Why periodic surfaces cannot be used to synthesize negative indices of refraction

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In 1968 Veselago wrote his now famous paper where he posed the question: What would happen if a material had both negative permittivity and permeability [1]. He concluded that $\overline{E}$, $\overline{H}$ and the propagation constant $\hat{s}$ would form a *left handed system* instead of the conventional *right handed system*. Perhaps the most striking conclusion was that the index of refraction, i.e., the propagation constant as well, would be negative. Veselago was quick to point out that such materials have not been found in nature; in fact, he prudently added that there were perhaps profound reasons for their absence.

Beginning in the 1990's, Pendry *et al* in a series of papers suggested that negative ε and later negative μ could be produced artificially by use of special elements in a periodic structure [2]. This was followed by numerous papers that claimed to have experimentally observed or theoretically produced negative indices of refraction with periodic structures [3-11, to mention just a few].

Most of these papers were either experimental or based on numerical solutions that revealed very little about what actually went on ***inside*** the metamaterials rather than just the scattered field ***outside***. A notable exception was a paper by Steve Cummer [12]. It actually measures the field inside the DNM media (wires and SRR as usual) but placed inside a waveguide. However, this "slab" of DNM has a one way attenuation of 10 dB, i.e. it takes us way inside the rim of the Smith chart (VSWR ~2:1). A typical input impedance of such a "slab" may look like that illustrated in Figure 1a, if the Smith chart is normalized to the intrinsic impedance Z_1 of the slab. Alternatively we can normalize such that the locus circle is entirely to the left of Z_0 as shown in Figure 1b. If we now observe the input impedance on the locus circle as a function of frequency from the center of the Smith chart, it will look like the impedance on the left side of the locus circle moves clockwise, while the impedance on the right side moves counter clockwise ("wrong" way). The last case is often perceived as an indication of a negative propagation constant. This is, of course, not the case as clearly illustrated for the same material in Figure 1a. Similarly, Figure 1c shows the case when the locus circle is located to the right of Z_0. More details will be given in a future paper [13]. Note that if the "slab" is lossless, the locus circle would be located on the rim of the Smith chart and no ambiguity would exist. In other words, Cummers' paper does not conclusively show negative propagation constant (or negative indices of refraction).

A thorough review of the literature encompassing periodic structures or frequency selective surfaces from 1960's to the present would quickly verify that periodic structures of this type have been studied rather extensively for several decades, which has resulted in a very rigorous theory that has been extensively tested and experimentally verified [14, 15 to mention a few].

More specifically, the precise expression for the H-field for an infinite array of infinitesimal elements, $\hat{p}dl$, with current I, is given by [16]:

$$d\overline{H} = \frac{Idl}{2D_xD_z}\sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{e^{-j\beta\overline{R}\cdot\hat{r}_\pm}}{r_y}\hat{p}\times\hat{r}_\pm \qquad y >< 0 \qquad (1)$$

where D_x and D_z are the interelement spacings of the array, $\hat{r}$ are the directions of the plane wave spectrum and

$$r_y = \sqrt{1-\left(s_x + k\frac{\lambda}{D_x}\right)^2 - \left(s_z + n\frac{\lambda}{D_z}\right)^2}$$

Similarly, the E-field is given by [16]:

$$d\overline{E} = Idl\frac{Z}{2D_xD_z}\sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{e^{-j\beta\overline{R}\cdot\hat{r}_\pm}}{r_y}\left[\hat{p}\times\hat{r}_\pm\right]\times\hat{r}_\pm, \quad y >< 0 \qquad (2)$$

Inspection of (1) and (2) shows that the H-fields are oriented along $\hat{p}\times\hat{r}_\pm$ and the E-fields along $\left[\hat{p}\times\hat{r}_\pm\right]\times\hat{r}_\pm$, i.e., all plane waves given by (1) and (2) are *right handed*. The simplicity of (1) and (2) might suggest that the currents have been approximated in the derivation, like for example, neglecting the interaction between elements. In fact, the Method of Moment Theory developed in [15] contains ***all*** mutual couplings between ***all*** elements regardless of shape and location. As an example see Figures 3.4, 3.6 and 3.7 in [15]. Equations (1) and (2) contain all the evanescent and propagating waves. In other words, they are valid all the way up to the surface of the elements, i.e., they represent the fields everywhere ***inside*** and ***outside*** of the meta-material, regardless of its complexity. Note further that magnetic dipoles are merely loops with ***electric*** currents, i.e. they are automatically incorporated in (1) and (2).

In case we use slots instead of dipoles, we will, of course, use "***magnetic***" currents. However, this does not change our fundamental argument.

Let us finally consider an infinite array of a finite number of segments $\hat{p}_1, \hat{p}_2, \cdots, \hat{p}_p$ with the reference vectors $\overline{R}_1, \overline{R}_2, \cdots, \overline{R}_p$ and current moments $I_1 dl_1, I_2 dl_2, \cdots, I_p dl_p$, respectively. We note that the plane wave directions $\hat{r}$ (spectrum) are the same and identically equal to $\hat{r}$ since all the array of segments $\hat{p}_p$ have the same inter-element spacings D_x and D_z. Thus, the total field H-field from an infinite array of arbitrary elements is according to (1) given by

$$\overline{H} = \frac{1}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta \overline{R}\cdot\hat{r}_\pm}}{r_y} \left[I_1 dl_1 \hat{p}_1 e^{-j\beta \overline{R}_1\cdot\hat{r}_\pm} + I_2 dl_2 \hat{p}_2 e^{-j\beta \overline{R}_2\cdot\hat{r}_\pm} + \cdots + I_p dl_p \hat{p}_p e^{-j\beta \overline{R}_p\cdot\hat{r}_\pm} \right] \times \hat{r}_\pm$$

or

$$\overline{H} = \frac{1}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta \overline{R}\cdot\hat{r}_\pm}}{r_y} \left[composite\ vector \right] \times \hat{r}_\pm, \qquad y >< 0 \qquad (3)$$

We have denoted the square bracket by the "composite vector". It is seen to consist of a sum of unit vectors $\hat{p}_p$ multiplied with the scalars $I_p dl_p$ and the individual phase advances $e^{-j\beta \overline{R}_p\cdot\hat{r}_\pm}$. At this time we shall leave the physical interpretation open, however interesting it might be. What matters is that we by application of (2) can obtain the total E-field for an infinite array of arbitrary elements:

$$\overline{E} = \frac{Z}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta \overline{R}\cdot\hat{r}_\pm}}{r_y} \left[\left[composite\ vector \right] \times \hat{r}_\pm \right] \times \hat{r}_\pm, \qquad y >< 0 \qquad (4)$$

From (3) and (4), we conclude that the total $\overline{E}$ - and H-fields from an infinite array of arbitrary elements can produce only **right handed waves** regardless of the interelement spacings D_x and D_z.

We finally emphasize that the location $\overline{R}_p$ of the individual segments $\hat{p}_p$ is completely arbitrary. Thus, some of the elements can readily be located in other arrays parallel with the original one (not tilted since it would lead to violation of Floquet's Theorem).

Thus, we have actually proven that a multilayered periodic structure can never produce a **left handed field**. Note further, that the evanescent waves are attenuated (as their name indicates) not amplified as they go through the periodic structure. More specifically, EM-waves originate on electric and/or magnetic conductors. They do not suddenly start behaving "strange" between the elements. For more detail, study the theory of periodic structures in [15]. This does not contradict Veselago [1] who made no claims about the existence of materials with negative μ and ε.

Note: It should finally be pointed out that some earlier papers are also skeptical regarding the existence of negative indices of refraction from periodic structures [18, 19, 20].

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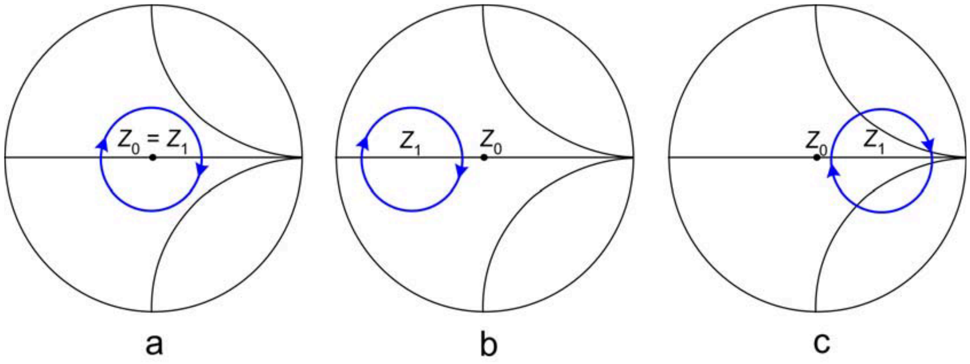

Figure 1. A typical input impedance as a function of frequency of a lossy slab with intrinsic impedance Z_1 terminated in free space. a) Z_1 normalized to the center Z_0 of the Smith chart. b) Normalized such that the locus circle is to the left of Z_0. c) Normalized such that the locus circle is to the right of Z_0. Seen from the center Z_0, it looks like the impedance moves the "wrong way" in the right half of the locus circle in case b, and on the left half in case c. If there is no loss in the slab, the input impedance will be on the rim of the Smith chart and no ambiguity exists.