An Inverse Scattering Method for Lossy Objects Using Time-Reversed Fields

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Abstract - **A time-domain inverse scattering approach using time-reversed fields for reconstructing electrical parameters of cylindrical objects based on gradient method is proposed. The approach does not require the explicit knowledge of incident fields illuminating the object and the time-reversed fields are calculated by time-reversing the equivalent current sources determined by the total fields recorded on a measurement line. This gradient based inverse approach can deal with a reconstruction of lossy object. Numerical simulations are carried out to show the effectiveness of the proposed method.**

Index Terms — **inverse scattering, gradient method, lossy material.**

1. Introduction

A significant number of electromagnetic imaging methods have been proposed last decades because of their theoretical interests and practical importance. There are two classes of imaging problems; reconstruction of electrical parameter distributions of scattering objects and detection of targets. The time-reversal (TR) technique was first proposed for acoustic waves and later has also been applied to electromagnetic waves. The main application of TR techniques is detection and identification of point-like targets. There are few papers that deal with reconstruction problem with TR techniques[1][2].

In this paper, a time-reversal approach for reconstructing of electrical parameters of lossy cylindrical objects based on gradient method without using explicitly information on the incident field is examined instead of the method based on Genetic algorithm(GA)[3]. In order to assess the effectiveness of the approach, some numerical simulations are carried out.

2. Formulation

2.1 Direct problem

Let us consider the scattering by an inhomogeneous object embedded in a homogeneous background medium. For simplicity, we consider two dimensional TM wave case where an electric field is parallel to the axis of the unknown cylindrical object. A short pulsed wave is generated by an electric current source $\mathbf{J}(\mathbf{r},t)=J_z(\mathbf{r},t)\hat{z}$ where $\mathbf{r}=(x,y)$ and $\hat{z}$ is the unit vector in the z direction. We assume that the transmitter source is turned on at time $t=0$ and there is no electromagnetic field before $t=0$. Then, the total fields $\mathbf{v}(\mathbf{r},t)$ satisfy Maxell's equations in the following matrix form:

$$\left[\bar{A}\frac{\partial}{\partial x}+\bar{B}\frac{\partial}{\partial y}-\bar{C}\frac{\partial}{\partial(ct)}-\bar{D}\right]\mathbf{v}=\mathbf{j} \qquad (1)$$

where

$$\mathbf{v}=\begin{pmatrix} v_1(\mathbf{r},t)\\ v_2(\mathbf{r},t)\\ v_3(\mathbf{r},t)\end{pmatrix}=\begin{pmatrix} E_z(\mathbf{r},t)\\ \eta H_x(\mathbf{r},t)\\ \eta H_y(\mathbf{r},t)\end{pmatrix} \quad (2a) \qquad \mathbf{j}=\begin{pmatrix}\eta J_z(\mathbf{r},t)\\ 0\\ 0\end{pmatrix} \quad (2b)$$

under the initial condition of zero fields ($\mathbf{v}(\mathbf{r},t)=0$, $t<0$). $\bar{A}$, $\bar{B}$, $\bar{C}$ are 3×3 constant matrices and $\bar{D}$ are 3×3 matrices contain electrical parameters (the relative permittivity $\varepsilon_r(\mathbf{r})$, relative permeability $\mu_r(\mathbf{r})$, and conductivity $\sigma(\mathbf{r})$) of the medium. For more details, please refer to [4]. c $(=1/\sqrt{\varepsilon_b\mu_b})$ and η $(=\sqrt{\mu_b/\varepsilon_b})$ are light speed and intrinsic impedance of the background media with permittivity ε_b, permeability μ_b, and conductivity $\sigma_b=0$.

We consider two regions which are denoted by Ω and $\bar{\Omega}$. They are separated by a measurement surface $\partial\Omega$. The impressed source $\mathbf{J}$ is assumed to be in the exterior region $\bar{\Omega}$, while a scattering object is in the interior region Ω. The total fields ($\mathbf{E}$, $\mathbf{H}$) are measured on $\partial\Omega$ during the measurement time from $t=0$ to T. By using the measured total fields ($\mathbf{E}$, $\mathbf{H}$), we can set up an interior equivalent problem where the electric and magnetic surface current sources given by $\mathbf{J}_s=\hat{n}\times\mathbf{H}$, $\mathbf{M}_s=\mathbf{E}\times\hat{n}$ (the unit vector $\hat{n}$ is inward normal to $\partial\Omega$) reproduce the same field in Ω and null field in $\bar{\Omega}$.

2.2 Time-reversed field

Provided that the total fields inside the region Ω are negligibly small after the time $t=T$, we can set up another interior equivalent problem which is the same as we described in the previous subsection except that the problem is solved backward in time. The time-reversed field $\mathbf{u}(\mathbf{r},t)$ of the problem is the solution of the following equations:

$$\left[\bar{A}\frac{\partial}{\partial x}+\bar{B}\frac{\partial}{\partial y}-\bar{C}\frac{\partial}{\partial(ct)}-\bar{D}\right]\mathbf{u}=\mathbf{s} \qquad (3)$$

where

$$\mathbf{u}=\begin{pmatrix} u_1(\mathbf{r},t)\\ u_2(\mathbf{r},t)\\ u_3(\mathbf{r},t)\end{pmatrix}, \quad (4a) \qquad \mathbf{s}=\begin{pmatrix} n_x\eta H_y-n_y\eta H_x\\ -n_yE_z\\ n_xE_z\end{pmatrix}\delta_{\partial\Omega}. \quad (4b)$$

under the final condition of zero fields ($\mathbf{u}(\mathbf{r},t)=0$, $t>T$). Here, $\delta_{\partial\Omega}$ is a delta function representing a source concentrated on the surface $\partial\Omega$. The source $\mathbf{s}$ produces the time reversed fields which are identical to the total fields of the original problem in Ω, while null field in $\bar{\Omega}$, i.e,

$$\mathbf{u}(\mathbf{r},t)=\begin{cases}\mathbf{v}(\mathbf{r},t) & \text{if } \mathbf{r}\in\Omega \\ 0 & \text{if } \mathbf{r}\in\bar{\Omega}\end{cases}. \tag{5}$$

In the equivalent problem, the primary source $\mathbf{j}(\mathbf{r},t)$ is not needed to calculate the time-reversed fields $\mathbf{u}(\mathbf{r},t)$. The time-reversed fields are calculated backward in time from $t=T$ to 0.

2.3 Inverse problem

The time-domain inverse scattering problem is considered for estimating $\varepsilon_r(\mathbf{r})$, $\mu_r(\mathbf{r})$ and $\sigma(\mathbf{r})$ with the knowledge of the tangential components of the original total fields $\mathbf{v}(\mathbf{r},t)$ measured on the surface $\partial\Omega$ but without explicit knowledge of the incident field. If the estimated electrical parameter is identical with true ones, the time-reversed fields cancel out the fields scattered by the object in $\bar{\Omega}$ during the interval $[0,T]$ as represented by eq.(5). On the other hand, an incorrectly estimated electrical parameter does not give null field in $\bar{\Omega}$. Based on this fact, we reduce the inverse problem discussed here to an optimization problem in which the following cost functional is minimized,

$$Q(\mathbf{p})=\sum_{n=1}^{N}\int_{0}^{cT}\int_{\bar{\Omega}}K(t)\left|\mathbf{u}_n(\mathbf{p};\mathbf{r},t)\right|^2 d\mathbf{r}\,d(ct) \tag{6}$$

where $\mathbf{p}=[\varepsilon_r(\mathbf{r}),\ \mu_r(\mathbf{r}),\ \eta\sigma(\mathbf{r})]^{\mathrm{T}}$ is a vector of the electrical parameters of the object. The superscript T denotes the transpose. The time factor $K(t)$ is a non-negative weighting function which takes a value of zero at $t=0$. N successive illuminations are assumed to explore the object. The vector $\mathbf{u}_n(\mathbf{p};\mathbf{r},t)$ represents the time-reversed fields for an estimated parameter vector $\mathbf{p}$ due to the n-*th* equivalent surface current $\mathbf{s}_n$, n=1,…N.

We apply a gradient method to minimization of the cost functional. The gradient of the functional (6) is given by

$$\mathbf{g}(\mathbf{r})=\left[g_\varepsilon(\mathbf{r}),\ g_\mu(\mathbf{r}),\ g_\sigma(\mathbf{r})\right]^{\mathrm{T}} \tag{7}$$

where

$$g_\varepsilon(\mathbf{r})=\int_0^{cT}\sum_{n=1}^{N}w_{1,n}(\mathbf{p};\mathbf{r},t)\frac{\partial u_{1,n}(\mathbf{p};\mathbf{r},t)}{\partial(ct)}d(ct) \tag{8a}$$

$$g_\mu(\mathbf{r})=\int_0^{cT}\sum_{m=2}^{3}\sum_{n=1}^{N}w_{m,n}(\mathbf{p};\mathbf{r},t)\frac{\partial u_{m,n}(\mathbf{p};\mathbf{r},t)}{\partial(ct)}d(ct) \tag{8b}$$

$$g_\sigma(\mathbf{r})=\int_0^{cT}\sum_{n=1}^{N}w_{1,n}(\mathbf{p};\mathbf{r},t)u_{1,n}(\mathbf{p};\mathbf{r},t)d(ct). \tag{8c}$$

The adjoint field vector $\mathbf{w}(\mathbf{p};\,\mathbf{r},\,t)$ is written as

$$\mathbf{w}=\left[w_1(\mathbf{p};\mathbf{r},t)\quad w_2(\mathbf{p};\mathbf{r},t)\quad w_3(\mathbf{p};\mathbf{r},t)\right]^T. \tag{9}$$

This field is calculated as [4].

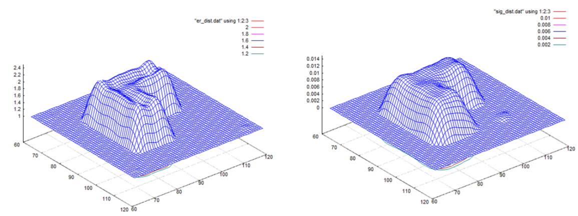
3. Numerical Results

In order to assess the proposed method, numerical simulations of reconstructing two-dimensional lossless dielectric objects are carried out. In the simulations presented here, the primary source is a z-directed line current source $\mathbf{J}=I(t)\hat{z}$ with the time factor

$$I(t)=\frac{d^3}{dt^3}\exp\left[-\alpha^2(t-\tau)^2\right] \tag{10}$$

where $\tau=\beta\Delta t$, $\alpha=5.6/(\beta\Delta t)$ and $\beta=132$ with the time step size $\Delta t=0.98(\Delta x/c\sqrt{2})=10.4$ ps and the cell size $\Delta x=\Delta y=\lambda/10\sqrt{4}=4.5$ mm . Here, $\lambda=90$ mm is the wavelength in free space at the highest frequency where the source spectrum is 1/20 of the maximum value. The computational domain is discretized into 180×180 cells. Twelve sources are placed on the boundary line of the 5λx5λ square and are successively used to probe the cylinder. The tangential components of the total electric fields are collected on the periphery of the 4λx4λ square surrounding the object. The reconstructed region is a 3λx3λ square.

The reconstructed results of L-shape cylinder (relative permittivity is 2.0 and conductivity is 0.01[S/m]) after 40 iterations by gradient method are shown in Fig.1. This numerical simulations show that the proposed method using time-reversed fields is very effective and deals with lossy objects.



(a) Relative permittivity (b) Conducitivity

Fig.1 construction results of L-shape cylinder

4. Conclusion

A gradient-based inverse scattering approach using time-reversed fields to reconstruct electrical parameters of lossy cylindrical objects without explicit knowledge of incident fields has been proposed. Numerical simulation has showed the usefulness of our inverse technique.

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