An Optimization Approach with Edge-Preserving Regularization to the Time-Domain Inverse Scattering Problem for an Anisotropic Cylindrical Object

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I. Introduction

In several areas of applied sciences such as geophysical exploration, medical imaging, and nondestructive testing, we often encounter nonlinear inverse scattering problems. A variety of inversion algorithms for the problems in both frequency and time domains have been proposed to reconstruct the shape, location, internal structure, and material parameters of scattering objects[1–8]. These algorithms have been applied to scattering objects of isotropic materials. Some materials such as ceramics, ferrites, crystals show dielectric and/or magnetic anisotropies and are of vital importance in the design of monolithic microwave and optical integrated circuits. There are, however, few works that deal with inverse scattering problems for anisotropic materials.

In this paper, we consider the problem of determining the shape, location and electrical parameters of an inhomogeneous anisotropic cylinder imbedded in a homogeneous medium from the knowledge of the incident fields and the measured field data. The previously proposed inversion method for isotropic material[8] is extended to treat the anisotropic case. The method is based on optimization technique and the gradient for an appropriate cost functional of anisotropic parameters is explicitly derived. Explict expression of the gradient makes the inversion algorithm efficient in cooperation with finite-difference time-domain method (FDTD).

II. Formulation of the Problem

Let us consider a cylindrical object of inhomogeneous and biaxially anisotropic material surrounded by a homogeneous isotropic medium as shown in Fig. 1. The material is assumed to be dispersionless, i.e, independent of frequency. The axis of the cylinder, z-direction is assumed to be one of the principal axes of the anisotropic material. We consider the TM case for which the magnetic field is perpendicular to the plane of incidence (i.e, the magnetic field is parallel to the axis of the cylinder). The TE fields can be treated in the same manner. The cylindrical object is successively illuminated by a short pulse wave generated by a current source located at a transmitter point $\boldsymbol{r}_m^t$, $(m = 1, 2, \cdots, M)$:

$$\boldsymbol{J}(\boldsymbol{r}, t) = \boldsymbol{I}(t)\delta(\boldsymbol{r} - \boldsymbol{r}_m^t) \tag{1}$$

where $\boldsymbol{r}$ denotes the position vector of a point (x, y), and $\delta(\boldsymbol{r})$ is the Dirac delta function. We assume that the transmitter is turned on at time $t = 0$ and there are no fields before time $t = 0$.

The TM fields satisfy Maxwell's equations in the following matrix form:

$$\boldsymbol{\mathcal{L}v} = \boldsymbol{J} \tag{2}$$

where

$$\boldsymbol{v} = \begin{bmatrix} E_x(\boldsymbol{r}, t) \\ E_y(\boldsymbol{r}, t) \\ \eta H_z(\boldsymbol{r}, t) \end{bmatrix}, \quad \boldsymbol{J} = \begin{bmatrix} J_x(\boldsymbol{r}, t) \\ J_y(\boldsymbol{r}, t) \\ 0 \end{bmatrix} \tag{3}$$

subject to zero initial condition

$$\boldsymbol{v}(r,0) = \boldsymbol{0}. \tag{4}$$

The quantity $\eta(=\sqrt{\mu_0/\epsilon_0})$ denotes the intrisic impedance in free space. The partial differential operator $\boldsymbol{\mathcal{L}}$ is defined by

$$\boldsymbol{\mathcal{L}} \overset{\text{def}}{=} \boldsymbol{A}\frac{\partial}{\partial x} + \boldsymbol{B}\frac{\partial}{\partial y} - \boldsymbol{C}\frac{\partial}{\partial(ct)} \tag{5}$$

where c is the speed of light in free space and

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} \epsilon_{r11}(\boldsymbol{r}) & \epsilon_{r12}(\boldsymbol{r}) & 0 \\ \epsilon_{r21}(\boldsymbol{r}) & \epsilon_{r22}(\boldsymbol{r}) & 0 \\ 0 & 0 & \mu_{r33}(\boldsymbol{r}) \end{bmatrix}. \tag{6}$$

The inverse scattering problem considered here is the one of determining the unknown constitutive parameters from the knowledge of the incident pulse wave and the time domain data of electromagnetic waves measured at receiver points .

Let us introduce the cost functional:

$$\begin{aligned} F(\boldsymbol{p},\boldsymbol{b}) \;\; &= \;\; F_1(\boldsymbol{p}) + F_2(\boldsymbol{p},\boldsymbol{b}) = \int_0^T \sum_{m=1}^{M}\sum_{n=1}^{N} K_m(\boldsymbol{r}_n^r,t)\left|\boldsymbol{v}_m(\boldsymbol{p};\boldsymbol{r}_n^r,t) - \widetilde{\boldsymbol{v}}_m(\boldsymbol{r}_n^r,t)\right|^2 dt \\ &\quad + \sum_{i=1}^{5}\lambda_i \iint_S \left\{ b_i \left|\frac{\nabla p_i}{\chi_i}\right|^2 + \psi\left(b_i\right)\right\} dS \end{aligned} \tag{7}$$

where $\boldsymbol{p} = [p_1, p_2, \cdots, p_5]^t = [\epsilon_{r11}(\boldsymbol{r}), \epsilon_{r12}(\boldsymbol{r}), \epsilon_{r21}(\boldsymbol{r}), \epsilon_{r22}(\boldsymbol{r}), \mu_{r33}(\boldsymbol{r})]^t$ (t denotes transpose) is the parameter vector and $\boldsymbol{b} = [b_1, b_2 \cdots, b_5]^t$. $\widetilde{\boldsymbol{v}}_m(\boldsymbol{r}_n^r, t)$ is the measured electromagnetic fields at the receiver point $\boldsymbol{r}_n^r$ due to the current source located at the transmitter point $\boldsymbol{r}_m^t$ and $\boldsymbol{v}_m(\boldsymbol{p};\boldsymbol{r}_n^r,t)$ is the calculated electromagnetic fields for an estimated parameter $\boldsymbol{p}$ at the same position under the same current excitation. The function $K_m(\boldsymbol{r}_n^r,t)$ is a nonnegative weighting function which takes a value of zero at $t = T$, where T is the duration of the measurement. The second term is the edge-preserving regularization term, which enhances the stability of the algorithm with respect to noise[4,9]. The parameters λ_i are regularization parameters, and the parameters χ_i fix the threshold level with respect to discontinuity. It is known that for a fixed $\boldsymbol{p}$ the regularization term $F_2(\boldsymbol{p},\boldsymbol{b})$ takes the minimum value when $b_i\left(|\nabla p_i/\chi_i|\right) = \varphi'\left(|\nabla p_i/\chi_i|\right)/\left(2|\nabla p_i/\chi_i|\right)$, where ψ is an 'edge-preserving potential function'. The value of b_i at the point tends to 0 when the position is located on an edge, and it tends to 1 when the position is located on a homogeneous area. In this paper we use $\varphi(t) = t^2/(1+t^2)$. The function $\psi(t)$ is convex and analytically defined from $\varphi(t)$.

III. Reconstruction algorithm

We employ a conjugate gradient method to find an ideal solution minimizing the functional $F(\boldsymbol{p})$. Following the same discussion as that in reference[8], the Fréchet differential $F'_{1\boldsymbol{p}}\delta\boldsymbol{p}$ of the functional $F_1(\boldsymbol{p})$ can be represented as

$$F'_{1\boldsymbol{p}}\delta\boldsymbol{p} = \iint_S \left[\sum_{i=1}^{5} \mathrm{g}_i \delta p_i\right] dS = \langle \mathbf{g_1}, \delta\boldsymbol{p}\rangle. \tag{8}$$

The gradient vector $\mathbf{g_1}$ is expressed in terms of the adjoint field vector $\boldsymbol{w}_m$ which satifies the adjoint equation:

$$\boldsymbol{\mathcal{L}}^*\boldsymbol{w}_m = \sum_{n=1}^{N} K_m(\boldsymbol{r}_n^r,t)\left[\boldsymbol{v}_m(\boldsymbol{p};\boldsymbol{r}_n^r,t) - \widetilde{\boldsymbol{v}}_m(\boldsymbol{r}_n^r,t)\right]\delta(\boldsymbol{r}-\boldsymbol{r}_n^r) \tag{9}$$

subject to

$$\boldsymbol{w}_m(\boldsymbol{p};\boldsymbol{r},T) = \boldsymbol{0}. \tag{10}$$

Here $\mathcal{L}^*$ is the adjoint opertor of $\mathcal{L}$ and is given by

$$\mathcal{L}^* = -\frac{\partial}{\partial x}\boldsymbol{A}^t - \frac{\partial}{\partial y}\boldsymbol{B}^t + \frac{\partial}{\partial (ct)}\boldsymbol{C}^t. \tag{11}$$

Since $\boldsymbol{A}$ and $\boldsymbol{B}$ are constant matrices and $\boldsymbol{C}$ is independent of time t, the adjoint operator $\mathcal{L}^*$ has the same form as the operator $\mathcal{L}$. Eq.(9) implies that the adjoint functions are the time-reversed fields generated by the residuals at the observation points at time $t = T$. Therefore, FDTD can be applied to compute the adjoint fields with backward time-stepping from $t = T$ to $t = 0$.

A simple calculation, provided that $\boldsymbol{b}$ is fixed, gives the Fréchet differential $F'_{2\boldsymbol{p}}\delta\boldsymbol{p}$ of the functional $F_2(\boldsymbol{p},\boldsymbol{b})$ in the inner product form:

$$F'_{2\boldsymbol{p}}\delta\boldsymbol{p} = -\iint_S \sum_{i=1}^{5} \delta p_i \frac{2\lambda_i \nabla \cdot \left[b_i\left(\left|\frac{\nabla p_i}{\chi_i}\right|\right)\nabla p_i\right]}{\chi_i^2} dS = \langle \mathsf{g}_2, \delta\boldsymbol{p}\rangle. \tag{12}$$

From Eqs. (8) and (12), we obtain the gradient vector $\mathsf{g} = \mathsf{g}_1 + \mathsf{g}_2$ for the total functional F.

IV. Numerical Results

The proposed reconstruction algorithm has been successfully tested with computer simulations. One of the simulations was done for a circular cylinder consisting of two concentric homogeneous anisotropic layers. The inner layer with a radius of $4.5\Delta x$ and the outer layer with a radius of $9.5\Delta x$ have material properties $(\epsilon_{r11}, \epsilon_{r12}, \epsilon_{r21}, \epsilon_{r22}, \mu_{r33}) = (4.875, 0.65, 0.65, 5.625, 1.0)$ and $(2.925, 0.39, 0.39, 3.375, 1.0)$, respectively. The FDTD solution space, bounded by an eight-cell perfectly matched layer (PML)[10], consists of 66×66 cells with cell size $\Delta x = \Delta y = 2.948\ mm$. The reconstruction area containing a scattering object is a square of 24×24 cells. The background medium is assumed to be free space. The cylindrical object is illuminated successively by a transmitter at four different positions. The scattering data for each illumination are measured at eight observation points. The time step size Δt is chosen to be 6.88 ps. The time excitation function is chosen to be

$$J_x(t) = J_y(t) = \tau \frac{d}{dt}\left[\left(\frac{t}{\tau}\right)^4 e^{-\frac{t}{\tau}}\right] \tag{13}$$

where τ is 0.125 ns. The time duration T of the measurement is $500\Delta t$. The weighting function is chosen to be $K_m = \cos(\pi t/2T)$. Initial guess for the electrical parameters are chosen as those of the background medium, i.e., free space. The true distributions and the reconstructed results after 2000 iterations are shown in Fig.2(a) and (b), respectively.

V. Conclusions

We have proposed a new optimization approach to the time-domain inverse scattering problem of inhomogeneous anisotropic cylindrical objects with edge-preserving regularization. The finite-difference time-domain (FDTD) method is used effectively as both forward and backward solvers. Numerical simulations have demonstrated the validity of the present time-domain propagation-backpropagation algorithm for a dielectric anisotropic cylinder with high-contrast permittivity tensor. The inversion algorithm can also be easily extended to lossy and/or dispersive anisotropic media.

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References

[1] W. H. Weedon and W. C. Chew, *Inverse Problem*, pp. 551-564, 1993.

[2] W. Yu and R. Mittra, *Microwave Opt. Technol. Lett.*, Vol.15, pp. 282-288, Aug. 1997.

[3] P. M. van den Berg and R. E. Kleinman, *Inverse Problems,* vol. 11, pp.L5-L10, 1995.

[4] C. Pichot, P. Lobel, C. Dourthe, L. Blanc-Féraud, and M. Barlaud, *IEICE Trans. Electron.,* vol. E80-C, no. 11, pp. 1343-1348, Nov. 1997.

[5] S. Y. Yang, H. K. Choi, and J. W. Ra, *Microwave Opt. Technol. Lett.,* vol. 16, pp. 17-21, Sept. 1997.

[6] F. Natterer and F. Wübbeling, *Inverse Problems,* no. 11, pp. 1225-1232, 1995.

[7] M. Gustafsson and S. He, *Mathemtics and Computers in Simulation,* vol. 50, pp. 541-551, 1999.

[8] T. Tanaka, T. Takenaka and S. He, *Microwave Opt. Technol. Lett.*, Vol.20, pp. 72-77, Jan. 1999.

[9] P. Charbonnier, L. B. Féraud, G. Aubert and M. Barlaud, *IEEE Trans. on Image Processing,* vol. 6, pp. 298-311, 1997.

[10] J. P. Berenger, *J. Comput. Phys.*, Vol.114, pp. 185-200, 1994.

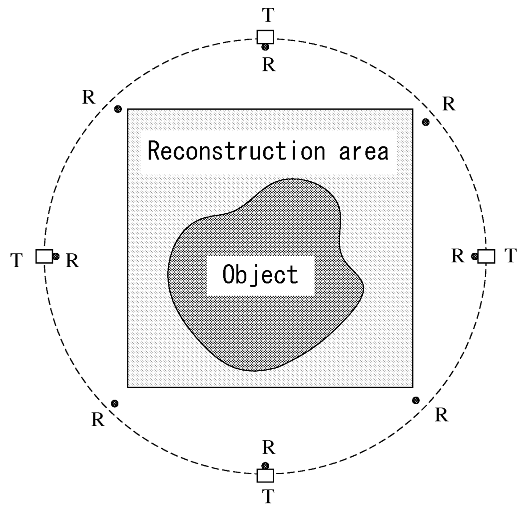

Fig.1 Geometrical configuration of the inverse problem. 'T' and 'R' indicate the positions for the transmitters and receivers, respectively.

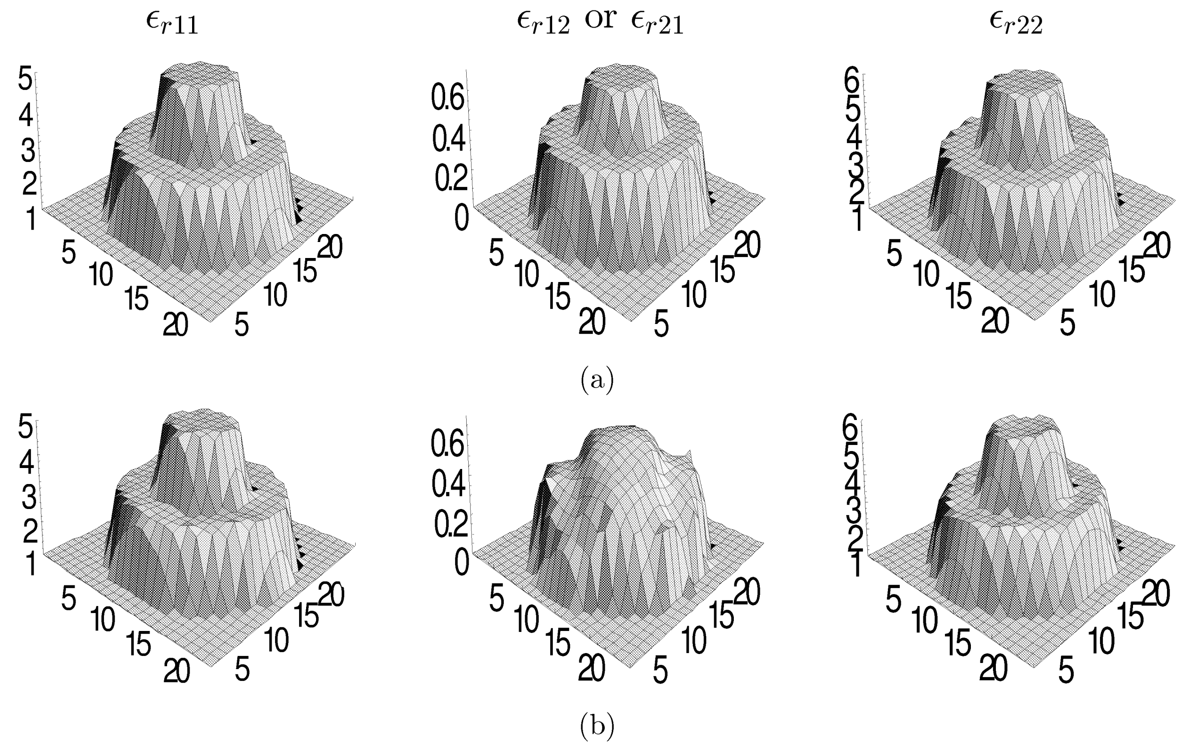

Fig.2 Reconstruction of an anisotropic cylinder with two layers. The first layer has a radius $4.5\Delta x$ and the second layer has $9.5\Delta x$. (a) True distributions. (b) Reconstructions after 2000 iterations.