A NEW DIRECTION FINDING TECHNIQUE FOR MAGNETOSPHERIC VLF WAVES OBSERVED ON THE GROUND

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Abstract. A new direction finding technique for the magnetospheric VLF waves observed on the ground is proposed on the basis of the wave energy distribution of those waves in wave number space. This method is essentially different from the previous methods influenced by the propagation characteristics of the waves and/or the source effects on the lower edge of the ionosphere. It is shown that the present method is effective even when two or more sources are simultaneously existed in the lower ionosphere and that the scale of the exit regions can be estimated. And also it is represented that this method is useful to investigate the magnetospheric propagation characteristics and the ionospheric transmission mechanism of VLF waves.

1. Introduction

Ground-based direction finding (DF) techniques have been used extensively to locate the ionospheric exit regions of magnetospheric VLF/ELF waves. The results have been applied to the investigation of the generation and propagation mechanisms of those waves and also to study the dynamics of magnetospheric plasma (Carpenter, 1980; Hayakawa *et al.*, 1986).

Several DF systems have so far been proposed; (1) goniometer triangulation (Bullough and Sagredo, 1973); (2) field-analysis method (Okada *et al.*, 1977); (3) Poynting vector method (Leavitt *et al.*, 1978). For the first goniometer method, the orthogonal loop aerials are used to deduce only the azimuthal direction, while an additional use of the vertical electric field is made for latter two systems which yields not only the azimuth but also the incident angle. In the principle of all these DF methods, we assume that a single plane wave arrives at the observing point with a definite wave normal direction.

Strangeways (1980) has numerically studied the error of all these DF systems due to the effects of multiple rays in the Earth-ionosphere waveguide and the wave polarization. It is found that the former effect is not so important for nearby sources such as a few hundred kilometers from the station and then field-analysis DF method is suitable in this range. However, he has adopted the ionospheric transmission model of VLF radio waves such that a point source at the lower edge of the ionosphere radiates waves with all emittance angles, resulting in the presence of multiple rays in the Earth-ionosphere waveguide. The point source itself remains as an important unsolved problem and furthermore the previous DF methods are all ineffective in the case of simultaneous presence of a few ducts. So, we are strongly required to develop a DF system which is effective for a wide source and the possibility of several simultaneous ducts. This kind of multiple sources or widely spread source region is a very common phenomenon. In conclusion, the estimation of the wave energy distribution of magnetospheric VLF waves will be the best DF system for these typical situation.

2. Estimation of wave energy distribution of VLF waves observed on the ground

The ground can be considered as a perfect conductor for electromagnetic waves at VLF, and hence the observable field components on the ground are two horizontal magnetic field components (B_x, B_y) and a vertical electric field component (E_z), which are used to construct the auto- and cross-power spectra, resulting in the estimation of the spectral matrix. Those measured field components are understood as being the sum of a lot of plane waves with appropriate amplitude, with different propagation direction and with their phase relationship being quite random. The properties of such an incoherent and random wave fields can be described statistically. Storey and Lefeuvre (1979) have proposed to characterize it in terms of a function called the wave distribution function (WDF) which specifies how the wave energy density is distributed with respect to wave number space or to the space of angular frequency ω and the direction of propagation (θ, ϕ). The element of spectral matrix $S_{ij}(\omega)$ at a frequency ω for the waves observed on the ground is related to the wave distribution function $F(\omega, \theta, \phi)$ by the following relation,

$$S_{ij}(\omega) = \frac{\pi}{2} \int_{-1}^{1} \int_{0}^{2\pi} a_{ij}(\omega, \theta, \phi; p)\, F(\omega, \theta, \phi)\, d(\cos\theta)\, d\phi \quad (i, j = x, y, z)$$

where $a_{ij}(\omega, \theta, \phi; p)$ is the element of integration kernel, $p = B_{\|}/B_{\perp}$ the polarization of the downcoming wave (see Fig. 1) and $F(\omega, \theta, \phi)$ is the WDF to be determined. This is an inversion problem in which we will estimate the WDF by means of the maximum entropy concept, using the spectral matrix constructed from three electromagnetic field components measured on the ground. Assuming that the ground is a perfect conductor, the kernels $a_{ij}(\omega, \theta, \phi; p)$ are given as follows:

$$\begin{aligned}
a_{xx} &= A(p)\,[\sin^2\phi + p\,p^* \cos^2\theta \cos^2\phi - (p + p^*) \cos\theta \sin\phi \cos\phi] \\
a_{xy} &= a^*_{yx} = A(p)\,[-(1 - p\,p^* \cos^2\theta) \sin\phi \cos\phi + (p \cos^2\phi - p^* \sin^2\phi) \cos\theta] \\
a_{xz} &= a^*_{zx} = A(p)\,[-(\sin\phi - p \cos\theta \cos\phi) \sin\theta] \\
a_{yy} &= A(p)\,[\cos^2\phi + p\,p^* \cos^2\theta \sin^2\phi + (p + p^*) \cos\theta \sin\phi \cos\phi] \\
a_{yz} &= a^*_{zy} = A(p)\,[(\cos\phi + p \cos\theta \sin\phi) \sin\theta] \\
a_{zz} &= A(p) \sin^2\theta
\end{aligned}$$

where $A(p) = 2/(1 + p\,p^*)$. In the derivation of these kernels, the direct ray is only considered, because the effect of multiple rays in the Earth-ionosphere waveguide is evidenced by Strangeways (1980), based on the point source assumption, to be negligible for the range where the field-analysis DF is effective. Of course, his point source assumption is a big problem to be investigated, and spreading of the source region is reasonably considered to be a more important factor than the multiple ray effect.

As shown by above equations, the kernels depend on the inversion model, that is, the polarization p and it is generally possible for the wave propagating in free space to have any value of polarization. However, VLF waves in the magnetosphere is polarized right-handed circularly. And the polarization of whistler waves observed at Yamaoka is found statistically to be right-handed circular (Ohta and Eguchi, 1986). We can know the specific value with respect to the polarization of the equivalent plane wave in terms of the spectral matrix deduced by the wave field components observed on the ground, although each wave has an individual value influenced by the conditions of the ionosphere,

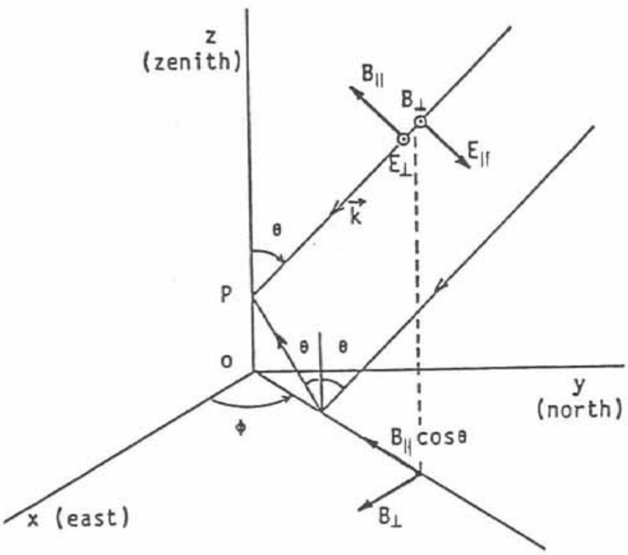

Fig. 1 The electromagnetic field components at the observing point P. The wave is incident with an incident angle θ and with an azimuthal angle ϕ, and the wave is composed of TE and TM modes.

propagation characteristics in the magnetosphere, observing location and so on.

3. Application to simulated data

We have applied the present method to simulated data; a single source model and a two source model. Each source is composed of many elementary plane waves; the average of the polarization of elementary plane waves being right-handed circular or right-handed elliptic, with some deviations of the amplitude and argument. It is found that the inversion on the basis of the kernels with the polarization corresponding to each source yields very satisfactory results for a single source model. An example of the results is given in Fig. 2: The broken line indicates the level of the assumed standard deviation of the source and the full line the levels of 75% and 50% of the maximum of the WDF. Also, we have very useful relationship between the assumed standard deviation of the source and the spread of 50% level of the WDF with the correlation coefficient of 0.94. And even if the source is composed of two peaks, the present method is able to resolve the two peaks when they are well-detached from each other.

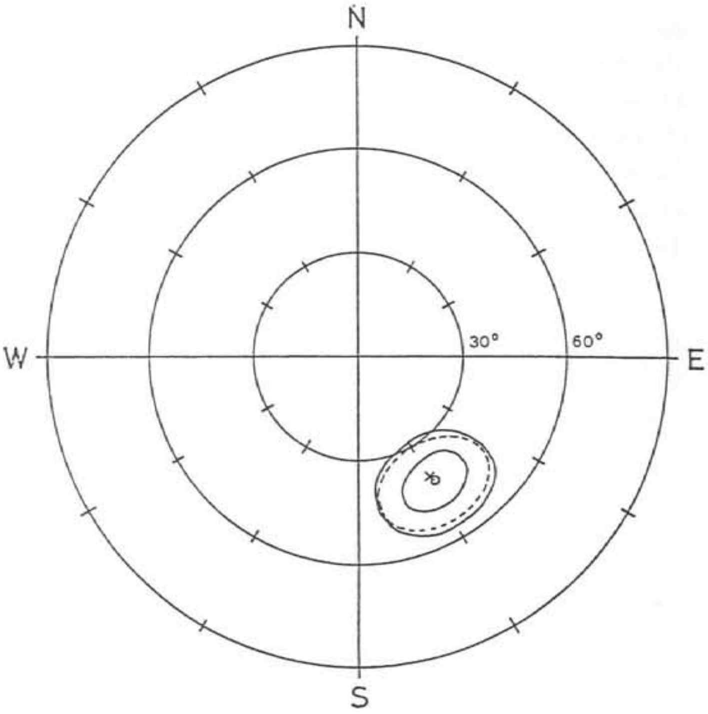

Fig. 2 An example of the WDF solutions: the broken line shows the level of the assumed standard deviation of the source and the full line the levels of 75% and 50% of the maximum WDF. The small circle indicates the direction of the maximum of the WDF and × the result obtained on the assumption of a single plane wave. The WDF solution is deduced by the right-handed circular polarization model; $Q = 0.11$; $P_r = 0.039$.

4. Application to actual data

We have applied the method to real measurements. Unfortunately, the data of VLF/ELF hiss, to which the present method is most effective, are not available, but we use the data of whistlers observed at Yamaoka (geomag. lat. 25° N), Japan. In this experiment, we have recorded the wave forms of three field components (B_x, B_y, E_z) whose center frequency is 5.0 kHz ($\Delta f = \pm 200$ Hz), which have been used for the automatic DF on the basis of field-analysis principle (Ohta *et al.*, 1984). Only one example of the WDF results on 15*th* January, 1982, is illustrated in Figs. 3(a) and (b). Figure 3(a) is the WDF result obtained by the

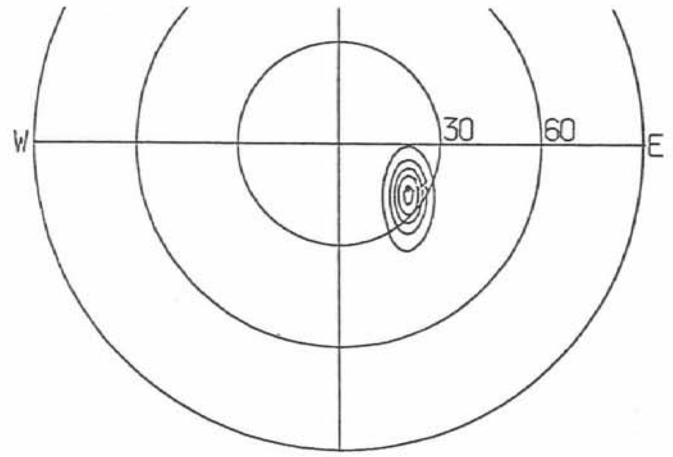

Fig. 3(a) WDF solution for one of daytime whistlers observed at Yamaoka on 15*th* January, 1982. The WDF solution is obtained by the right-handed circular polarization model; $Q = 3.67$; $P_r = 4.91$.

right-handed circular model, Fig. 3(b) the WDF by the right-handed elliptic model on the basis of the polarization of the equivalent plane wave. The stability parameter Q and prediction parameter P_r are the quantities which indicate the acceptability of the WDF solution, and the solution with $Q \leq 1$ and $P_r \leq 9$ is considered to be acceptable for the ground observation. Comparing (a) with (b), the changes of the WDF shape, stability and prediction parameters might lead us to image the effect of the multiply reflected waves in the Earth-ionosphere waveguide in spite of small incident angle. In conclusion, The proposed WDF method is more effective compared to the previous DF methods and also will provide us with further information concerning the magnetospheric propagation (such as duct characteristics) and ionospheric transmission mechanism.

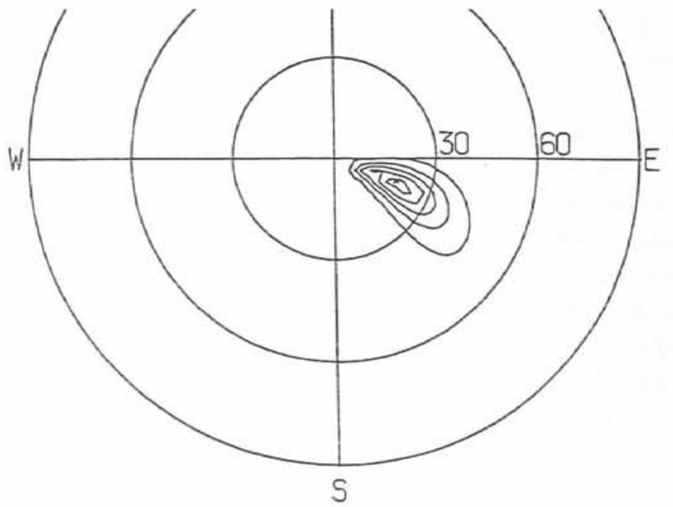

Fig. 3(b) WDF solution for daytime whistler corresponding to Fig. 3(a), which is obtained by the right-handed elliptic polarization model; $Q = 0.507$; $P_r = 0.536$.

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