Formulation of Two-Dimensional Electromagnetic Scattering from Circular Cylinder Backed by Periodic Circular Cylinder Array

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Abstract

The present paper formulates the electromagnetic scattering by a structure consisting of a periodic array of circular cylinder and another circular cylinder located in front of the array. Periodicity of the structure is locally collapsed and the standard formulations based on Floquet's theorem cannot be applied. In this paper, pseudo-periodic Fourier transform is formally introduced to analyze the problem. This transform makes the field components pseudo-periodic and they are then expressed in the Rayleigh expansion outside the cylinders. The recursive transition-matrix algorithm with the help of Yasumoto et al's formula for the lattice sums is used to calculate the scattering by the periodic cylinder array.

1. Introduction

Electromagnetic scattering from periodic structures has been extensively studied for a long time as wavelength and polarization selective components in microwave, millimeter-wave, and optical wave regions. Recently, structures in which the periodicity is locally collapsed have also received much interest because they induce distinct properties. One of the most promising properties is the field localization caused by defects in the electromagnetic bandgap structures that allows realizing essential optical elements of wavelength size. For example, a lump of defects may behave as a microcavity resonator and a line defect may behave as a waveguide. The computation of such structures has been mainly performed with the finite differences time-domain method, the beam propagation method, the method of fictitious sources, and the recursive transition-matrix algorithm (RTMA)[1]. However, these approaches are able to apply to the problems of scatterers with finite extent, and the approaches for scatterer with infinite extent has been limited to fully periodic structure with plane wave incidence.

This paper proposes a novel formulation of the electromagnetic scattering from a circular cylinder backed by periodic array of circular cylinders. The cylinders are infinitely long and parallel to each other with identical separations, while one cylinder is added in front of the array. To approach the problem, we introduce a new idea, which is the pseudo-periodic Fourier transform (PPFT). This transform converts any field component into a pseudo-periodic function, which is a product of a periodic function and an exponential phase factor, and all transformed components can be expanded in the generalized Fourier series[2]. Hence, the transformed fields outside the cylinder objects can be expressed by a superposition of plane waves. The reflected and the transmitted waves of the periodic cylinder array for plane wave incidence are derived by RTMA with the help of Yasumoto et al's formula for the lattice sums[3], [4] though the scattering by the additional cylinder is analyzed by the standard process of RTMA. The proposed formulation is applied to a periodic cylinder array with an additional cylinder for a line source excitation, and we present the field intensity distribution near the additional cylinder.

2. Formulation

A. Setting of the Problem

The geometry under consideration is schematically shown in Fig. 1. The structure consists of a circular cylinder and a periodic array of circular cylinders that are infinitely long in the z-direction and situated parallel to each other. The periodic array consists of the identical cylinders with homogeneous and isotropic media described by the permittivity ε_p, the permeability μ_p, and the radius a_p. One cylinder in the periodic array is located at the origin and the other cylinders

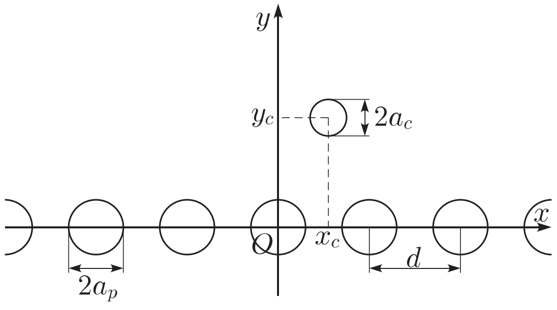


Fig. 1: Circular cylinder backed by periodic array of circular cylinders.

are periodically spaced with a common distance d in the x-direction. An additional cylinder with the permittivity ε_c, the permeability μ_c, and the radius a_c is located at $(x, y) = (x_c, y_c)$ $(y_c > a_p + a_c)$. The surrounding region is filled by a lossless, homogeneous, and isotropic material with the permittivity ε_s and the permeability μ_s. We deal with only time-harmonic fields assuming a time-dependence in $e^{-i\omega t}$, and the electromagnetic fields are supposed to be uniform in the z-direction. Two fundamental polarizations are expressed by TM and TE, in which the H and the E fields are respectively perpendicular to the z-axis. The total field is expressed by the sum of the incident field $\psi^{(i)}(x, y)$, the scattered field from the periodic cylinder array $\psi_p^{(s)}(x, y)$, and the scattered field from the additional cylinder $\psi_c^{(s)}(x, y)$ in the following form:

$$\psi(x, y) = \psi^{(i)}(x, y) + \psi_p^{(s)}(x, y) + \psi_c^{(s)}(x, y). \tag{1}$$

The incident field $\psi^{(i)}(x, y)$ is assumed to illuminate the cylinders from the upper or lower regions and there exists no source inside the structure $-a_p \le y \le y_c + a_c$.

B. Pseudo-Periodic Fourier Transform and Rayleigh Expansion

Let $\psi(x, y)$ be a two-dimensional wave function satisfying the following Helmholtz equation outside the cylinders:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + {k_s}^2\right)\psi(x, y) = 0 \tag{2}$$

where k_s is the wavenumber in the surrounding media and supposed to be real and constant. Here we introduce a transform defined by

$$\overline{\psi}(x; \xi, y) = \sum_{m=-\infty}^{\infty} \psi(x - m\,d, y)\, e^{i\,m\,d\,\xi} \tag{3}$$

which is implicitly assumed to be converge, and we call this transform the pseudo-periodic Fourier transform (PPFT). The inverse of PPFT (IPPFT) is formally given as

$$\psi(x, y) = \frac{d}{2\pi}\int_{-\pi/d}^{\pi/d} \overline{\psi}(x; \xi, y)\, d\xi. \tag{4}$$

The differential operators in Eq. (2) are unchanged by PPFT, and then the equation is transformed into the same form:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + {k_s}^2\right)\overline{\psi}(x; \xi, y) = 0. \tag{5}$$

The transformed field $\overline{\psi}(x; \xi, y)$ has a pseudo-periodic property with the pseudo-period d in terms of x:

$$\overline{\psi}(x - d; \xi, y) = \overline{\psi}(x; \xi, y)\, e^{-i\,d\,\xi}, \tag{6}$$

and therefore can be expressed in the Rayleigh expansion[2]:

$$\begin{aligned}\overline{\psi}(x; \xi, y) = \boldsymbol{f}^{(+)}(x; \xi, y - y')^t\, \overline{\boldsymbol{a}}^{(+)}(\xi, y') \\ + \boldsymbol{f}^{(-)}(x; \xi, y - y')^t\, \overline{\boldsymbol{a}}^{(-)}(\xi, y')\end{aligned} \tag{7}$$

where $\overline{\boldsymbol{a}}^{(+)}(\xi, y')$ and $\overline{\boldsymbol{a}}^{(-)}(\xi, y')$ denote the column matrices generated by the amplitude of plane waves propagating in the positive and the negative y-direction, respectively, and $\boldsymbol{f}^{(\pm)}(x; \xi, y)$ are the column matrices of the plane wave basis sets given as

$$\left(\boldsymbol{f}^{(\pm)}(x; \xi, y)\right)_n = e^{i(\alpha_n(\xi)\,x \pm \beta_n(\xi)\,y)} \tag{8}$$

with

$$\alpha_n(\xi) = \xi + n\,\frac{2\pi}{d} \tag{9}$$

$$\beta_n(\xi) = \sqrt{{k_s}^2 - \alpha_n(\xi)^2}. \tag{10}$$

C. Scattering by Periodic Cylinder Array

First, we consider the scattering by the periodic cylinder array located at $y = 0$. The transformed field $\overline{\psi}(x; \xi, y)$ near the cylinder array is decomposed into the incident and the scattered fields:

$$\overline{\psi}(x; \xi, y) = \overline{\psi}_p^{(i)}(x; \xi, y) + \overline{\psi}_p^{(s)}(x; \xi, y), \tag{11}$$

where the first term is PPFT of $\psi^{(i)}(x, y) + \psi_c^{(s)}(x, y)$ given in Eq. (1).

Using Eq. (7), the incident field is expressed as

$$\begin{aligned}\overline{\psi}_p^{(i)}(x; \xi, y) = \boldsymbol{f}^{(-)}(x; \xi, y)^t\, \overline{\boldsymbol{a}}^{(-)}(\xi, +0) \\ + \boldsymbol{f}^{(+)}(x; \xi, y)^t\, \overline{\boldsymbol{a}}^{(+)}(\xi, -0)\end{aligned} \tag{12}$$

Here, we introduce the cylindrical wave expansions. Let Z denote the cylinder functions J or $H^{(1)}$, and $\boldsymbol{g}^{(Z)}(x, y)$ be column matrices generated by the corresponding cylindrical waves in such a way that its nth entries are given as

$$\left(\boldsymbol{g}^{(Z)}(x, y)\right)_n = Z_n(k_s\,\rho(x, y))\, e^{i\,n\,\phi(x,y)} \tag{13}$$

with

$$\rho(x, y) = \sqrt{x^2 + y^2} \tag{14}$$

$$\phi(x, y) = \arg(x + i\,y). \tag{15}$$

Then the incident field $\overline{\boldsymbol{\psi}}_p^{(i)}(x; \xi, y)$ in Eq. (12) is rewritten as

$$\overline{\psi}_p^{(i)}(x; \xi, y) = \boldsymbol{g}^{(J)}(x, y)^t\, \overline{\boldsymbol{b}}_0^{(i)}(\xi) \tag{16}$$

where $\overline{\boldsymbol{b}}_0^{(i)}(\xi)$ denotes the column matrix giving the expansion coefficients and derived as

$$\overline{\boldsymbol{b}}_0^{(i)}(\xi) = \boldsymbol{P}^{(-)}(\xi)^t\, \overline{\boldsymbol{a}}^{(-)}(\xi, +0) + \boldsymbol{P}^{(+)}(\xi)^t\, \overline{\boldsymbol{a}}^{(+)}(\xi, -0) \tag{17}$$

with

$$\left(\boldsymbol{P}^{(\pm)}(\xi)\right)_{n,m} = \left(\frac{i\,\alpha_n(\xi) \pm \beta_n(\xi)}{k_s}\right)^m. \tag{18}$$

On the other hand, the scattered field can be expressed by the sum of the outgoing wave from each scatterer. Therefore, the scattered fields can be written as follows:

$$\overline{\psi}_p^{(s)}(x; \xi, y) = \sum_{l=-\infty}^{\infty} \boldsymbol{g}^{(H^{(1)})}(x - l\,d, y)^t\, \overline{\boldsymbol{b}}_l^{(s)}(\xi). \tag{19}$$

Because of the pseudo-periodicity of the transformed field, Floquet's theorem yields the following relation

$$\overline{\boldsymbol{b}}_l^{(s)}(\xi) = \overline{\boldsymbol{b}}_0^{(s)}(\xi)\, e^{i\,l\,d\,\xi}. \tag{20}$$

Then a RTMA technique developed by Yasumoto *et al*.[3], [4] gives the relation between $\overline{\boldsymbol{b}}_0^{(s)}(\xi)$ and $\overline{\boldsymbol{b}}_0^{(i)}(\xi)$ in the following form:

$$\overline{\boldsymbol{b}}_0^{(s)}(\xi) = \left(\boldsymbol{T}_p^{-1} - \boldsymbol{L}(k_s\, d, \xi\, d)\right)^{-1} \overline{\boldsymbol{b}}_0^{(i)}(\xi) \tag{21}$$

with

$$(\boldsymbol{L}(\zeta,\eta))_{n,m} = \sum_{l=1}^{\infty} H_{n-m}^{(1)}(l\,\zeta)\left[e^{i\,l\,\eta} + (-1)^{n-m}\,e^{-i\,l\,\eta}\right]. \tag{22}$$

The entries of $\boldsymbol{L}(\zeta,\eta)$ is given by the lattice sums, which are known to converge very slowly. An efficient calculation of lattice sums has been developed by Yasumoto and Yoshitomi[3], and we use it for practical computation. Also, the square diagonal matrix $\boldsymbol{T}_p$ is the transition-matrix (T-matrix) of the unit cylinder of the array in isolation. Concrete representation of the T-matrix depends on the polarization, and the (n,m)th-entries are given by

$$\begin{aligned}(\boldsymbol{T}_p)_{n,m} &= \delta_{n,m} \\ &\times \frac{\sqrt{\frac{\varepsilon_p}{\mu_p}} J_n(k_s a_p) J_n'(k_p a_p) - \sqrt{\frac{\varepsilon_s}{\mu_s}} J_n'(k_s a_p) J_n(k_p a_p)}{\sqrt{\frac{\varepsilon_s}{\mu_s}} H_n^{(1)\prime}(k_s a_p) J_n(k_p a_p) - \sqrt{\frac{\varepsilon_p}{\mu_p}} H_n^{(1)}(k_s a_p) J_n'(k_p a_p)}\end{aligned} \tag{23}$$

for TM polarization, and

$$\begin{aligned}(\boldsymbol{T}_p)_{n,m} &= \delta_{n,m} \\ &\times \frac{\sqrt{\frac{\mu_p}{\varepsilon_p}} J_n(k_s a_p) J_n'(k_p a_p) - \sqrt{\frac{\mu_s}{\varepsilon_s}} J_n'(k_s a_p) J_n(k_p a_p)}{\sqrt{\frac{\mu_s}{\varepsilon_s}} H_n^{(1)\prime}(k_s a_p) J_n(k_p a_p) - \sqrt{\frac{\mu_p}{\varepsilon_p}} H_n^{(1)}(k_s a_p) J_n'(k_p a_p)}\end{aligned} \tag{24}$$

for TE polarization, where k_p denotes the wavenumber inside the cylinders in the periodic array and $\delta_{n,m}$ denotes Kronecker's delta.

The scattered field $\overline{\psi}_p^{(s)}(x;\xi,y)$ in Equation (19) can be expressed in the Rayleigh expansion:

$$\overline{\psi}^{(s)}(x;\xi,y) = \begin{cases} \boldsymbol{f}^{(+)}(x;\xi,y)^t\,\boldsymbol{Q}^{(+)}(\xi)\,\overline{\boldsymbol{b}}_0^{(s)}(\xi) & \text{for } y>0 \\ \boldsymbol{f}^{(-)}(x;\xi,y)^t\,\boldsymbol{Q}^{(-)}(\xi)\,\overline{\boldsymbol{b}}_0^{(s)}(\xi) & \text{for } y<0 \end{cases} \tag{25}$$

with

$$\left(\boldsymbol{Q}^{(\pm)}(\xi)\right)_{n,m} = \begin{cases} \frac{2}{\beta_n(\xi)\,d}\left(\frac{-i\,\alpha_n(\xi)\pm\beta_n(\xi)}{k_s}\right)^m & \text{for } m\geq 0 \\ \frac{2}{\beta_n(\xi)\,d}\left(\frac{i\,\alpha_n(\xi)\pm\beta_n(\xi)}{k_s}\right)^{-m} & \text{for } m<0 \end{cases}. \tag{26}$$

Then the scattering matrix that relate the Rayleigh coefficients of the incident and the scattered fields is given as

$$\begin{pmatrix}\overline{\boldsymbol{a}}^{(+)}(\xi,+0) \\ \overline{\boldsymbol{a}}^{(-)}(\xi,-0)\end{pmatrix} = \begin{pmatrix}\boldsymbol{S}_{p,11}(\xi) & \boldsymbol{S}_{p,12}(\xi) \\ \boldsymbol{S}_{p,21}(\xi) & \boldsymbol{S}_{p,22}(\xi)\end{pmatrix}\begin{pmatrix}\overline{\boldsymbol{a}}^{(-)}(\xi,+0) \\ \overline{\boldsymbol{a}}^{(+)}(\xi,-0)\end{pmatrix} \tag{27}$$

with

$$\boldsymbol{S}_{p,11}(\xi) = \boldsymbol{Q}^{(+)}(\xi)\left(\boldsymbol{T}_p^{-1} - \boldsymbol{L}(k_s d,\xi d)\right)^{-1}\boldsymbol{P}^{(-)}(\xi)^t \tag{28}$$

$$\boldsymbol{S}_{p,12}(\xi) = \boldsymbol{I} + \boldsymbol{Q}^{(+)}(\xi)\left(\boldsymbol{T}_p^{-1} - \boldsymbol{L}(k_s d,\xi d)\right)^{-1}\boldsymbol{P}^{(+)}(\xi)^t \tag{29}$$

$$\boldsymbol{S}_{p,21}(\xi) = \boldsymbol{I} + \boldsymbol{Q}^{(-)}(\xi)\left(\boldsymbol{T}_p^{-1} - \boldsymbol{L}(k_s d,\xi d)\right)^{-1}\boldsymbol{P}^{(-)}(\xi)^t \tag{30}$$

$$\boldsymbol{S}_{p,22}(\xi) = \boldsymbol{Q}^{(-)}(\xi)\left(\boldsymbol{T}_p^{-1} - \boldsymbol{L}(k_s d,\xi d)\right)^{-1}\boldsymbol{P}^{(+)}(\xi)^t. \tag{31}$$

D. Scattering by Additional Cylinder

Next, we consider the scattering by the additional cylinder located at $(x,y)=(x_c,y_c)$. The transformed field $\overline{\psi}(x;\xi,y)$ near the additional cylinder is decomposed into the incident and the scattered fields in the following form:

$$\overline{\psi}(x;\xi,y) = \overline{\psi}_c^{(i)}(x;\xi,y) + \overline{\psi}_c^{(s)}(x;\xi,y). \tag{32}$$

The incident field for the cylinder $\overline{\psi}_c^{(i)}(x;\xi,y)$ is PPFT of $\psi^{(i)}(x,y)+\psi_p^{(s)}(x,y)$ by using the notations in Eq. (1), and also it is possible to express in the Rayleigh expansion and the cylindrical wave expansion as follows:

$$\begin{aligned}\overline{\psi}_c^{(i)}(x;\xi,y) &= \boldsymbol{f}^{(-)}(x;\xi,y-y_c)^t\,\overline{\boldsymbol{a}}^{(-)}(\xi,y_c+0) \\ &\quad + \boldsymbol{f}^{(+)}(x;\xi,y-y_c)^t\,\overline{\boldsymbol{a}}^{(+)}(\xi,y_c-0) \qquad (33) \\ &= \boldsymbol{g}^{(J)}(x-x_c,y-y_c)^t\,\overline{\boldsymbol{b}}_c^{(i)}(\xi) \qquad (34)\end{aligned}$$

where relation between $\overline{\boldsymbol{a}}^{(\pm)}(\xi,y_c-0)$ and $\overline{\boldsymbol{b}}_c^{(i)}(\xi)$ are given by

$$\begin{aligned}\overline{\boldsymbol{b}}_c^{(i)}(\xi) &= \boldsymbol{P}^{(-)}(\xi)^t\,\boldsymbol{F}(x_c;\xi,0)\,\overline{\boldsymbol{a}}^{(-)}(\xi,y_c+0) \\ &\quad + \boldsymbol{P}^{(+)}(\xi)^t\,\boldsymbol{F}(x_c;\xi,0)\,\overline{\boldsymbol{a}}^{(+)}(\xi,y_c-0)\end{aligned} \tag{35}$$

with

$$(\boldsymbol{F}(x;\xi,y))_{n,m} = \delta_{n,m}\,e^{i(\alpha_n(\xi)\,x+\beta_n(\xi)\,y)}. \tag{36}$$

The scattered field is expressed by the sum of the outgoing wave from the cylinder and written as

$$\overline{\psi}_c^{(s)}(x;\xi,y) = \boldsymbol{g}^{(H^{(1)})}(x-x_c,y-y_c)^t\,\overline{\boldsymbol{b}}_c^{(s)}(\xi). \tag{37}$$

The expansion coefficients are obtained as

$$\overline{\boldsymbol{b}}_c^{(s)}(\xi) = \boldsymbol{T}_c\,\overline{\boldsymbol{b}}_c^{(i)}(\xi) \tag{38}$$

where $\boldsymbol{T}_c$ is the T-matrix of the cylinder and given by replacing the subscript p in Eqs. (23) and (24) by c. IPPFT yields

$$\psi_c^{(s)}(x,y) = \boldsymbol{g}^{(H^{(1)})}(x-x_c,y-y_c)^t\,\boldsymbol{b}_c^{(s)} \tag{39}$$

with

$$\boldsymbol{b}_c^{(s)} = \frac{d}{2\pi}\int_{-\pi/d}^{\pi/d}\overline{\boldsymbol{b}}_c^{(s)}(\xi)\,d\xi. \tag{40}$$

PPFT is applied to the scattered field Eq. (39), and we obtain

$$\begin{aligned}&\overline{\psi}^{(s)}(x;\xi,y) \\ &= \begin{cases}\boldsymbol{f}^{(+)}(x-x_c;\xi,y-y_c)^t\boldsymbol{Q}^{(+)}(\xi)\,\boldsymbol{b}_c^{(s)} & \text{for } y>y_c \\ \boldsymbol{f}^{(-)}(x-x_c;\xi,y-y_c)^t\boldsymbol{Q}^{(-)}(\xi)\,\boldsymbol{b}_c^{(s)} & \text{for } y<y_c\end{cases}.\end{aligned} \tag{41}$$

Then the scattering matrix is given in the following form:

$$\begin{aligned}\begin{pmatrix}\overline{\boldsymbol{a}}^{(+)}(\xi,y_c+0) \\ \overline{\boldsymbol{a}}^{(-)}(\xi,y_c-0)\end{pmatrix} &= \begin{pmatrix}\overline{\boldsymbol{a}}^{(+)}(\xi,y_c-0) \\ \overline{\boldsymbol{a}}^{(-)}(\xi,y_c+0)\end{pmatrix} \\ &+\int_{-\pi/d}^{\pi/d}\begin{pmatrix}\boldsymbol{S}_{c,11}(\xi,\xi') & \boldsymbol{S}_{c,12}(\xi,\xi') \\ \boldsymbol{S}_{c,21}(\xi,\xi') & \boldsymbol{S}_{c,22}(\xi,\xi')\end{pmatrix}\begin{pmatrix}\overline{\boldsymbol{a}}^{(-)}(\xi',y_c+0) \\ \overline{\boldsymbol{a}}^{(+)}(\xi',y_c-0)\end{pmatrix}d\xi'\end{aligned} \tag{42}$$

with

$$\boldsymbol{S}_{c,11}(\xi,\xi') = \frac{d}{2\,\pi}\,\boldsymbol{F}(-x_c;\xi,0)\,\boldsymbol{Q}^{(+)}(\xi)\,\boldsymbol{T}_c \cdot \boldsymbol{P}^{(-)}(\xi')^t\,\boldsymbol{F}(x_c;\xi',0) \tag{43}$$

$$\boldsymbol{S}_{c,12}(\xi,\xi') = \frac{d}{2\,\pi}\,\boldsymbol{F}(-x_c;\xi,0)\,\boldsymbol{Q}^{(+)}(\xi)\,\boldsymbol{T}_c \cdot \boldsymbol{P}^{(+)}(\xi')^t\,\boldsymbol{F}(x_c;\xi',0) \tag{44}$$

$$\boldsymbol{S}_{c,21}(\xi,\xi') = \frac{d}{2\,\pi}\,\boldsymbol{F}(-x_c;\xi,0)\,\boldsymbol{Q}^{(-)}(\xi)\,\boldsymbol{T}_c \cdot \boldsymbol{P}^{(-)}(\xi')^t\,\boldsymbol{F}(x_c;\xi',0) \tag{45}$$

$$\boldsymbol{S}_{c,22}(\xi,\xi') = \frac{d}{2\pi}\,\boldsymbol{F}(-x_c;\xi,0)\,\boldsymbol{Q}^{(-)}(\xi)\,\boldsymbol{T}_c \cdot \boldsymbol{P}^{(+)}(\xi')^t\,\boldsymbol{F}(x_c;\xi',0). \tag{46}$$

E. Scattering from Entire Structure

When the fields are computed over the entire structure, numerical experiments may show, in many cases, numerical instabilities because of the growing exponential functions. To avoid this difficulty, we use the method proposed in Ref. [5]. Equations (27) and (42) that relate the Rayleigh coefficients must be satisfied for arbitrary $\xi \in [-\pi/d, \pi/d]$. Here we take L sample points $\{\xi_l\}$, and the integration in Eq. (42) is approximated by an appropriate numerical integration scheme in the following form:

$$\begin{pmatrix}\overline{\boldsymbol{a}}^{(+)}(\xi_l, y_c+0)\\ \overline{\boldsymbol{a}}^{(-)}(\xi_l, y_c-0)\end{pmatrix} = \begin{pmatrix}\overline{\boldsymbol{a}}^{(+)}(\xi_l, y_c-0)\\ \overline{\boldsymbol{a}}^{(-)}(\xi_l, y_c+0)\end{pmatrix} + \sum_{l'=1}^{L} w_{l'} \begin{pmatrix}\boldsymbol{S}_{c,11}(\xi_l,\xi_{l'}) & \boldsymbol{S}_{c,12}(\xi_l,\xi_{l'})\\ \boldsymbol{S}_{c,21}(\xi_l,\xi_{l'}) & \boldsymbol{S}_{c,22}(\xi_l,\xi_{l'})\end{pmatrix}\begin{pmatrix}\overline{\boldsymbol{a}}^{(-)}(\xi_{l'}, y_c+0)\\ \overline{\boldsymbol{a}}^{(+)}(\xi_{l'}, y_c-0)\end{pmatrix} \tag{47}$$

where w_l denotes the weight factor. To treat the discretized Rayleigh coefficients systematically, we introduce the following column matrices:

$$\widehat{\boldsymbol{a}}^{(\pm)}(y) = \begin{pmatrix}\overline{\boldsymbol{a}}^{(\pm)}(\xi_1, y)\\ \vdots \\ \overline{\boldsymbol{a}}^{(\pm)}(\xi_L, y)\end{pmatrix} \tag{48}$$

and then Eqs. (27) and (47) are rewritten as follows:

$$\begin{pmatrix}\widehat{\boldsymbol{a}}^{(+)}(+0)\\ \widehat{\boldsymbol{a}}^{(-)}(-0)\end{pmatrix} = \begin{pmatrix}\widehat{\boldsymbol{S}}_{p,11} & \widehat{\boldsymbol{S}}_{p,12}\\ \widehat{\boldsymbol{S}}_{p,21} & \widehat{\boldsymbol{S}}_{p,22}\end{pmatrix}\begin{pmatrix}\widehat{\boldsymbol{F}}\,\widehat{\boldsymbol{a}}^{(-)}(y_c-0)\\ \widehat{\boldsymbol{a}}^{(+)}(-0)\end{pmatrix} \tag{49}$$

$$\begin{pmatrix}\widehat{\boldsymbol{a}}^{(+)}(y_c+0)\\ \widehat{\boldsymbol{a}}^{(-)}(y_c-0)\end{pmatrix} = \begin{pmatrix}\widehat{\boldsymbol{S}}_{c,11} & \widehat{\boldsymbol{S}}_{c,12}\\ \widehat{\boldsymbol{S}}_{c,21} & \widehat{\boldsymbol{S}}_{c,22}\end{pmatrix}\begin{pmatrix}\widehat{\boldsymbol{a}}^{(-)}(y_c+0)\\ \widehat{\boldsymbol{F}}\,\widehat{\boldsymbol{a}}^{(+)}(+0)\end{pmatrix} \tag{50}$$

with

$$\widehat{\boldsymbol{F}} = \begin{pmatrix}\boldsymbol{F}(0;\xi_1,y_c) & & \boldsymbol{0}\\ & \ddots & \\ \boldsymbol{0} & & \boldsymbol{F}(0;\xi_L,y_c)\end{pmatrix} \tag{51}$$

$$\widehat{\boldsymbol{S}}_{p,nm} = \begin{pmatrix}\boldsymbol{S}_{p,nm}(\xi_1) & & \boldsymbol{0}\\ & \ddots & \\ \boldsymbol{0} & & \boldsymbol{S}_{p,nm}(\xi_L)\end{pmatrix} \tag{52}$$

$$\widehat{\boldsymbol{S}}_{c,nm} = \begin{pmatrix}w_1\boldsymbol{S}_{c,nm}(\xi_1,\xi_1) & \cdots & w_L\boldsymbol{S}_{c,nm}(\xi_1,\xi_L)\\ \vdots & \ddots & \vdots\\ w_1\boldsymbol{S}_{c,nm}(\xi_L,\xi_1) & \cdots & w_L\boldsymbol{S}_{c,nm}(\xi_L,\xi_L)\end{pmatrix} + \begin{cases}\boldsymbol{0} & \text{for } n=m\\ \boldsymbol{I} & \text{for } n\neq m\end{cases} \tag{53}$$

where we used also another relation between the Rayleigh coefficients:

$$\overline{\boldsymbol{a}}^{(\pm)}(\xi, y_p) = \boldsymbol{F}(0;\xi,\pm(y_p-y_q))\,\overline{\boldsymbol{a}}^{(\pm)}(\xi, y_q). \tag{54}$$

for $0 < y_p, y_q < y_c$. From Eqs. (49) and (50), we finally obtain the following relation:

$$\begin{pmatrix}\widehat{\boldsymbol{a}}^{(+)}(y_c+0)\\ \widehat{\boldsymbol{a}}^{(-)}(-0)\end{pmatrix} = \widetilde{\boldsymbol{S}}\begin{pmatrix}\widehat{\boldsymbol{a}}^{(-)}(y_c+0)\\ \widehat{\boldsymbol{a}}^{(+)}(-0)\end{pmatrix} \tag{55}$$

where the scattering matrix of the entire structure is derived as

$$\widetilde{\boldsymbol{S}} = \begin{pmatrix}\widehat{\boldsymbol{S}}_{c,12}^{-1} & -\widehat{\boldsymbol{F}}\,\widehat{\boldsymbol{S}}_{p,11}\,\widehat{\boldsymbol{S}}_{p,21}^{-1}\\ -\widehat{\boldsymbol{F}}\,\widehat{\boldsymbol{S}}_{c,22}\,\widehat{\boldsymbol{S}}_{c,12}^{-1} & \widehat{\boldsymbol{S}}_{p,21}^{-1}\end{pmatrix}^{-1} \cdot \begin{pmatrix}\widehat{\boldsymbol{S}}_{c,12}^{-1}\,\widehat{\boldsymbol{S}}_{c,11} & \widehat{\boldsymbol{F}}\Big(\widehat{\boldsymbol{S}}_{p,12}-\widehat{\boldsymbol{S}}_{p,11}\widehat{\boldsymbol{S}}_{p,21}^{-1}\widehat{\boldsymbol{S}}_{p,22}\Big)\\ \widehat{\boldsymbol{F}}\Big(\widehat{\boldsymbol{S}}_{c,21}-\widehat{\boldsymbol{S}}_{c,22}\widehat{\boldsymbol{S}}_{c,12}^{-1}\widehat{\boldsymbol{S}}_{c,11}\Big) & \widehat{\boldsymbol{S}}_{p,21}^{-1}\,\widehat{\boldsymbol{S}}_{p,22}\end{pmatrix}. \tag{56}$$

3. Numerical Example

To validate the present formulation, we consider a specific example excited by a line source situated parallel to the z-axis at $(x,y) = (x_s,y_s)$ for $y_s > y_c + a_c$. The incident field is expressed as

$$\psi^{(i)}(x,y) = H_0^{(1)}(k_s\,\rho(x-x_s, y-y_s)). \tag{57}$$

and the transformed incident field for $y < y_s$ can be expressed by a superposition of the downward propagating plane waves as:

$$\overline{\psi}^{(i)}(x;\xi,y) = \boldsymbol{f}^{(-)}(x;\xi,y-y_c)^t\,\overline{\boldsymbol{a}}^{(-)}(\xi,y_c+0). \tag{58}$$

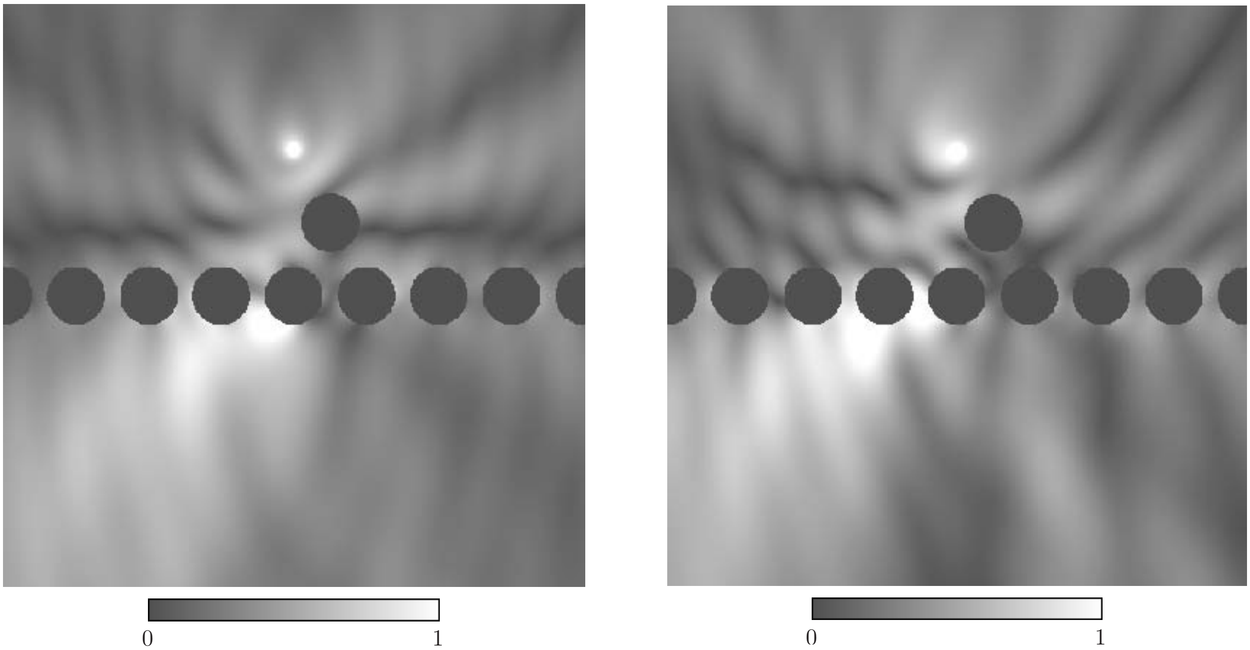
The Rayleigh coefficients of the incoming plane waves are given by

$$\Big(\overline{\boldsymbol{a}}^{(-)}(\xi,y_c+0)\Big)_n = \frac{2}{\beta_n(\xi)d}\,e^{-i[\alpha_n(\xi)\,x_s-\beta_n(\xi)(y_s-y_c)]} \tag{59}$$

$$\Big(\overline{\boldsymbol{a}}^{(+)}(\xi,-0)\Big)_n = 0 \tag{60}$$

and the coefficients of the outgoing plane waves are then calculated by using the scattering matrix given in Eq. (56).

Figure 2 shows the distribution of the total field intensity outside the cylinders. The parameters are chosen as following values: $\varepsilon_s = \varepsilon_0$, $\varepsilon_p = \varepsilon_c = 4\,\varepsilon_0$, $\mu_s = \mu_p = \mu_c = \mu_0$, $d = 0.8\,\lambda_0$, and $a_p = a_c = 0.4\,d$. The additional cylinder is located at $(x_c, y_c) = (0.5\,d, d)$, and the line source is located at $(x_s,y_s) = (-0.5\,d, 2\,d)$. In the presented numerical computation, each cylindrical wave expansion wave truncated 11 terms (from -5th- to 5th-order waves) and the scattered fields are calculated as a sum of the outgoing cylindrical waves from 21 cylinders (axes of the cylinders are situated



(a) TM polarization

(b) TE polarization

Fig. 2: Distribution of the total field intensity outside the cylinders for line source excitation.

at $(x, y) = (0, 0), \ldots, (\pm 10\, d, 0))$ and an additional cylinder. Also, the Gauss-Legendre scheme with the order $L = 100$ is applied to the numerical integration of the convolutions and IPPFT. Figures 2(a) and 2(b) are E_z and H_z distributions for TM and TE polarizations, respectively, and it is observed that reliable results are obtained.

4. CONCLUSION

This paper formulates a novel approach to the two-dimensional electromagnetic scattering of an infinitely periodic cylinder array with an additional cylinder. The field components are converted to pseudo-periodic functions by PPFT, and RTMA is applied with the help of Yasumoto et al's formula for the lattice sums. The proposed formulation is applied to a specific example and numerical experiments have provided reliable results. In this paper, we calculated the fields outside the cylinders but, of course, the fields inside them can be easily obtained. Also, we have dealt with the scattering by dielectric cylinders only. However, this approach is easily arranged to the problems of perfectly conducting cylinders by replacing the T-matrices of the unit cylinders given in Eqs. (23) and (24). Anyway, the present formulation shows an ability of PPFT that enables the mediation between the electromagnetic problems on fully periodic and non-periodic structures.

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