

# Comparing Between Current and Field Basis Functions in Moment Method Solutions

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**ABSTRACT** —The center fed disk antenna is used for comparing between the known moment method (MM) solution based on expanding the surface current density  $J_p(\rho)$  on the disk, and the proposed MM expansion of the field component  $E_p(\rho)$  outside the disk, on the disk plane. It is shown that for  $R < \lambda$ , 3 current basis functions or 3 field basis functions are enough for excellent convergence of the solution. However, for  $R > \lambda$ , more current basis functions are needed for getting a reasonable solution, while 3 field basis functions are enough for getting accurate solutions, where  $R$  is the radius of the disk and  $\lambda$  is the wavelength. CST software simulations are in good agreement with the MM results.

**Index Terms** — moment method, basis functions, infinite domain.

## 1. Introduction

The moment method (MM) is used frequently in solving electromagnetic problems. However, it is well known that generally the basis functions are defined on the body area, and when the electric size of the body is increased – more basis functions are needed in order to get a reasonable solution. [1], for example, proposed to divide an electrical large body to sub-domains, and to use the MM for each sub-domain, taking into account the boundary conditions between the relevant domains.

In this paper, we compare the MM solution for the center-fed microstrip disk antenna, based on carefully selected electric field basis functions describing  $E_p(\rho)$  on the disk plane ( $z = h$ ), to the known MM solution based on current basis functions [2], and to the CST software simulations for the same problem. The proposed field basis functions for  $E_p(\rho)$  are entire-domain, continuous functions, defined on the total disk plane, exactly contain the physical behaviour near the edge of the disk and at infinity.

The structure of the paper is as follows: Section II describes the formulation of the problem and includes the MM solution based on the field basis functions. Section III deals with the selection of these basis functions, and numerical results are presented in Section IV. Finally section V concludes the work.

## 2. Moment Method Formulation

The geometry for the patch disk antenna model is illustrated in Fig.1. An infinite metallic ground plane is covered by an infinite layer of relative dielectric constant equal to 1 in this case, and thickness  $h$ . Above the dielectric layer there is a metallic circular disk which is excited at its center by a thin cylinder with a constant current  $I_0$  contacting the disk. The disk radius is  $R$ , the probe radius is  $a$ , and its height is  $h$ .

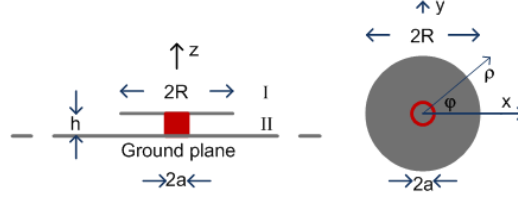


Figure 1: Side view (left) and upper view (right) of the disk antenna.

The subscript I denotes the region above the patch while the subscript II denotes the region under the patch. The current density produced by the probe is modelled by a vertical current density  $\mathbf{J}_V$  inside the dielectric layer which is expressed as

$$\mathbf{J}_V = \hat{\mathbf{z}}J_V(x, y) = \hat{\mathbf{z}}I_0 \frac{\delta(\rho - a)}{2\pi a} \quad (1)$$

The formulation of the fields in each region is exactly as in [2] and will not be repeated here. The difference between the MM solution in [2] and the present formulation is that here we formulate the MM equation for field basis functions. The moment method equation is given by

$$k_0^2 \int_0^\infty \sum_{n=1}^N a_n \tilde{f}_n(k_t) \tilde{f}_i^*(k_t) k_t \left[ \frac{j \cot(\gamma h) - 1}{\gamma} \right] dk_t = \int_0^\infty k_t^2 \frac{j \omega \mu_0 \tilde{J}_V(k_t)}{\gamma^2} \tilde{f}_i^*(k_t) dk_t \quad i=1,2,3\dots \quad (2)$$

where  $k_0$  is the wavenumber and  $k_t^2 = k_x^2 + k_y^2$ , and where

$$\gamma = \sqrt{k_0^2 - k_t^2}, k_0 > k_t, \quad -j\sqrt{k_t^2 - k_0^2}, k_t > k_0 \quad (3)$$

$$k_0 = \omega \sqrt{\mu_0 \epsilon_0} = 2\pi / \lambda_0 \quad (4)$$

We expand  $E_\rho$  outside the disk by the MM solution

$$E_\rho(\rho, z = h) = \sum_{n=1}^N a_n f_n(\rho), \quad n=1,2,3,\dots,N, \rho > R \quad (5)$$

and  $f_n(\rho)$  are zero for  $\rho < R$ .  $f_n(\rho)$  are the field expansion (basis) functions, and  $a_n$  are unknown coefficients. Here Galerkin moment method was used.

### 3. Choosing the Basis Functions

The basis functions are chosen to exactly obey the physical behaviour of near the wedge of the disk and at infinity.

$$E_\rho \sim \alpha_1(\rho - R)^{-1/2} + \alpha_2(\rho - R)^{1/2} + \alpha_3(\rho - R)^{3/2} + \dots, \rho \rightarrow R \quad (6)$$

$$E_\rho \sim \exp(-jk_0\rho) \left( \frac{a}{\rho^3} + \frac{b}{\rho^4} + \frac{c}{\rho^5} + \dots \right), \quad \rho \rightarrow \infty \quad (7)$$

where an example of a subset is

$$f_n(\rho) = \frac{\exp(-jk_0\rho)}{\rho^{n+1}\sqrt{\rho^2 - R^2}}, \quad n=1,2,3.. \quad (8)$$

The above functions contain all the needed powers near the edge of the disk and at infinity, with no undesired powers.

#### 4. Numerical Results

For simplicity we choose  $a=0.001\text{m}$ ,  $h=0.0025\text{m}$ ,  $R=0.05\text{m}$  and  $\epsilon_r = 1$ . The MM matrix elements were numerically calculated by Matlab Software. The surface current density on the disk at the first resonance is shown in figure 2. The blue line is for 1 field basis function, the green line is for 2 field basis functions, and the red line is for 3 field basis functions. The turquoise circles are for 3 current basis functions [2]. Figure 3 presents the input impedance of the antenna. The blue and magenta lines are for the real and imaginary values of the input impedance, respectively. The green circles are for 2 current basis functions, and the red circles are for 3 current basis functions. Figure 4 shows the CST software results, which needs about 30 million mesh cells and a lot of computing time.

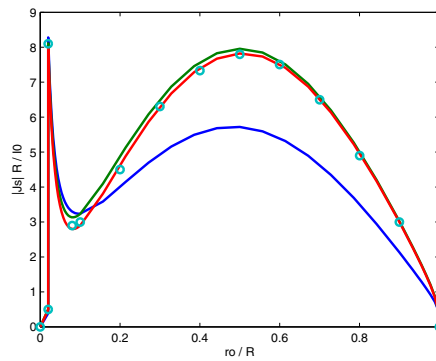


Figure 2. Absolute value the surface current density on the disk.

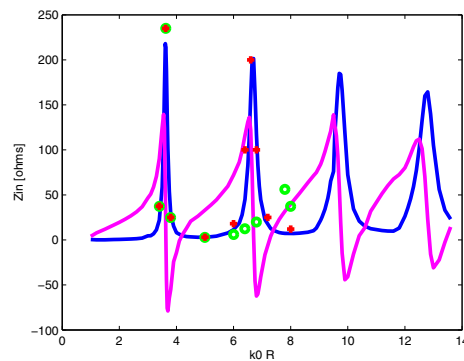


Figure 3. Input impedance in ohms as function of frequency.

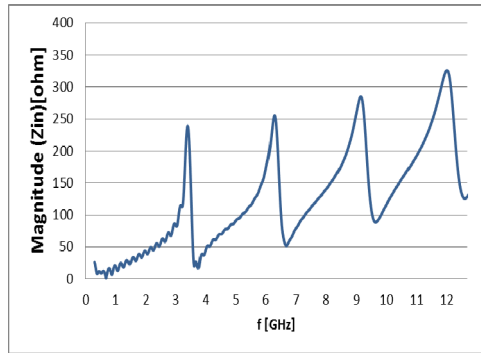


Figure 4. Absolute value of the input impedance in ohms as function of frequency - CST Microwave Studio.

## 5. Conclusions

Carefully selected field basis functions defined in the entire infinite  $z = h$  plane were introduced. It is shown that these types of function are far more efficient than current basis functions for electrical large bodies. The reason is that the field basis functions include more physical content than the current basis functions - the relevant frequency. Much more research has to be done in learning these basis functions.

## References

- [1] Lacente, E., A. Monorchio and R. Mittra, "An Iteration-Free MoM Approach Based on Excitation Independent Characteristic Basis Functions for Solving Large Multiscale Electromagnetic Scattering Problems", IEEE Trans. AP, Vol. 56, No. 4, 999-1007, 2008.
- [2] S. Pinhas, S. Shtrikman and D. Treves, "Moment Method Solution of the Center Fed Microstrip Disk Antenna Invoking Feed and Edge Current Singularities", IEEE Trans. AP-37, pp. 1516 – 1522, 1989.