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Rotationally Symmetric (RS)-LOD-FDTD with CPML for Analysing Resonant Structures

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1. Introduction

Structures with rotational symmetry (RS) are commonly encountered in many wireless systems that involve antennas and microwave filters. Such rotationally symmetric structures have body of revolution (BOR) symmetry which allows one to analytically extract the known azimuthal behaviour of the fields around the axis of symmetry, and to project the original three-dimensional problem to a numerically solvable two-dimensional plan, reducing the computationally burden substantially in the process. The rotationally symmetric resonant structures have been analysed using various analytical and numerical methods of electromagnetics such as the mode matching method, integral equation technique, the finite element method, and the finite difference time domain method [1]-[2]. The rotationally symmetric finite-difference time domain (RS-FDTD) method has also been used effectively for treating electromagnetic problems in time domain, involving structures with circular symmetry [2]. However, RS-FDTD suffers from Courant-Friedrich-Lewy (CFL) stability constraint and as a result, finer grid sizes and smaller time steps are required to retain the stability which will cause significant increase in computational time.

To eliminate dependence on CFL stability constraint, implicit FDTD methods viz., ADI-FDTD and LOD-FDTD have been introduced [3], [5]-[6]. The LOD formulation is a simple type of split-step approach [5] and is first order accurate in time [6], whereas the ADI formulation is second order accurate in time. A body of revolution version of the ADI-FDTD technique has also been proposed in [4]. Recently, the LOD scheme, [5]-[6] has been extended for LOD-BOR-FDTD method [7]-[8]. However, so far only conventional absorbing boundary conditions such as MUR and PML have been used with the LOD-BOR-FDTD method in the literature [7]-[8] for bodies with rotational symmetry. It has been established that employing conventional absorbing boundary conditions such as PML etc.do not lead to improved solution accuracies. Hence, there is a need to derive the new convolutional perfectly matched layer (CPML) absorbing boundary condition for rotationally symmetric version of LOD-FDTD. Further, when analysing cylindrical resonant structures using LOD-BOR-FDTD method, it is also important to introduce improved spectral estimation techniques to reduce the computational time.

In this paper, we present a RS-LOD-FDTD formulation for rotationally symmetric resonant cavities and derive the convolutional perfectly matched layer (CPML) absorbing boundary condition for this method. We also extend digital signal processing based spectral estimation technique to efficiently analyse the resonant frequencies of cylindrical resonant structures. The numerical results show that the RS-LOD-FDTD-CPML method with improved spectral estimation provides accurate results while reducing the computational time.

2. Analysis

2.1 RS-LOD-FDTD Method

In RS-LOD-FDTD method, it is assumed that the angular variation of the electromagnetic fields has either $\sin(m\varphi)$ or $\cos(m\varphi)$ which may be factored out of Maxwell's equations. As a result, the fields at any arbitrary $\varphi = \varphi_0$ plane may be related to its corresponding value in the reference coordinates. To obtain the 2-D RS-LOD-FDTD lattice begin with the 3-D Yee cell in cylindrical coordinates. This is then projected onto the r-z plane to give a 2-D lattice. Note that in

the 2-D cell ($E_z\,, H_r$) and ($E_r\,, H_z$) share the same positions. So, according to the LOD-FDTD principle [5]-[6], the updating equations of RS-LOD-FDTD for TE_{01} mode can be written as follows. For brevity, we will examine the case for m=0 and provide only one step.
First step:

$$\begin{aligned} H_r\,|^{n+1/2}_{i+1/2,j} = H_r\,|^{n}_{i+1/2,j} &+ \frac{\Delta t}{2\mu\Delta z}\left(E_\varphi\,|^{n+1/2}_{i+1/2,j+1/2} - E_\varphi\,|^{n+1/2}_{i+1/2,j-1/2}\right) \\ &+ \frac{\Delta t}{2\mu\Delta z}\left(E_\varphi\,|^{n}_{i+1/2,j+1/2} - E_\varphi\,|^{n}_{i+1/2,j-1/2}\right) \end{aligned} \tag{1a}$$

$$\begin{aligned} E_\varphi\,|^{n+1/2}_{i+1/2,j+1/2} = E_\varphi\,|^{n}_{i+1/2,j+1/2} &+ \frac{\Delta t}{2\varepsilon\Delta z}\left(H_r\,|^{n+1/2}_{i+1/2,j+1} - H_r\,|^{n+1/2}_{i+1/2,j}\right) \\ &+ \frac{\Delta t}{2\varepsilon\Delta z}\left(H_r\,|^{n}_{i+1/2,j+1} - H_r\,|^{n}_{i+1/2,j}\right) \end{aligned} \tag{1b}$$

where $E\,, H$ are electric and magnetic field intensities.

2.2 CPML Absorbing Boundary Condition for RS-LOD-FDTD Method

So far in the literature, the convolutional perfectly matched layer (CPML) absorbing boundary condition has not been considered with LOD-BOR-FDTD method [7]-[8]. But in this paper, we derive CPML absorbing boundary condition for the RS-LOD-FDTD method. The CPML is highly effective in absorbing evanescent waves with long time signature and can be placed closer to the objects within the problem space so as to gain time and memory savings. The CPML formulation for RS-LOD-FDTD method for TE wave is given below. For brevity, in this paper we provide only one step for the explanation of the method for RS-LOD-FDTD CPML.
First step:

$$\begin{aligned} H_r\,|^{n+1/2}_{i+1/2,j} = H_r\,|^{n}_{i+1/2,j} &+ \frac{\Delta t}{2\mu\Delta z k(j)}\left(E_\varphi\,|^{n+1/2}_{i+1/2,j+1/2} - E_\varphi\,|^{n+1/2}_{i+1/2,j-1/2}\right) \\ &+ \frac{\Delta t}{2\mu\Delta z k(j)}\left(E_\varphi\,|^{n}_{i+1/2,j+1/2} - E_\varphi\,|^{n}_{i+1/2,j-1/2}\right) + \frac{\Delta t}{\mu\Delta z k(j)}\psi_{hrz}\,|^{n}_{i+1/2,j} \end{aligned} \tag{2a}$$

$$\begin{aligned} E_\varphi\,|^{n+1/2}_{i+1/2,j+1/2} = E_\varphi\,|^{n}_{i+1/2,j+1/2} &+ \frac{\Delta t}{2\varepsilon\Delta z k(j)}\left(H_r\,|^{n+1/2}_{i+1/2,j+1} - H_r\,|^{n+1/2}_{i+1/2,j}\right) \\ &+ \frac{\Delta t}{2\varepsilon\Delta z k(j)}\left(H_r\,|^{n}_{i+1/2,j+1} - H_r\,|^{n}_{i+1/2,j}\right) + \frac{\Delta t}{\varepsilon\Delta z k(j)}\psi_{e\varphi z}\,|^{n}_{i+1/2,j+1/2} \end{aligned} \tag{2b}$$

Where, ψ_{hrz} and $\psi_{e\varphi z}$ are discrete variables which may have non-zero values only in some CPML regions but are necessary to implement the absorbing boundary [1].

2.3 Spectral Estimation Method

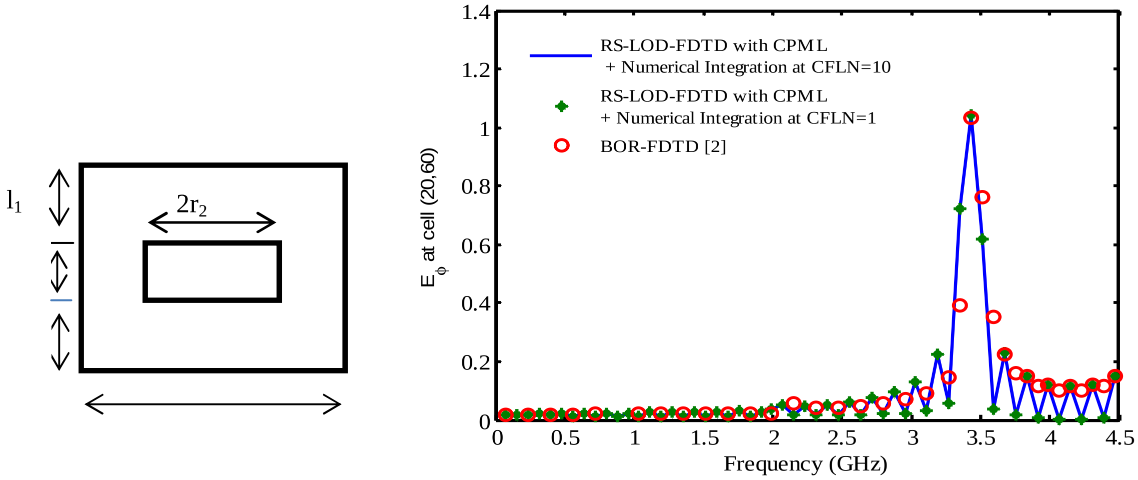
In order to transform the time domain data to the frequency domain, spectral estimation methods are required. Spectral estimation technique with explicit FDTD [10] is well known for calculating the resonant frequencies of cylindrical structures. Here, we employ spectral estimation technique with implicit RS-LOD-FDTD. The digital filtering and estimation as described in [10] has been implemented for this analysis. The merit of this technique is that it removes ambiguities sometimes encountered with the discrete Fourier Transform, due to narrow band signal components with the centre frequency.

3. Numerical Results

To show the validity of the RS-LOD-FDTD technique described in the previous section, the resonant frequencies of the cylindrical resonators were calculated and compared with the published results using BOR-FDTD as well as with the measured results. The first structure which was analysed was a circular cavity with a dielectric disc (ε_r =35.74) placed at the centre of the cavity. The cross section of the structure is shown in Fig.1 (a). Table 1 shows comparison of the results calculated using our RS-LOD-FDTD method with CPML and spectral estimation method with the results obtained with BOR-FDTD which are taken from [2]. The size of the spatial mesh employed for the RS-LOD-FDTD simulation was $\Delta r = 0.17272$ mm and $\Delta z = 0.1524$ mm. Fig. 1 (b) shows the strength E_φ versus frequency, in which CFLN=10 is used for RS-LOD-FDTD method. From the Table 1 and Fig. 1 (b), it is seen that the result calculated using spectral estimation with RS-LOD-FDTD agrees reasonably well with the published results in the literature [2].

Table 1: Comparison of TE_{01} Resonant Frequencies Calculated and BOR-FDTD [2] with b=2.5908, $2r_2$=1.7272, l_1=0.381, l_2=0.762, l_3=0.381, (units are in cm) and ε_r=35.74

Mode	RS-LOD-FDTD with CPML	BOR-FDTD [2]	% Error
TE_{01}	3.433	3.435	0.06



(a) (b)

Figure 1: (a) Cross section of the cylindrical cavity with a dielectric disk filling (ε_r =35.74), (b) E_φ versus frequency for the cavity.

The second example is the dielectric ring resonator loaded in a circular waveguide taken from [9]. The discretization values are $\Delta r = 0.127$ mm and $\Delta z = 0.127$ mm. The results obtained from using our RS-LOD-FDTD-CPML algorithm for this structure is compared with the measured result [9] and published BOR-FDTD results [2] are tabulated in Table 2. Again, the calculated results are in good agreement with the measured ones.

Table 2: Comparison of TE_{01} Resonant Frequencies Calculated and those Measured in [9] with 2a= 0.25, 2b=0.8, 2c=1.2, l_1=0.205, l_2=0.28, l_3=0.295 (units are in inches) and ε_r=36.

Mode	RS-LOD-FDTD with CPML	BOR-FDTD [2]	Measured [9]	% Error
TE_{01}	3.080	3.081	3.086	0.2

To compare performances of our RS-LOD-FDTD method with ADI-BOR-FDTD method, we discuss the number of arithmetic operations required. It is worth mentioning that only two equations are solved in each half step for RS-LOD-FDTD method. This leads to a reduction in the number of arithmetic operations for the updating equations as compared to the ADI-BOR-FDTD, as shown in Table 3, in which M/D and A/S indicate multiplication/division and addition/ subtraction operations, respectively. It can be observed that the number of arithmetic operations for the RS-LOD-FDTD is substantially reduced. This results in a simple algorithm, with a subsequent reduction in the computational time. The proposed method takes 25.8 s which can achieve up to

40% saving in CPU time in comparison with ADI-BOR-FDTD method which takes 45.44 s for calculating the resonant frequency of the dielectric resonator.

Table 3: Number of Arithmetic Operations

	Implicit		Explicit		Total	
	M/D	A/S	M/D	A/S	M/D	A/S
ADI-BOR-FDTD [4]	14+14	20+20	13+13	20+20	54	80
RS-LOD-FDTD	11+11	10+10	6+6	10+10	34	40

4. Conclusions

In this paper, RS-LOD-FDTD coupled with CPML and spectral estimation technique is presented for computing the resonant frequencies of rotationally symmetric cylindrical resonators. The proposed method is unconditionally stable and the numerical results agree reasonably well with the results available in the literature as well as with the measured results. Comparing with ADI-BOR-FDTD, the proposed method is characterized by lighter computational burden. The proposed method will be extended to model more complex rotationally symmetry structures in the near future.

Acknowledgments

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