3-D RAY TRACING OF WHISTLER MODE WAVES IN A NON-DIPOLAR MAGNETOSPHERE

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1. Introduction

Ray tracing with a computer, of the whistler mode waves in the magnetosphere surrounding the earth has been often made to interpret various whistler mode phenomena observed on the ground and on satellites. In these calculations, analytical models of the geomagnetic field and ion densities in the medium are required. Thus far, a dipole model has been assumed for the geomagnetic field and a diffusive equilibrium model for plasma density. Most of the ray tracing has been confined to the meridian plane.

The actual geomagnetic field is significantly non-dipolar, especially in certain longitudes as shown later in Fig. 1. The plasma density depends on local time or longitudes leading to sharp density gradients in the longitudinal direction, particularly at around the sunrise and sunset hours. Under such circumstances, 3-D ray tracing using a more realistic geomagnetic field and plasma density models is necessary for detailed studies of whislter mode wave phenomena, e.g., tracing the ray paths of a ground based VLF signal observed by scientific satellites.

In the present study, the ray tracing program is modified to accomodate a more realistic geomagnetic field model, including higher spherical harmonic moments in addition to the dipole. A more realistic electron density model is used as well.

2. Fundamental 3-D ray tracing equations in the earth magnetosphere

3-D ray tracing equations are represented by the following (Kimura, 1966)

$$
\begin{aligned}
dr/dt &= (\rho_r - \mu\partial\mu/\partial\rho_r)/\mu^2 \\
d\theta/dt &= (\rho_\theta - \mu\partial\mu/\partial\rho_\theta)/r\mu^2 \\
d\phi/dt &= (\rho_\phi - \mu\partial\mu/\partial\rho_\phi)/r\mu^2\sin\theta \\
d\rho_r/dt &= (\partial\mu/\partial r)/\mu + \rho_\theta d\theta/dt + \rho_\phi \sin\partial\ d\phi/dt \\
d\rho_\theta/dt &= [(\partial\mu/\partial\theta)/\mu - \rho_\theta dr/dt + r\rho_\phi\cos\theta\ d\phi/dt]/r \\
d\rho_\phi/dt &= [(\partial\mu/\partial\phi)/\mu - \rho_\phi\sin\theta\ dr/dt - r\rho_\phi\cos\theta\ d\theta/dt]
\end{aligned}
\qquad (1)
$$

where r, θ and ϕ are the geocentric distance, geomagnetic colatitude and geomagnetic longitude. μ is the refractive index and ρ_r, ρ_θ and ρ_ϕ are r, θ and ϕ components of the refractive index vector $\underline{\rho}$ ($/\underline{\rho}/= \mu$'). The refractive index μ is determined by the double quadratic equation

$$A\mu^4 - B\mu^2 + C = 0, \qquad (2)$$

in which A, B and C are the functions of plasma and cyclotron frequencies of electron and ions, and of the angle ψ between the geomagnetic field direction and the refractive index vector $\underline{\rho}$. The angle ψ is given by:

$$\cos\psi = [(B_r\rho_r + B_\theta\rho_\theta + B_\phi\rho_\phi)/B\mu] \qquad (3)$$

where B_r, B_θ, B_ϕ are r, θ, ϕ components of the geomagnetic field B. In the dipole model, B_r and B_θ are represented by a simple function of θ only and B_ϕ = 0.

The electron and ion densities in the space above the top of the ionosphere are often approximated by the diffusive equilibrium model inside the plasmasphere and collisionless model outside the plasmapause, both of which represent electron and ion density profiles along the geomagnetic field lines (e.g.

Aikyo and Ondoh, 1971).

In general, if all terms in the right hand side of eq. (1) can be calculated using plasma parameters and geomagnetic field parameters as mentioned above, the ray paths can be traced by assigning first an initial wave normal direction, and by numerically integrating (1), such as by Adam's predictor-corrector method.

3. Non-dipolar model of geomagnetic field

The earth's magnetic field can be represented in the geographical coordinate system (r, θ, Φ) by the following geomagnetic potential V

$$V = a\sum_{n}(a/r)^{n+1}\sum_{m}[g_n^m \cos m\Phi + h_n^m \sin m\Phi]P_n^m(\theta) \qquad (5)$$

where a is the mean earth radius, $P_n^m(\theta)$ is the associated Legendre spherical harmonic functions of order m and degree n, g_n^m and h_n^m are the Gauss coefficients. The r, θ, Φ of the field (B_r, B_θ, B_Φ) are given by $B = -\nabla V$. The Gauss coefficients are adjusted to fit the observed geomagnetic field. In one such geomagnetic field model, called IGRF (International Geomagnetic Reference Field) model, the coefficients g_n^m and h_n^m are known to order and degree of 10 (see e.g. Peddie, 1982).

By integrating the direction cosines of the geomagnetic field vector, field lines can be traced. Fig. 1 illustrates a longitudinal dependence of the geomagnetic field lines starting at geomagnetic latitude 50° on the earth surface, which are calculated by using the Gauss coefficients of IGRF 1980. (a) shows the projection of the all traced field lines on one geomagnetic meridian plane. (b) is the projection of the field lines on the geomagnetic equatorial plane as it is viewed from the geomagnetic north pole. The difference of the field lines from the dipole model that are also shown in the figures, is evident. Especially longitudinal deviation becomes great between geomagnetic longitudes of 0° and $+90^\circ$, due to Brazilian anomaly.

In calculating the right hand side of (1), the derivatives of B_r, B_θ, B_Φ with respect to r, θ, Φ are required, so that the 1st and 2nd derivatives of V with respect to r, θ, Φ must be calculated. The θ derivatives of P_n^m which appear in the 2nd derivatives of V can be calculated analytically.

3. Processes in calculating paths under a non-dipolar model

All necessary quantities associated with the geomagnetic field can be calculated at any point in the course of ray tracing by giving the geographical coordinates and assuming an appropriate set of Gauss coefficients, such as IGRF. However, in calculating the plasma density there, we need the electron density, ion density, and their temperatures at the foot (reference altitude) of the corresponding field line, because the plasma density represented by the diffusive equilibrium and collisionless model is the density profile along a geomagnetic field line.

If the plasmapause effect is taken into account, the L value is also needed at each step of the ray path. In the present paper, the L value of a field line is defined as the ratio of the longest radial distance of each field line to the earth radius.

In the course of ray tracing, the field line foot point and L value are required every step of ray tracing. Practically, the following procedure is used, in order to reduce the computational time: a necessary number of field line tracings are made in advance before the ray tracing process, starting from the earth's surface. On each field line, several mesh points are distributed, at which necessary information of the field line is registered. In the course of ray tracing the information necessary for determining plasma densities is obtained by interpolation from the mesh points.

In order to determine the mesh points, field line tracing is made prior to ray tracing, in an adopted geomagnetic field model, such as IGRF. Field lines traced from the earth's surface every 1° in geomagnetic latitude and every 5° in geomagnetic longitude suffice. We draw latitudinal planes (rigorously curved surfaces) at every 5° of geomagnetic latitude. All crossing points of the above mentioned field lines and the above latitudinal surfaces become the mesh points. At all mesh points along a same field line, the parameters, i.e. geomagnetic latitude (θ_0) and longitude (ϕ_0) of the field line at the reference radial distance r_0, and L value of the field line are common and are, therefore, commonly registered. The geographical coordinates of each mesh point are written in the computer memory, with other necessary information.

4. Examples of ray paths in the IGRF model

Some examples of ray paths calculated in the IGRF 1980 model are shown in Fig. 2 (a), (b). In this case, an electron density model without latitudinal dependence of the electron density at the reference altitude is adopted for ray tracing, so that the interpolation of parameters using the mesh points was not necessary. Namely (a) of the figure shows the projection on a meridian plane of the ray paths of 5 KHz starting from an altitude of 300 km at 45° geomagnetic latitude and at every 30° of geomagnetic longitudes. (b) is the projection on the geomagnetic equatorial plane as viewed from the north pole. In (a) and (b), a ray path in the dipole model is also shown for comparison.

Fig. 3(a) illustrates three ray paths of a 5 kHz signal starting from geomagnetic latitudes 55°, 60° and 65° on the ground of the Siple station (64.78°S) geomagnetic longitude (7.64°W) under a plasmaspheric model with the plasmapause at L = 3.2. In this figure, all ray paths are projected on the Siple geomagnetic meridian plane. Fig. 3(b) shows the same ray paths projected on the geomagnetic equatorial plane seen from the north pole. As is evident from this figure, the ray paths deviate eastwards by about 15°, even though the initial wave normal direction at the bottom of the ionosphere is assumed as vertical. This eastward deviation appears to be consistent with the results of the EXOS-B (Jikiken) satellite observation (Kimura et al., 1983), in which there was a tendency that the Siple signal was observed more frequently and with stronger intensity in the east side from Siple station than the west side.

For ray paths starting from the +90° geomagnetic meridian plane, the paths show westward deviation due to the deviation of magnetic field lines from those in the diple model. This effect is consistent with the observational results that the Omega signal transmitted from Aldra, Norway has been more frequently observed in the west side of the geomagnetic meridian of Aldra, when it was observed on the ISIS satellites over the Antarctic Syowa Station.

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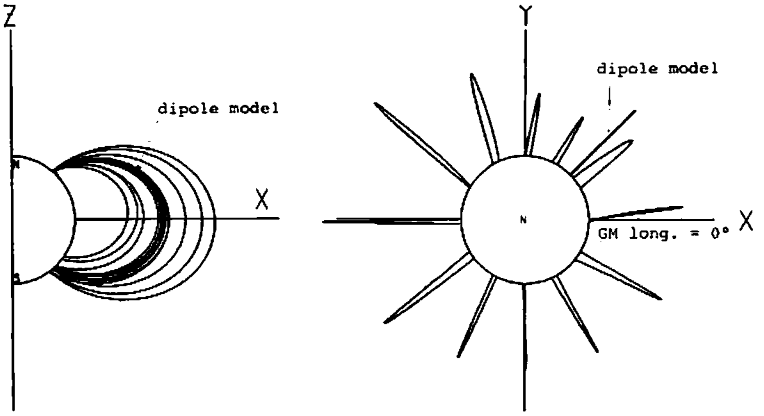

Fig. 1. Longitudinal dependence of magnetic field lines in IGRF model, starting from a geomagnetic latitude of 50° on the ground.

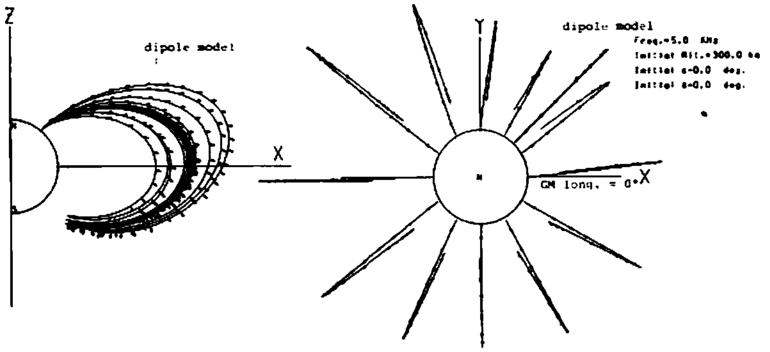

Fig. 2. Longitudinal dependence of 3-D ray paths of 5 kHz signals starting from 50° geomagnetic latitude at an altitude of 120 km, with vertical initial wave normal direction, in the IGRF model.

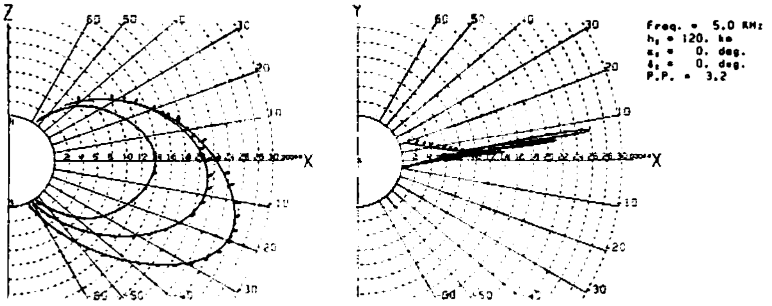

Fig. 3. 3-D ray paths of 5kHz signals starting from the Siple geomagnetic longitude (7.64°W) at geomagnetic latitudes of 55°, 60° and 65°S in the IGRF model.