

## A RADIATIVE TRANSFER APPROACH TO SCATTERING FROM THE RANDOMLY ROUGH SEA SURFACE WITH FOAM SCATTERERS

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Abstract: By using iterative method to solve the vector radiative transfer equation for a layer of scatterers with randomly rough underlying boundary, the backscattering coefficient is derived, and is applied to the two-scale model of rough sea surface with foam scatterers. Employing a newly-modified Cox and Munk's probability density function and a semi-empirical Pierson's sea spectrum, numerical results of polarized backscattering are calculated. The functional dependences on wind speed and direction, and other parameters are discussed, and theoretical results are well compared with experimental data.

### 1. INTRODUCTION

Electromagnetic backscattering from randomly rough sea surface and the functional dependence on wind (speed and direction) and the parameters of sea surface have been interested in the ocean remote sensing. Numerical approach of the two-scale model has been widely applied to the study of scattering from complex rough sea surface, which is modeled as a combination of large and small scales.

However, it has been observed that foam and whitecaps over sea surface driven by strong wind can significantly effect the emissivity of sea surface. Droppleman (1970, *J. Geophys. Res.*) modeled foam as a homogeneous layer with a mean dielectric constant. Rosencratz and Staelin (1972, *J. Geophys. Res.*) proposed a model of a series of plane, thin films parallel to the smooth sea surface. All of these models are far from the actual picture of randomly rough sea surface. Moreover, in the active remote sensing of backscattering from sea surface, the effect of foam scatterers has not been studied.

In this paper, we present a model of a layer of discrete scatterers over a two-scale randomly rough surface. By using iterative method to the vector radiative transfer (VRT) equation and the boundary conditions of rough surface, the polarized backscattering coefficients are derived. The solutions include the scattering from foam scatterers, rough surface, and coupling contributions of scatterers and surface. Employing Pierson's sea spectrum and a newly-modified Cox and Munk's probability density function, numerical results of backscattering coefficients at large incidence angles are obtained. The functional depen-

dences on wind speed and direction, and parameters such as polarization, observation angle, frequency, etc. are discussed. Theoretical results are well compared with experimental measurements.

## 2. ITERATIVE SOLUTION OF VRT EQUATION

We are concerned with the problem as shown in Fig. 1. The vector VRT equation of spherical scatterers is

$$\cos\theta \frac{d}{dz} \bar{I}(\theta, \Phi, z) = -K_e \bar{I}(\theta, \Phi, z) + \int_0^\pi d\theta' \sin\theta' \int_0^{2\pi} d\Phi' \bar{P}(\theta, \Phi; \theta', \Phi') \cdot \bar{I}(\theta', \Phi', z). \quad (1)$$

The boundary conditions are written as ( $0 < \theta < \pi/2$ ):

$$\bar{I}(\pi - \theta, \Phi, z=d) = \bar{I}_{\text{oi}} \delta(\cos\theta - \cos\theta') \delta(\Phi - \Phi'), \quad (2a)$$

$$\bar{I}(\theta, \Phi, z=0) = \int_0^{\pi/2} d\theta' \sin\theta' \int_0^{2\pi} d\Phi' \bar{R}(\theta, \Phi; \theta', \Phi') \cdot \bar{I}(\pi - \theta', \Phi', z=0), \quad (2b)$$

where  $\bar{I}_{\text{oi}}$  is the incidence intensity,  $\bar{R}$  is the bistatic reflectivity at the rough boundary  $z=0$ , whose elements  $R_{pq}$  ( $p, q = \text{vertical polarization v, or horizontal polarization h}$ ) is expressed by the bistatic scattering coefficient  $\gamma_{pq}(\theta, \Phi; \theta', \Phi')$  as

$$R_{pq}(\theta, \Phi; \theta', \Phi') = \frac{1}{4\pi} \gamma_{pq}(\theta, \Phi; \theta', \Phi') \frac{\cos\theta'}{\cos\theta}. \quad (3)$$

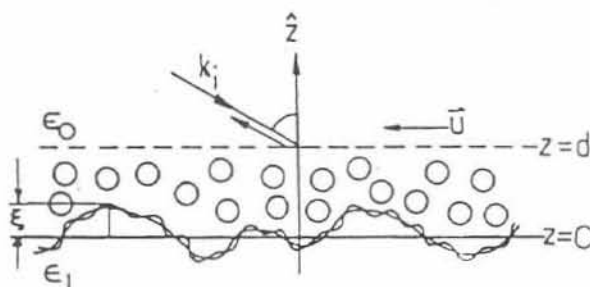


Fig. 1 Geometry of the problem

We use iterative method to solve Eq. (1) and the boundary conditions (2a, b). We obtain the zeroth- and first-order solutions of backscattering coefficients; respectively:

$$\sigma_{pq}^{(0)}(\theta_i) = \sigma_{pq0}(\theta_i) \exp(-2k_e d \sec\theta_i), \quad (4)$$

where  $\sigma_{pq0}(\theta_i)$  is the backscattering coefficient of rough surface without the top layer of scatterers; the factor  $\exp(-2K_e d \sec\theta_i)$  is the attenuation due to the scattering and absorption through the layer depth  $d$ .

$$\begin{aligned}
\sigma_{hh}^{(1)}(\theta_i) = & \frac{2\pi}{K_e} \cos\theta_i P_{hh}(\theta_i, \Phi_i; \pi - \theta_i, \Phi_i) [1 - \exp(-2K_e d \sec\theta_i)] \\
& + \frac{4\pi}{K_e} \cos\theta_i \exp(-2K_e d \sec\theta_i) \int_0^{\pi/2} d\theta' \sin\theta' \int_0^{2\pi} d\Phi' \\
& \sum_{q=v, h} P_{hq}(\theta_i, \pi + \Phi_i; \theta', \Phi') R_{qh}(\theta', \Phi'; \pi - \theta_i, \Phi_i) \\
& \times \frac{\sec\theta_i}{K_e(\sec\theta' - \sec\theta_i)} \left\{ 1 - \exp[-K_e d(\sec\theta' - \sec\theta_i)] \right\} \\
& + \frac{4\pi}{K_e} \cos\theta_i \exp(-2K_e d \sec\theta_i) \int_0^{\pi/2} d\theta' \sin\theta' \int_0^{2\pi} d\Phi' \\
& \sum_{q=v, h} R_{hq}(\theta_i, \pi + \Phi_i; \pi - \theta', \Phi') P_{qh}(\pi - \theta', \Phi'; \pi - \theta_i, \Phi_i) \\
& \times \frac{\sec\theta'}{K_e(\sec\theta' - \sec\theta_i)} \left\{ 1 - \exp[-K_e d(\sec\theta' - \sec\theta_i)] \right\} \\
& + \frac{4\pi}{K_e} \cos\theta_i \exp(-2K_e d \sec\theta_i) \int_0^{\pi/2} d\theta' \sin\theta' \int_0^{2\pi} d\Phi' \\
& \sum_{q=v, h} R_{hq}(\theta_i, \pi + \Phi_i; \pi - \theta', \Phi') \int_0^{\pi/2} d\theta'' \sin\theta'' \int_0^{2\pi} d\Phi'' \\
& \sum_{q=v, h} P_{qi}(\pi - \theta', \Phi'; \theta'', \Phi'') R_{ih}(\theta'', \Phi''; \pi - \theta_i, \Phi_i) \\
& \times \frac{\sec\theta'}{K_e(\sec\theta' + \sec\theta'')} \left\{ 1 - \exp[-K_e d(\sec\theta' + \sec\theta'')] \right\} . \tag{5}
\end{aligned}$$

In the same way, we can obtain  $\sigma_{vv}^{(1)}$ ,  $\sigma_{vh}^{(1)}$ , and  $\sigma_{hv}^{(1)}$ .

### 3. BACKSCATTERING FROM TWO-SCALE ROUGH SEA SURFACE

It has been studied (Fung and Lee, 1982, IEEE J. Oceanic. Eng.) that, at large incidence angle, backscattering from two-scale randomly rough surface is mainly contributed by the large slope average of the small perturbation solution:

$$\sigma_{pp0}(\theta_i, \Phi_0) = \iint \sigma_{pp}^{spa}(\theta_{il}, \Phi_0) (1 + \xi_x \tan\theta_i) P(\xi_x, \xi_y) d\xi_x d\xi_y, \tag{6}$$

where all notations follow Fung and Lee (1982). The azimuthal angle  $\Phi_0 = 0^\circ$  is upwind direction,  $= 90^\circ$  or  $270^\circ$ , crosswind, and  $= 180^\circ$  downwind. It has been well known that  $\sigma_{pp}^{spa}(\theta_{il}, \Phi_0)$  is proportional to the sea spectrum  $W(2k \sin\theta_{il}, \Phi_0)$ , which we employ a semi-empirical Pierson spectrum (Fung and Lee, 1982).

We modify the Cox and Munk's large slope probability density function to take account of the difference between upwind and downwind backscattering caused by the skewness of surface contour. The function is written as

$$P(\xi_x, \xi_y) = \frac{1}{2\pi\sigma_u\sigma_e} \exp\left[ -(\xi_x - s_1)^2 / \sigma_u^2 - \xi_y^2 / \sigma_e^2 \right] (1 + f), \tag{7}$$

where the upwind and crosswind slope variance  $\sigma_u$ ,  $\sigma_c$  and the peakness function  $f$  follow the Cox and Munk's results. The skewness function  $s_1$  in Eq. (7) is chosen from our data matching as

$$s_1(u) = 0.007 + 0.0015u. \quad (8)$$

#### 4. NUMERICAL RESULTS

Numerical results of  $\sigma_{pp}(\theta_i) = \sigma_{pp}^{(0)}(\theta_i) + \sigma_{pp}^{(1)}(\theta_i)$  are shown in Fig. 2–5. Following the observation and simulations in the previous works, the depth  $d$  is in cm order, and foam particle radius is in mm order. We take

$$f_s d = 0.01 \exp(0.06u) - 0.011, \quad (u > 2 \text{ m/s}) \quad (9)$$

where  $f_s$  is the fractional volume of foam scatterers.

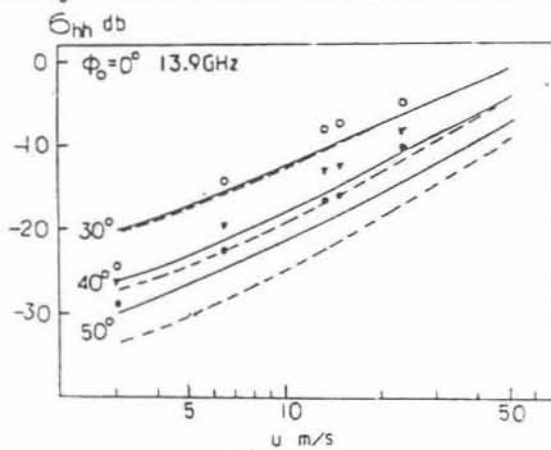


Fig.2

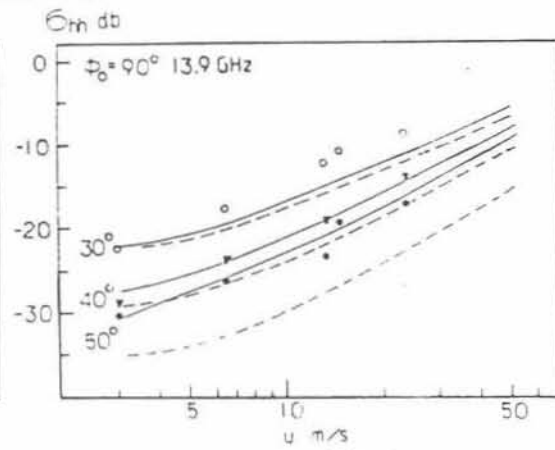


Fig.3

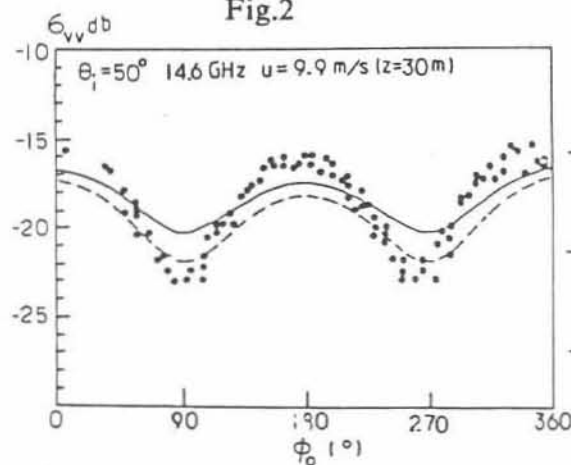


Fig.4

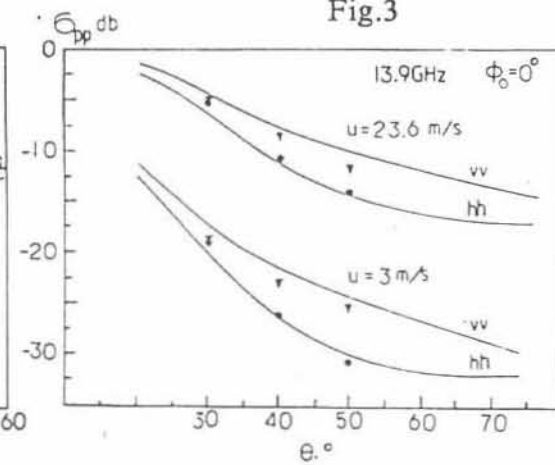


Fig.5

#### ACKNOWLEDGEMENT

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