplitude	100 0.06m, 0.34m 0.06m, 0.34m
Δs : Length of a cell Δt : Time increment	0.001m 1.5ps
N : Number of raindrops in a subspace	8 (5mm/h), 16 (10mm/h), 33 (20mm/h) 66 (40mm/h), 83 (50mm/h)
a_i : Length of a side of a raindrop n_r^* : Complex refractive index of raindrops	1mm 6.46-j2.81

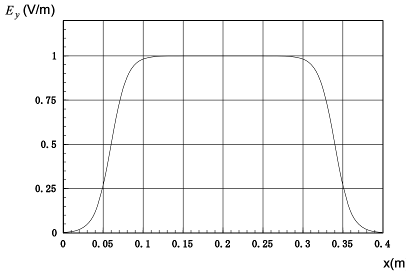
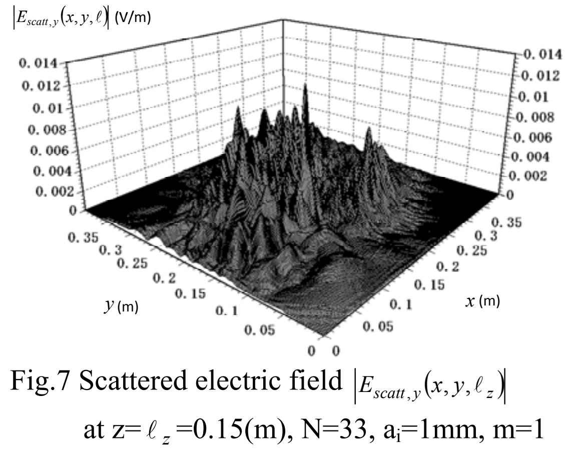


Fig.5 Spatial distribution of incident electric field at z=0(m)



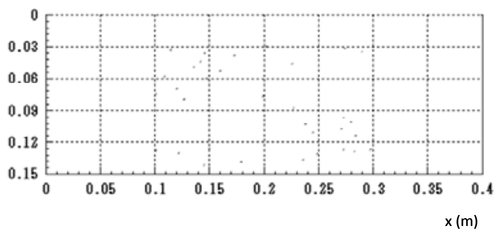


Fig.6 Cross sectional view of raindrop Distribution(N=33, a_i=1mm, m=1, s=1, ℓ_z=0.15m)

Fig.7 Scattered electric field $|E_{scatt,y}(x,y,\ell_z)|$ at z= ℓ_z=0.15(m), N=33, a_i=1mm, m=1

5. Statistical Analysis of Rain Attenuation by FDTD

In FDTD simulation, rain attenuation α for propagation length ℓ (m) and Specific attenuation by rain A (dB/km) are calculated by receiving power $P(\ell)$ and $P_0(\ell)$ given by incident field $\mathbf{E}_{inc}$,

$$A = \alpha \cdot 10^3 / \ell \,, \ \alpha = -10\log_{10}\frac{P(\ell)}{P_0(\ell)}, \ P(\ell) = \int_S \overline{W}(x, y, \ell)dS \ , \ \overline{W}(x, y, \ell) = \frac{1}{T}\int_{t'}^{t'+T}\left(E_x H_y - E_y H_x\right)dt \quad (3)$$

where ℓ =0.15(m) is propagation distance of analysis space. Rain rate R(mm/h) is obtained by $R = \frac{N}{V} vta^3 \cdot 10^{-6}$.

N is a number of raindrops in one subspace with volume $V = \ell_x \ell_y \ell_z$ =0.024(m^3), v is the terminal velocity of raindrops and t=1(hr)=3600(sec). When v =4(m/s), t=1(hr), V =0.024(m^3) and a =1(mm) are assumed, R is given by R =0.6 N . Fig.8 shows specific rain attenuation obtained by FDTD using these parameters. Here, five realizations of random media with different positions of raindrops under the same number of raindrops are analyzed by FDTD method and specific rain attenuations are evaluated using eq. (3). Table 2 shows statistical evaluation of specific attenuation A(dB/km) for rainfall rate R(mm/h). Here, m=1, 2, ･･･, M is a number of samples of random media, under the same condition and M is the number of samples and M=5 in this analysis. From Table 2, tendency that the variance becomes large when rain rate R is heavy is observed. It is considered that in random media with many raindrops, electromagnetic waves in an observation plane are fluctuated strongly due to the interference of multiple scattered waves by many raindrops.

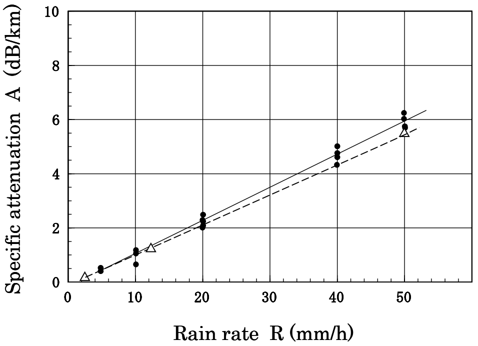


Fig. 8 Specific rain attenuation by FDTD.
△: Approximate calculation by [2]

Table 2 Statistical evaluation of specific rain attenuation

R (mm/h)	$E[A]$	$Va[A]\times10^2$
5	0.52	0.31
10	1.04	3.13
20	2.28	2.17
40	4.67	6.24
50	5.88	6.01

6. Conclusions

In this paper, electromagnetic scattering and specific rain attenuation by rain are evaluated by using FDTD method. When average distance of raindrops is less than 0.1(m) or rainfall rate is greater than 14mm/h, the interferences of multiple scattering waves are observed and FDTD simulation is effective to evaluate microwave attenuation by rain correctly. Evaluation of rain attenuation at z=$2\ell_z$ and $3\ell_z$ will be considered by using parallel and successive computation of FDTD. Estimating correlations between rain rates and microwave attenuations for several simulation models, these results by FDTD method should be compared with experimental measured results. Based on these computer simulations, the optimum measurement systems of raindrop using microwave techniques may be accomplished.

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