PROPAGATION OF ELF PULSES IN THE EARTH-IONOSPHERE WAVEGUIDE.

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The problem of the ELF pulse propagation in the earth-ionosphere waveguide has been focussing attention for a long time in connection with the slow-tail portion of atmospherics, generated by lightning discharges. Already in 1953 Hepburn and Pierce [1] deduced effective values of the conductivity of the lower edge of the ionosphere from experimental data, assuming that the mean separation between the oscillatory head of an atmospheric and the maximum of a slow-tail amplitude varies with distance to the source according to a linear law. The most detailed consideration of this problem was made by Wait [2-4]. Wait computed the transient response of the earth-ionosphere waveguide for a dipole pulse source in the form of a δ - function and pulses of certain analitical forms. The ionosphere was represented as a homogeneous conductor (with some effective conductivity σ), located above $h \sim 70$ km for daytime and $h \sim 90$ km for nighttime. Wait found, that in the framework of the accepted model the slow-tail separation t_Δ varies with distance ρ to the source, which is a vertical dipole, according to the law

$$t_\Delta^{1/2} = A + B\rho \tag{1}$$

where constant A is determined by the pulse width of the source, and constant B depends on parameters h and σ ; the width of the ELF pulse increasing with distance due to dispersion by square-law.

In this report the transient response of the earth-ionosphere waveguide is computed for a more adequate model of the daytime ionosphere, which has been considered according to ELF wave propagation by Greifinger and Greifinger [5-6]. The medium is assumed to be horizontally stratified with the exponential conductivity profile

$$\sigma(z)/\varepsilon_0\omega = exp\left\{(z-h_0)/\xi_0\right\} \tag{2}$$

Here $h_0(\omega)$ is altitude at which the conductivity currents of the ω frequency become equal to the displacement currents, ξ_0 is parameters of the conductivity change in the vicinity of altitudes $\sim h_0$.

The source is a vertical electric dipole at the perfectly conducting earth's surface ($z = 0$) with the current moment $J(t)d\ell$, moreover $J(t) = 0$ at $t < 0$. We put

$$\mathcal{J}(p) = \mathcal{L}\, J(t) = \int_0^\infty dt\; e^{-pt} J(t) \tag{3}$$

where symbol $\mathcal{L}$ is the Laplace transform operator and p is the transform variable formally equivalent to $-i\omega$. Then for the transient response of the zero-order waveguide mode for $z = 0$ we have

$$e_z(t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} dp\, e^{pt} E_z(p) \tag{4}$$

where

$$E_z(p) = \frac{J(p)\, d\ell\, p\, S_0^2(p)}{2\pi \varepsilon_0 c^2 h_0(p)} K_0\left(p \frac{\rho}{c} S_0(p)\right) \tag{5}$$

In the last formula $S_0(p)$ corresponds to the TEM wave propagation constant $S_0(\omega)$, the real part of which determines the phase velocity of the wave and the imaginary part does the same for the attenuation index, the $h_0(p)$ magnitude corresponds to altitude $h_0(\omega)$.

Let us represent the expression (2) as

$$\sigma(z) = \varepsilon_0 \omega_z(H) \exp\left\{(z-H)/\zeta_0\right\}, \tag{6}$$

where H is certain fixed altitude. Then the solution [5] for $S_0(\omega)$ will look as

$$S_0^2(\omega) = \frac{H + \zeta_0 \ln \dfrac{c^2}{4\zeta_0^2 \omega_z(H)\omega} + \dfrac{i\pi}{2}}{H - \zeta_0 \ln \dfrac{\omega_z(H)}{\omega} - \dfrac{i\pi}{2}} \tag{7}$$

and, accordingly,

$$S_0^2(p) = \frac{H_0 + \zeta_0 \ln \dfrac{c}{2\zeta_0 p}}{H_0 - \zeta_0 \ln \dfrac{c}{2\zeta_0 p}} \tag{8}$$

$$h_0(p) = H_0 - \zeta_0 \ln \frac{c}{2\zeta_0 p}$$

where

$$H_0 = H + \zeta_0 \ln \frac{c}{2\zeta_0\, \omega_z(H)} \tag{9}$$

Let us calculate the transient response for source $J(t) = J_0 \tau_0 \delta(t)$. In frequency range ω, that interests us, and that determines the ELF pulse form the inequality $\mu = |(\zeta_0/H_0)\ln(c/2\zeta_0 p)| < 1$ is valid. Using the asymptotics of the modified Bessel function at a large argument, let us make an expansion over parameter $\mu < 1$ in (5) and (8). At this, in the expression for $S_0(p)$ we shall restrict ourselves by regarding the terms of the μ^2 ordes

$$S_0(p) \simeq 1 + \frac{\zeta_0}{H_0} \ln \frac{c}{2\zeta_0 p} + \frac{1}{2}\left(\frac{\zeta_0}{H_0}\right)^2 \ln^2 \frac{c}{2\zeta_0 p} \tag{10}$$

Then

$$e_z(t) \simeq \frac{J_0 \tau_0\, d\ell}{4\pi \varepsilon_0 H_0 \zeta_0^{1/2} \rho^{3/2}} \sum_{m=0}^{2} \sum_{n=0}^{\infty} a_{mn} R_{mn}(\tau) \tag{11}$$

Here

$$R_{mn}(\tau) = \frac{1}{n!}\left(\frac{\zeta_0}{H_0}\right)^{m+n} \sum_{k=0}^{\left[\frac{m+n}{2}\right]} (-1)^{m+k} C_{m+n}^{2k} \pi^{2k} \left(\frac{\partial}{\partial\alpha}\right)^{m+n-2k} \left\{\frac{\Gamma(\alpha)}{\pi^{1/2}}\left(\frac{\rho}{c}\frac{\partial}{\partial\tau}\right)^{n+1}\left[\left(\frac{2\zeta_0}{c\tau}\right)^{\alpha}\theta(\tau)\right]\right\}\Bigg|_{\alpha=1/2} \tag{12}$$

$$a_{on} = 1 \ , \ a_{1n} = \frac{n+5}{2} \ , \ a_{2n} = \frac{10n+29}{8}$$

$\tau = t - \rho/c$, $\theta(\tau)$ is unity Heaviside function.

The expression (11) coincides with the space-time Greene function of the considered problem with precision to the numerical factor and the argument additivity. In particular, for a "standard" source used by Wait [2-3]

$$\mathcal{J}(t) = \frac{\mathcal{J}_0}{2\pi^{1/2}} \left(\frac{\tau_0}{t}\right)^{3/2} e^{-\frac{\tau_0}{4t}} \theta(t) \tag{13}$$

the transient response $e_z(t)$ is found from (11) via convolution

$$\int_0^\infty dt' \frac{\mathcal{J}(t')}{\mathcal{J}_0 \tau_0} \left(\frac{\partial}{\partial \tau}\right)^{n+1} \frac{\theta(\tau - t')}{(\tau - t')^\alpha} = \frac{\Gamma(1-\alpha)}{\pi^{1/2}} \tau^{-\alpha-n-1} e^{-\frac{\tau_0}{4\tau}} \Psi\left(-\alpha-n-\frac{1}{2}, \frac{1}{2}, \frac{\tau_0}{4\tau}\right) \tag{14}$$

where $\Psi(a, b, z)$ is confluent hypergeometric function.

Although the expressions obtained are rather awkward, the series in (11) converge fairly quickly due to the presence of a small parameter $\zeta_0/H_0 \ll 1$. For $t \gg \rho/c$ in the sum over n we can restrict ourselves by one term $n = 0$. Nevertheless, for $0 < \tau \leq \rho/2c$ it is necessary to take into account a large number of terms in the expressions (11)-(12). For such τ values it is possible to make an asymptotic estimation of the Laplace integral by the ρ parameter, similar to that obtained by Wait [4]. Omitting standard calculations, we find for source $\mathcal{J}(t)$

$$e_z(t) \simeq \frac{\mathcal{J}(p_1)\, d\ell}{4\pi \varepsilon_0 c \rho (\zeta_0 H_0)^{1/2}} p_1 S_0^2(t)\, e^{-\frac{\zeta_0}{H_0} \frac{\rho}{c} p_1 S_0(t)} \tag{15}$$

where

$$p_1 = \frac{c}{2\zeta_0} \exp\left(-\frac{H_0}{\zeta_0} \cdot \frac{S_0^2 - 1}{S_0^2 + 1}\right)$$

$$S_0(t) \simeq \frac{ct}{\rho}\left(1 + \frac{\zeta_0}{H_0}\right) \tag{16}$$

and, in particular, for the source (13) $\mathcal{J}(p_s) = \mathcal{J}_0 \tau_0 \exp\left\{-(p_s \tau_0)^{1/2}\right\}$.

The expression (15) describes a ELF pulse at distances $\rho \geq c/p_s$ in a certain domain of values τ near maximum $e_z(t)$. What permits to make an aproximate calculation of a separation interval t_s . It is easy to show that for the source (13)

$$t_s \simeq \frac{\zeta_0}{H_0} \frac{\rho}{c} \left\{ \ln \frac{\rho/2H_0}{\left(\sqrt{1 + \frac{c\tau_0}{16\rho} \frac{H_0}{\zeta_0}} - \sqrt{\frac{c\tau_0}{16\rho} \frac{H_0}{\zeta_0}}\right)^2} - 1 \right\} \tag{17}$$

and differs essentially from the square law (1) following from the model of a sharply confined ionosphere.

Fig. 2 illustrates the broadening of the ELF pulse with the increasing ρ distance for the "standard" source (13) shown in Fig. 1.

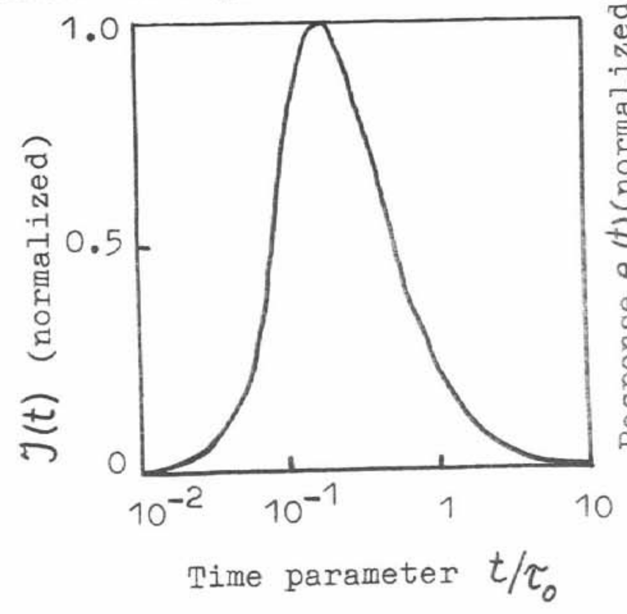

Fig. 1

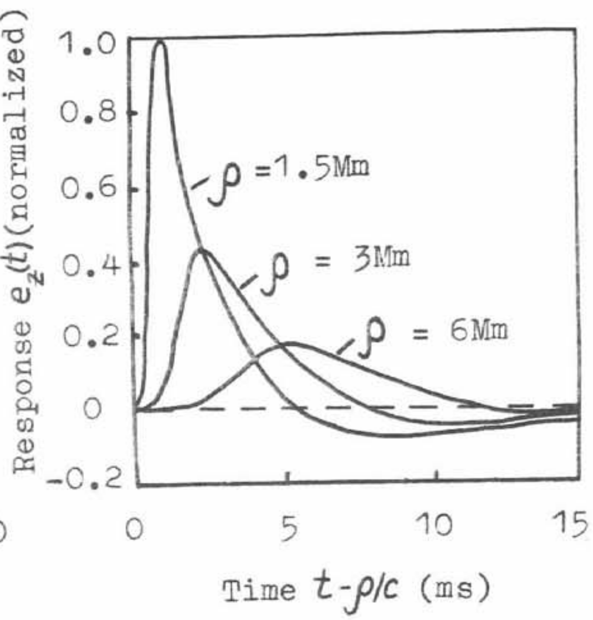

Fig. 2

While computing the pulse form, we assumed the parameters typical for daytime ionosphere [7] : $\zeta_0 = 3{,}5$ km, $H = 74$ km, $\omega_z(H) = 2{,}5 \cdot 10^5$, and $\tau_0 = 5$ ms. The response $e_z(t)$ is normalized to the unity by dividing by the quantity $7{,}3 \cdot 10^{-10} J_0 d\ell$ (V/m).

Refrences

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