Least square image reconstruction method for sparse array radar system

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***Abstract* – We demonstrate a method to arrange antennas for the 2-D sparse array system for 3-dimensional imaging for NDI application. A radar system consists from eight transmitters and eight receivers. It uses principle of uniformly distributed middle points between each pair of transmitter and receiver antennas to form equilateral triangles from them. Least square method as the image reconstruction method is implemented with support of calculations on graphical processor unit. Mathematical description of data preparation for the L2 norm fitting is shown. 8 transmitter and 8 receiver antennas were arranged on 2-D array of 60cm x 60cm area, and image reconstruction method were tested by a SFCW radar, that operates at frequency from 3814 MHz to 8067 MHz. From the images we obtained, we can distinguish target in 1m x 1m area with a resolution about 5 cm.**

***Index Terms* — Step Frequency Continuous Wave Radar, Sparse array, Least square method, Calculation on graphical processor unit.**

1. Introduction

We are developing Non-Destructive Inspection (NDI) method using radar technique for investigation of the wall conditions of houses after natural disasters such as tsunami and earthquakes. The radar systems we want to develop must have capability of measurement of large areas in limited time without scanning. Also, it should be compact and portable with a limited weight. In order to achieve these requirements, we started to develop sparse array subsurface radar system [1].

Stepped-Frequency Continuous Wave (SFCW) radar with sparse array configuration does not require physical movement as well as it consists of low number of antennas. In addition, the SFCW method allows us to measure Doppler Effect. These facts make this type of the radars very perspective [2].

The radar system we are developing is shown in Fig. 1.

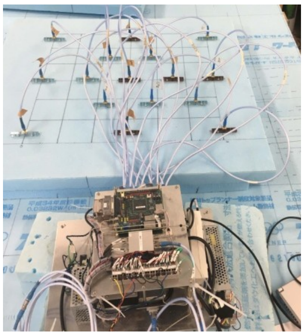


Fig. 1. SFCW sparse array radar

This radar system is a SFCW transmitter receiver system with switches for eight transmitters and eight receivers. The operating frequency of the radar is from 3814 MHz to 8067 MHz. Each pair of the transmitter and receiver antennas represents one channel. Consistently using all the transmitters, we can get totally 8x8=64 channels. The example of one signal after signal processing and background subtraction is shown in Fig. 2.

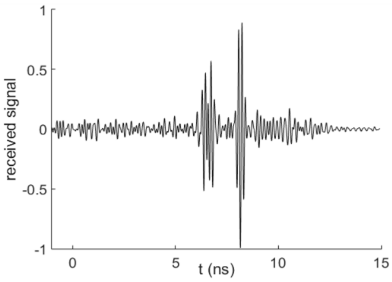
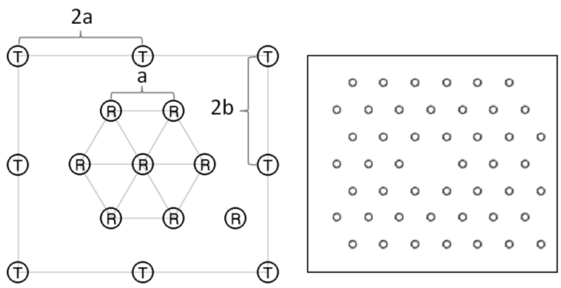


Fig. 2. Radar raw signal

2. Sparse array configuration

We considered optimization of antenna configuration. Our goal is 3D imaging, so the antennas should be placed on the 2D surface.

The first obvious principle to find the optimal antenna configuration is to spread evenly the middle points between any pair of transmitters and receivers. Important, that middle points are better to form as many as possible equilateral triangles. Equilateral triangles provide best stability of the result image due fluctuation of time delay of the received signal. An antenna configuration that satisfies these principles is shown in Fig. 3a. There “T” indicates transmitter position; “R” indicates receiver position.



(a) (b)

Fig. 3. Sparse array antenna pattern (a), and corresponding to this pattern distribution of the middle points (b)

On this pattern one antenna type is located on the rectangular grid, another antenna type on the grid of equilateral triangles, and $b = a\sqrt{3}/2$. Corresponding to proposed antenna pattern, distribution of middle points between every pair of the transmitters and receivers are shown in Fig. 3b. Although some middle points are overlapping each other, proposed pattern satisfy both principles mentioned above.

3. Least square method for sparse array system

Diffraction stacking method produces additional artifacts for the sparse radar system in image reconstruction. The least square method or L2 norm fitting method can deal with this system well. Assume X_{kl} to be target data (reconstructed image), R_{ij} is radar data after basic signal processing, Z_{ij}^{kl} is simulation matrix, which transform object distribution X_{kl} to the radar data. In other words

$$R_{ij} = \sum_{k,l} Z_{ij}^{kl} X_{kl} \qquad (1)$$

In practice, we know measured data matrix R and simulation matrix S, and we want to know X. If N and M are the lengths of the some dimensions of Z matrix, the matrix indexes could be reduced to the vectors

$$S_{i+Nj}^{k+Ml} = Z_{ij}^{kl};\ \vec{x}_{k+Ml} = X_{kl};\ \ \vec{r}_{i+Nj} = R_{ij} \qquad (2)$$

The exact solution of the system $\vec{r}_k = S_k^i \vec{x}_i$ is known to be impossible to find due very poor condition number of the matrix S.

We apply conjugate gradient method with Tikhonov regularization to implement least square method [3], [4]. It allows minimizing following expression

$$\|S\vec{x} - \vec{r}\|_{L2}^2 + a\|x\|_{L2}^2 \rightarrow min \qquad (3)$$

Conjugate gradient iterative method is correct only for symmetric and positive determined matrixes. To solve this problem, we can replace our system of equations to $S^T S\vec{x} = S^T\vec{r}$. The new system of equations has symmetric and positive determined matrix S^TS.

The calculation time can be often reduced by using graphical processor unit. Especially, the algorithm can be drastically boosted, if it uses operations with support high parallelization. Operations on matrices and calculation of the matrix S elements can be parallelized very well. So, we used Compute Unified Device Architecture (CUDA) for conjugate gradient method computation.

4. The experiment and the result

We have validated our design by experiment. The scheme of the target object distribution in the experiment is shown in Fig. 4. Two metallic thin plates with a right-angle corner and one linear metallic object were located under the sparse array antennas. A horizontal slice of the image reconstruction results is shown in Fig. 5.

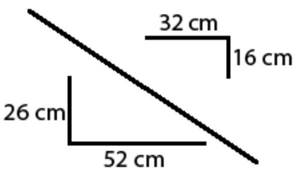


Fig. 4. Target object distribution in the experiment

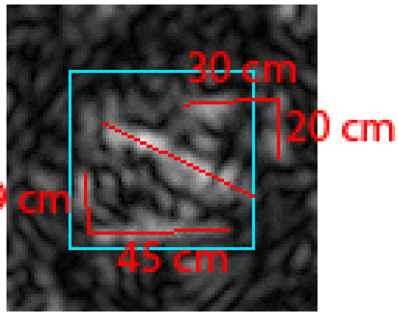


Fig. 5. Result of image reconstruction

Blue line indicates overall size of the sparse array system 60 cm x 60 cm. The distance from antenna feeding points to the targets was 89 cm. The red lines represent metallic objects, which were detected by the radar. The red captions are the lengths of the different sides of the right-angle corners in centimeters, which was measured from the radar results. The artifacts that we observe in the figure are caused by low quality of the raw signal. We can distinguish all three objects and see their relative position, which matched well with our expectations. Difference of measured sizes and actual sizes of the target objects is 5 cm in the mean. In addition, we can detect some object that is located out of the area, where is sparse array located.

5. Conclusion

We proposed antenna configuration optimization base on the principle of uniformly distributed equilateral triangles. Least square method was implemented to reconstruct image of sparse array system and minimize the number of artifacts. We carried out the experiment with 8 transmitters and 8 receivers, which were arranged in the area 60 cm x 60 cm. The operating frequency of SFCW radar is from 3814 MHz to 8067 MHz. On the obtained image we can distinguish all objects and measure their size with mean precision of 5 cm.

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