NONLINEAR AMPLIFICATION OF WHISTLER MODE WAVES IN AN INHOMOGENEOUS MAGNETIC FIELD

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1. Introduction

Nonlinear amplification of a monochromatic VLF whistler mode wave propagating along the geomagnetic field is studied via computer simulations. Natural and controlled whistler-mode signals have been used at Siple station, Antarctica (and its conjugate, Roberval, Quebec) to study nonlinear mechanism of wave growth in the magnetosphere [Helliwell, 1979]. The wave growth is believed to be caused by a whistler mode wave-particle interaction at the equatorial magnetosphere. However, the detailed mechanism of the growth is not fully understood because of the strong nonlinear features involved in the interactions. In this paper basic processes of the wave growth are investigated with the aid of self-consistent computer simulations. The simulation method used in the present study is Long-Time-Scale (LTS) code [Rathmann et al., 1978, Matsumoto and Omura, 1985] where cold plasma is treated as a fluid and hot plasma is treated as particles.

In a homogeneous magnetic field a whistler mode wave is amplified at the linear growth rate and then reaches a nonlinear saturation [Omura and Matsumoto, 1982]. In this paper effects of inhomogeneous magnetic field is studied. Basic processes of the interactions are discussed with an emphasis on roles of trapped and untrapped resonant electrons in a nonuniform dipole magnetic field. Based on this analysis the mechanism of a nonlinear growth only possible in an inhomogeneous medium is discussed. Assuming a realistic particle distribution in the magnetosphere, nonlinear amplification of a whistler mode wave in a dipole magnetic field is studied in detail.

2. Basic Equations for Whistler Interaction

We assume a purely transverse whistler mode wave which propagates along the geomagnetic field line and interacts with counter-streaming high energy resonant electrons existing in the magnetosphere. The evolution of the wave amplitude, wavenumber and frequency are given by [Omura and Matsumoto, 1982]

$$\frac{\partial B_w}{\partial t} = V_g\left(\frac{\partial B_w}{\partial z} + \frac{1}{2}\mu_0 J_E \right) \qquad (1)$$

$$\frac{\partial k}{\partial t} = -\frac{\partial \omega}{\partial z} \qquad (2)$$

$$\omega = \frac{k(k - \mu_0 J_B/B_w)}{k(k - \mu_0 J_B/B_w) + \Pi_e^2/c^2}\,\Omega_e \qquad (3)$$

where V_g is the group velocity given by

$$V_g = 2k(\ \Omega_e - \omega\ )/(\ k^2 + \Pi_e^2/c^2\ ) \tag{4}$$

and B_w, ω and k are the wave amplitude, frequency and wavenumber, respectively. J_E and J_B are the components of the transverse resonant current parallel to the wave electric field E_w and the wave magnetic field B_w, respectively. J_E and J_B are calculated by tracing motions of a large number of electrons in the fields determined by the above equations. As is seen in (1) and (3), J_E causes a change of the wave amplitude B_w, and J_B modifies the frequency ω.

3. Trajectories of Resonant Electrons

We introduce coordinates (z, θ, $v_\perp$, ζ) to describe motion of a resonant electrons in a monochromatic whistler mode wave. z is a distance along the geomagnetic field line measured from the equatorial plane. θ is defined by $\theta = k(v_\parallel - V_R)$, and ζ is a relative phase angle of $v_\perp$ and the wave magnetic field B_w. $v_\parallel$ and $v_\perp$ are velocity component parallel and perpendicular to the static magnetic field B_0, respectively. V_R is a resonance velocity defined by $V_R = (\omega - \Omega_e)/k$. The equations of motion are then expressed as

$$\frac{d\theta}{dt} = \omega_t^{\ 2}\ (\ \sin\zeta + R\ ) \tag{5}$$

$$\frac{dv_\perp}{dt} = \frac{\Omega_w}{k}\ (\ \Omega_e - \theta\ )\ \sin\zeta + \frac{1}{2}\ (\ V_R + \frac{\theta}{k}\ )\ \frac{v_\perp}{\Omega_e}\ \frac{\partial\Omega_e}{\partial z} \tag{6}$$

$$\frac{d\zeta}{dt} = \frac{\Omega_w}{kv_\perp}\ (\ \Omega_e - \theta\ )\ \cos\zeta + \theta \tag{7}$$

$$\frac{dz}{dt} = v_\parallel \tag{8}$$

where $\Omega_w = eB_w/m$ (e : electron charge, m : electron mass), and ω_t is a trapping frequency defined by $\omega_t = (kv_\perp\Omega_w)^{1/2}$. R is an inhomogeneity ratio defined by [Karpman, et al., 1974, Vomvoridis and Denavit, 1979].

$$R = \frac{1}{2\omega_t^2}\ \{\ 3v_\parallel - \frac{kv_\perp^2}{\Omega_e}\ \}\ \frac{\partial\Omega_e}{\partial z} \tag{9}$$

Configuration of resonant electron orbits in the velocity phase space is strongly modified by the inhomogeneity ratio R. Figure 1 shows trajectories of resonant electrons in $v_\parallel$ and ζ phase space [Omura and Matsumoto, 1982]. The shaded area in the $v_\parallel$-ζ plane is the trapping region in which trapped electrons oscillate around the resonance velocity V_R. It is noted that the range of ζ where trapped particles can occupy at $v_\parallel = V_R$ is limited when $0 < |R| < 1$, while they can occupy the whole range of ζ in the equatorial region where $R = 0$. On the other hand, untrapped electrons also occupy a limited phase range at $v_\parallel = V_R$ when $0 < |R| < 1$. Therefore, both untrapped and trapped electrons can be phase-bunched and form resonant currents.

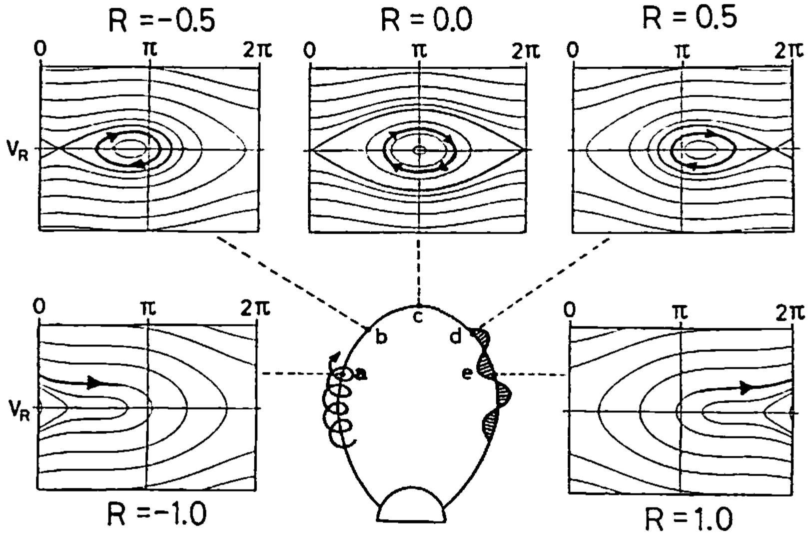

Figure 1

4. Roles of Trapped and Untrapped Resonant Electrons

We use a model of the simulation in which the wave propagating from the southern hemisphere encounters with counter-streaming hot electrons in the vicinity of the equator. This model simulates the Siple active experiment [Helliwell and Katsufrakis, 1974]. The geomagnetic field is assumed to be a dipole field. The initial amplitude and frequency of the wave are 10^{-5} ~ $10^{-1}B_{0EQ}$ and $1/2\Omega_{eEQ}$, where B_{0EQ} and Ω_{eEQ} are the geomagnetic field and the electron cyclotron frequency at the equator. The cold plasma density is assumed to be constant and taken as $\Pi_e/\Omega_{eEQ} = 10$.

As the phase-space configuration of the trapping region in the northern hemisphere is symmetrically reversed from that in the southern hemisphere as seen Figure 1, trapped and untrapped electrons exchange their roles in wave growth and damping in the opposite hemisphere. In order to clarify their roles, the simulation region is taken in either the southern or the northern hemisphere in different computer runs. We performed four different runs, through which we verify that trapped electrons cause wave growth in the southern hemisphere and wave damping in the northern hemisphere, and that untrapped resonant electrons cause wave damping in the southern hemisphere and wave growth in the northern hemisphere.

5. Nonlinear Growth in a Dipole Magnetic Field

In an inhomogeneous medium trapped and untrapped resonant electrons play different roles as discussed in the preceding sections. Therefore, energy change between the wave and the particles depends on the difference between the distribution function of trapped and untrapped resonant electrons in the region of $0 < |R| < 1$. It is noted that the mechanism of the growth in an inhomogeneous medium is qualitatively different from that in a homogeneous medium. In a homogeneous medium the wave growth takes place as predicted by the linear theory but soon it reaches a saturation within a half of the initial trapping period and the velocity distribution at the resonance velocity approaches to a plateau form, where neither trapped nor untrapped electrons causes wave growth/damping because of the symmetric trajectories around $\zeta = 180^0$ in the $v_\parallel$-ζ plane. In an inhomogeneous medium, however, the symmetry breaks as seen in Figure 1, and wave growth/damping occurs if there exists a situation where either trapped or untrapped electrons are dominant. We call this wave growth due to the strong nonlinearity of the resonant particle motion "nonlinear growth" in comparison with the linear growth in a homogeneous medium.

To investigate the nonlinear growth in a realistic situation in the magnetosphere, a large scale simulation with a realistic particle distribution has been performed, and its result is shown in Figure 2. The simulation region is across the equator as $z = -2000 \sim 2000$km and the wave is initially placed all over the simulation region and propagates to the right at a group velocity $V_g \simeq 480$ km/T_t. The inhomogeneity ratio is $|R| < 1$ in the region $|z| < 1200$km. Wave growth takes place all over the region, though the growth rate is different from position to position. Time history of the amplitude at z = 900, 0 and 900 km are shown in Figure 3. At the equator (z = 0), the linear growth occurs and stops about a half of the trapping time because the gradient of $v_{\parallel}$ distribution becomes small. In the region away from the equator, however, the wave continues to grow and this is interpreted as the nonlinear growth.

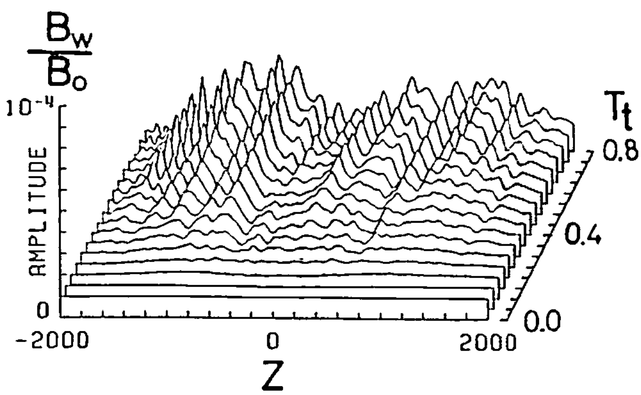

Figure 2

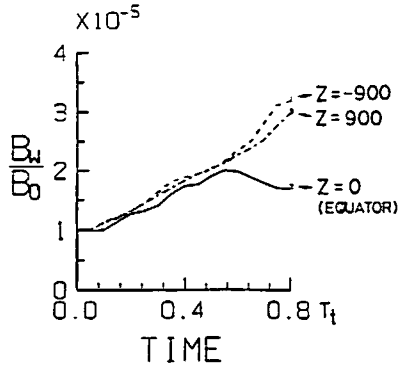

Figure 3

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