Error Prediction of a 3-D Mode Matching Technique for a Simple Geometry

#Shinichiro Ohnuki[1], Kenichiro Kobayashi [1], Tsuneki Yamasaki[1]

[1]Department of Electrical Engineering, College of Science and Technology, Nihon University

1-8-14 Surugadai, Kanda, Chiyoda-ku, Tokyo, 101-8308, Japan

E-mail: ohnuki.shinichiro@nihon-u.ac.jp

1. Introduction

It is important for radar cross section reduction and target identification to investigate electromagnetic scattering from various objects [1]. Our mode matching technique can analyze electromagnetic scattering problems with a high degree of accuracy [2]. In recent years, we have proposed an error prediction of our technique for two dimensional problems [3,4].

In this paper, we extend the idea to three dimensional problems and investigate computational accuracy of our technique. Compared with the reference solution, our prediction method for three dimensional problems is verified. Furthermore, we clarify the computational error due to the placement of sampling points.

2. Formulation

To investigate computational accuracy of our mode matching technique, electromagnetic scattering from a simple geometry, dielectric sphere, is studied. The incident wave is a plane wave propagating toward $+z$ direction as shown in Figure 1. In the spherical coordinate systems, the θ and ϕ components of the incident electric field can be written as

$$E_\theta^i = \frac{E_0}{k_0 r}\cos\phi\left[j\sum_{n=1}^{N} a_n \hat{J}_n'(k_0 r)\sin\theta P'^1_n(\cos\theta) - \frac{1}{\sin\theta}\sum_{n=1}^{N} a_n \hat{J}_n(k_0 r) P_n^1(\cos\theta)\right], \quad (1)$$

$$E_\phi^i = \frac{E_0}{k_0 r}\sin\phi\left[j\frac{1}{\sin\theta}\sum_{n=1}^{N} a_n \hat{J}_n'(k_0 r) P_n^1(\cos\theta) - \sum_{n=1}^{N} a_n \hat{J}_n(k_0 r)\sin\theta P'^1_n(\cos\theta)\right], \quad (2)$$

where $a_n = j^{-n}(2n+1)/(n(n+1))$, $\hat{J}_n(k_0 r) = \sqrt{\pi k_0 r/2}\cdot J_{n+1/2}(k_0 r)$, $J_n()$ is the n-th order of the Bessel function, $P_n^1()$ is the n-th order of the Legendre function, N is the truncation mode number, and k_0 is the wave number in vacuum. The time dependence is $e^{j\omega t}$ and suppressed throughout the paper.

In our method, we need to divide the whole physical space into some regions for which model expansion can be easily performed [2]. For this simple geometry, we divide the whole space into two regions, which are outside and inside the sphere. The electromagnetic fields in these regions are expressed as follows:

Region S_0 : Outside the sphere

The scattered field can be represented by using the Hankel function which satisfies the radiation condition. Using a finite sum of modes, it can be approximated as

$$E_\theta^{S_0} = -\frac{E_0}{k_0\, r}\cos\phi \left[j\sum_{n=1}^{N} b_n \hat{H}_n^{(2)\prime}(k_0 r)\sin\theta P'^{1}_{n}(\cos\theta) - \frac{1}{\sin\theta}\sum_{n=1}^{N} c_n \hat{H}_n^{(2)}(k_0 r) P_n^1(\cos\theta) \right] , \quad (3)$$

$$E_\phi^{S_0} = -\frac{E_0}{k_0\, r}\cos\phi \left[j\frac{1}{\sin\theta}\sum_{n=1}^{N} b_n \hat{H}_n^{(2)\prime}(k_0 r) P_n^1(\cos\theta) - \sum_{n=1}^{N} c_n \hat{H}_n^{(2)}(k_0 r)\sin\theta P'^{1}_{n}(\cos\theta) \right] , \quad (4)$$

where $\hat{H}_n^{(2)}(k_0 r) = \sqrt{\pi k_0 r/2} \cdot H_{n+1/2}^{(2)}(k_0 r)$, $H_n^{(2)}()$ is the n-th order of the second kind of the Hankel function.

Region S_1 : Inside the sphere

The electromagnetic field in this region can be represented by using the Bessel function, such as

$$E_\theta^{S_1} = \frac{E_0}{k_1 r}\cos\phi \left[j\sum_{n=1}^{N} d_n \hat{J}'_n(k_1 r)\sin\theta P'^{1}_{n}(\cos\theta) - \frac{1}{\sin\theta}\sum_{n=1}^{N} e_n \hat{J}_n(k_1 r) P_n^1(\cos\theta) \right] , \quad (5)$$

$$E_\phi^{S_1} = \frac{E_0}{k_1 r}\sin\phi \left[j\frac{1}{\sin\theta}\sum_{n=1}^{N} d_n \hat{J}'_n(k_1 r) P_n^1(\cos\theta) - \sum_{n=1}^{N} e_n \hat{J}_n(k_1 r)\sin\theta P'^{1}_{n}(\cos\theta) \right] , \quad (6)$$

where k_1 is the wave number in the homogeneous dielectric sphere.

The unknown coefficients b_n, c_n, d_n, and e_n are determined to satisfy the continuity condition at the sampling points on the surface of the sphere. The placement of sampling points will be discussed in Sec.3.

2.1 Error Prediction

To predict the computational accuracy of our method, we investigate the convergence rate of the Bessel function for varying the truncation mode number. The rate can be evaluated in terms of the excess bandwidth formula [5, 6]. Recently, we have proposed the error prediction which is given by [2, 3]

$$C_P = 10^{-2d_0} , \quad (7)$$

$$d_0 = \left\{ \frac{N - k_1 a}{1.8(k_1 a)^{1/3}} \right\}^{3/2} , \quad (8)$$

where $N > k_1 a$.

We can obtain the modified expression for the error prediction of the RCS as

$$C_I = \frac{D_I(N_{\max}) - D_I(N)}{D_I(N_{\max})} , \quad (9)$$

$$D_I(N) = S^2 , \quad (10)$$

$$S = \sum_{n=1}^{N} 10^{-2d_0} , \quad (11)$$

$$d_0 = \left\{ \frac{N - k_1 a}{1.8(k_1 a)^{1/3}} \right\}^{3/2} , \quad (12)$$

where $n > k_1 a$, $N_{\max}$ is the largest truncation mode number N for the reference solution.

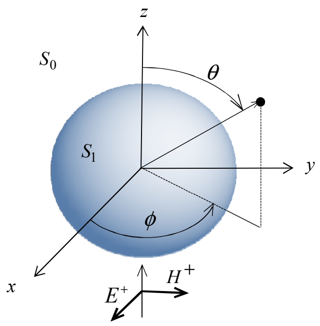


Figure 1: Geometry of the scatterer.

3. Computational Results

We investigate the computational accuracy due to the placement of sampling points as shown in Figure 2. The selection of intervals of the sampling points $\Delta\theta_n$ and $\Delta\phi_n$ are as follows:

(a) Case 1: $\begin{cases} \Delta\theta_n : 180^\circ n \times rand(n) \\ \Delta\phi_n : 0 \end{cases}$,

(b) Case 2: $\begin{cases} \Delta\theta_n : 180^\circ n/(N+1) \\ \Delta\phi_n : 0 \end{cases}$,

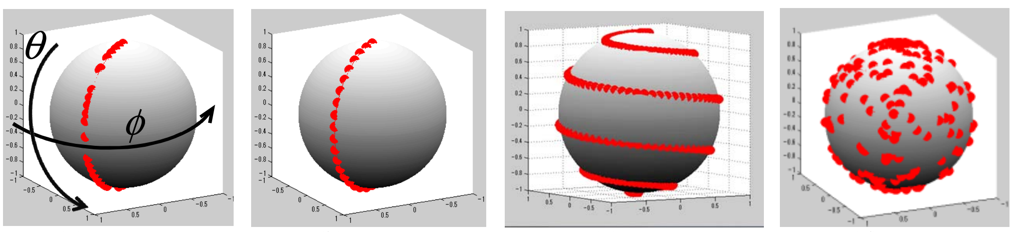
(c) Case 3: $\begin{cases} \Delta\theta_n : 180^\circ n/(N+1) \\ \Delta\phi_n : 360^\circ n/(N+1) \end{cases}$,

(d) Case 4: $\begin{cases} \Delta\theta_n : 180^\circ n/(N+1) \\ \Delta\phi_n : 360^\circ n \times rand(n) \end{cases}$,

where $n=1$ to N and $rand()$ is a random number from 0 to 1.

Figure 3 plots the relative error of the RCS for changing the placement of sampling points. The normalized frequency is $k_1 a = 12\pi$ and the observation point is at $\theta = 0^\circ$ and $\phi = 0^\circ$. Compared with the case 1, we can obtain higher accuracy for the cases 2, 3, and 4 in which sampling points are placed at the same θ interval. Hereafter, we perform numerical analysis using the sampling points at the same interval in θ direction.

Figure 4 shows the relative error of the RCS obtained by our method. Dots indicate the error for a dielectric sphere with $k_1 a = 6\pi$ and triangles indicate the error for a dielectric sphere with $k_1 a = 12\pi$. The solid line indicates the modified error prediction given by Eq. (9). The numerical results and the error prediction are in good agreement.



(a) case 1 (b) case 2 (c) case 3 (d) case 4

Figure 2: Placement of sampling points.

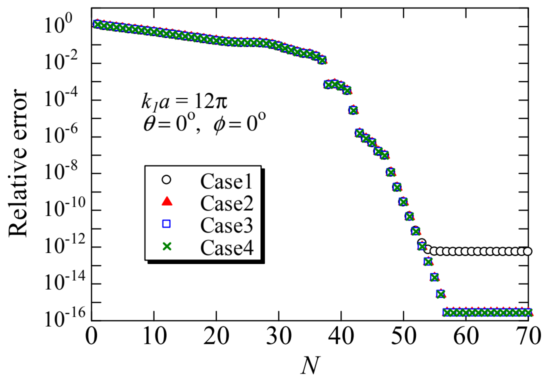


Figure 3: Relative error for various placements of sampling points.

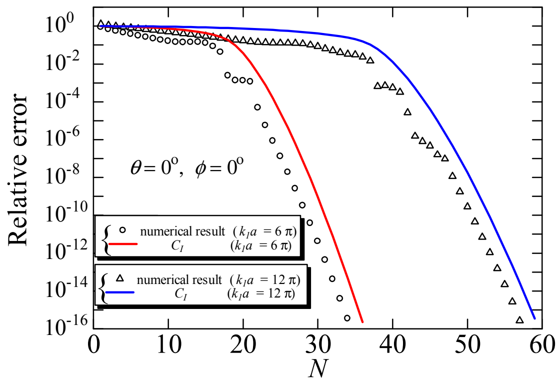


Figure 4: Error prediction for varying the truncation mode number *N*.

4. Conclusions

We study electromagnetic scattering from a dielectric sphere by using a mode matching technique. A novel error prediction method is proposed and verified. We also clarify the computational accuracy due to the placement of sampling points.

References

[1]E. F. Knott, J. F. Shaefer, and M. T. Tuley, Radar Cross Section: Second Edition, Boston: Artech House, 1993.

[2] S. Ohnuki, R. Ohsawa, and T. Yamasaki : "Electromagnetic Scattering from Rectangular Cylinders with Various Wedge Cavities and Bumps", *IEICE Trans*., Vol.E93-C, No.1, pp77-80, Jan, 2010.

[3] S. Ohnuki, T. Mochizuki, and T. Yamasaki : "Error Prediction of the Point Matching Method for EM scattering from a Conducting Rectangular Cylinder" , *IEEJ Trans*., Vol. 129, No.10, pp727-728, Oct, 2009.

[4] S. Ohnuki, T. Mochizuki, K. Kobayashi and T. Yamasaki : "Optimization of Field Decomposition for a Mode Matching Technique," *IEICE Trans*.,Vol. E95-C, No.1, pp.101-104, Jan, 2012.

[5] W. C. Chew, J. M. Jin, E. Michielssen and J. M. Song eds., Fast and Efficient Algorithms in Computational Electromagnetics, Artech House, 2001.

[6] S. Ohnuki and W. C. Chew "Truncation Error Analysis of Multipole Expansion," SIAM J. Scientific Computing, vol. 25, no. 4, pp. 1293-1306, 2003.

Ackowledgemetns

This work was partly supported by Nihon University Strategic Projects for Academic Research.