

Improved Absorbing Boundary Condition for TE_{n0} Multi-Mode Rectangular Waveguide Analysis Using FDTD Method

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Abstract: Finite-Difference Time-Domain (FDTD) method is applied to a modal analysis of a multi-mode rectangular waveguide. To absorb the unknown tangential electromagnetic field at the waveguide terminal, first the electric fields at one cell apart from the terminal are expanded numerically into a finite set of eigen modes, next Mur's absorbing boundary condition (ABC) is applied to each propagating mode and the analytical method using the modal attenuation constant is also applied to each evanescent mode. Finally, all modes are linearly combined at the terminal. The validity of the proposed method is confirmed by comparison with the mode matching method on the scattering power in a rectangular waveguide. This method is effective only in the single frequency analysis.

Key words: FDTD, Absorbing boundary condition, Multi-mode, Rectangular waveguide, Modal analysis

1. Introduction

It is essential to take into account higher modes along with the dominant mode on microwave circuit analysis or EMC design when much accuracy is needed especially in the high frequency range where the higher modes can propagate easily. For example, in the shielding problem with regard to the narrow gap having much wide dimension in its perpendicular direction, several higher modes having an electric field component perpendicular to the gap can exist and the mode coupling between the incident dominant mode and another modes is expected if a certain discontinuity is included in the path. Then to quantify the ratio of induced higher mode transmitting power or suppressed higher mode one to the dominant mode, which is commonly known as modal analysis, provides the effective information.

In this paper, the Finite-Difference Time-Domain (FDTD) method has been employed for the modal analysis of a rectangular broad waveguide where multi-TE_{n0} modes exist. To apply the FDTD method to the general waveguide analysis, it is necessary to model the infinite length waveguide having no reflection at each terminal for port connection. However, there is a difficulty of handling absorbing boundaries when the waveguide is

operated much above than the lowest mode's cut-off frequency because the waveguide operation corresponds to the multi-line transmission line. In recent study, some termination methods such as Higdon's ABC [1] which was applied to the specific high angle incidence, improved ABC structure [2] which surrounds the terminal like U shape by Mur's ABC [3], Berenger's basic PML [4] with scattering material [5], and improved PML [6][7] which modify PML constants to absorb the evanescent mode were reported. These methods are discussed mainly in terms of the absorption of the propagating modes near the cut-off frequency and also the evanescent modes under the cut-off frequency. However, there are some weaknesses in determination of the optimum condition for numerical stability, memory consumption or complicated differential equations in proportional to the accuracy. This paper presents a simple ABC based on the mode expansion technique to absorb the propagating modes as well as the evanescent modes.

2. Methodology

2.1 Extended ABC for Multi-Propagating Mode

Consider the configuration shown in Fig.1, where dimension a is the width of the waveguide and the ABC is applied to $k = 0$ plane. Let u represent a tangential electric field component and let $u(t = n\Delta t, x = i\Delta x, y = j\Delta y, z = k\Delta z) \equiv u_{ijk}^n$. The Mur's ABC is defined (1)

$$u_{ij0}^n = u_{ij1}^{n-1} - \frac{\Delta z - v\Delta t}{\Delta z + v\Delta t} (u_{ij1}^n - u_{ij0}^{n-1}) \quad (1)$$

if the waveguide is assumed as single mode operation, infinite length waveguide is readily modeled with relative high approximate accuracy using equation (1) substituted the phase velocity v . However, in the case of dimension a is much longer than the cut-off dimension where the waveguide is in multi-mode operation, equation (1) cannot be used because the phase velocity cannot be determined one. To remove this limitation, we find each eigen mode amplitude $U_1^n(1, 2, 3, \dots, N)$ from electric fields at one cell apart from the terminal u_{ij1}^n using the eigen mode expansion. Where (N) denotes the N^{th} higher mode and modal amplitude U which has

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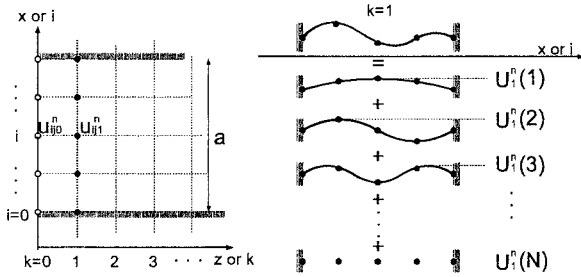


Fig.1 FDTD grid of a semi-infinite rectangular waveguide (left) and its eigen modes at $k = 1$ (right).

no information on xy position is suffixed by z position only. The arbitrary order modal amplitude U is calculated from the orthogonal integration (2) and this is readily implemented numerically such as Simpson's formula because the u_{ij1}^n term is discretized in advance. And the suffix j is arbitrary for TE_{n0} mode operation.

$$U_1^n(N) = \frac{2}{a} \int_0^a u_{ij1}^n \sin \frac{N\pi i \Delta x}{a} dx \quad (2)$$

The N^{th} higher mode waveform $u_{ij1}^n(N)$ at one cell apart from the terminal is reproduced by (3) using (2).

$$u_{ij1}^n(N) = U_1^n(N) \sin \frac{N\pi i \Delta x}{a} \quad (3)$$

The phase velocity of N th propagating mode $v(N)$ is

$$v(N) = \frac{\omega}{\sqrt{k^2 - (N\pi/a)^2}} \quad (4)$$

Then, we can apply Mur's one way equation to each higher propagating mode of arbitrary order with no modification using equation (3) and (4).

2.2 Improved ABC for Multi-Evanescent Mode

In the case of the evanescent mode Mur's ABC cannot be used because the corresponding part to the phase velocity of the propagating mode becomes pure imaginary. However, the terminal field is derived from the adjoining cell value taking into account the attenuation constant α because there is no phase factor in evanescent mode. Let $U_0^n(N)$ represent the N th evanescent modal amplitude at the terminal, $U_1^n(N)$ represent the one at one cell apart from the terminal and $\alpha(N)$ represent the modal attenuation constant, then $U_0^n(N)$ is expressed by equation (5).

$$U_0^n(N) = U_1^n(N) e^{-\alpha(N)\Delta z} \quad (5)$$

where $\alpha(N) = \sqrt{(N\pi/a)^2 - k^2}$

The terminal field is found with no differential approximation in the case of the evanescent mode. Fig.2 shows the procedure of improved ABC for propagating and evanescent mode based on the eigen mode expansion. Another way to absorb the evanescent mode is to

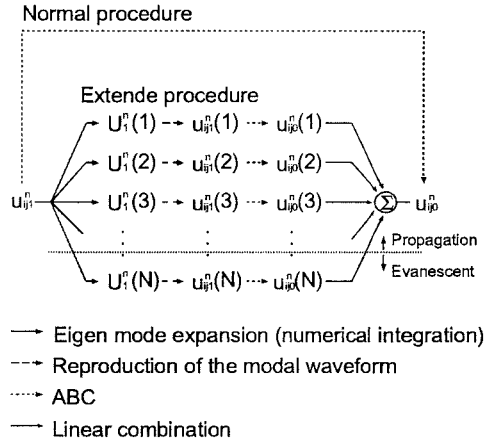


Fig.2 Calculation procedure of the multi-mode absorbing boundary condition based on the eigen mode expansion.

lengthen the distance between the discontinuity plane and the ABC plane to which the lowest evanescent mode is attenuated by insignificant level.

2.3 Modal Scattering Power

The modal expansion procedure used in the proposed ABC is also useful for derivation of the modal scattering power. To calculate the modal forward and backward real power the tangential electric and magnetic field are both expanded into finite propagating modes at the power reference plane $k = p$. N^{th} modal amplitude $U_p^n(N)$ is given by equation (2) substituted suffix $k = 1$ for $k = p$. The term u_{ijp}^n in the integration (2) must be discriminated between electric field e and magnetic field h because there is half step shift between e and h in both time and space. It is calculated by equation (6) for e and equation (7) for h .

$$u_{ijp}^n = e_{ijp}^n \quad (6)$$

$$u_{ijp}^n = \frac{1}{4} (h_{ijp}^{n-1} + h_{ijp-1}^{n-1} + h_{ijp}^n + h_{ijp-1}^n) \quad (7)$$

The N th modal amplitude of electric field $E_p^n(N)$ is separated into forward and backward portion [8] by equation (8) and (9).

$$E_p^n(N)^+ = \frac{E_p^n(N) - Z_{TE} H_p^n(N)}{2} \quad (8)$$

$$E_p^n(N)^- = \frac{E_p^n(N) + Z_{TE} H_p^n(N)}{2} \quad (9)$$

Finally the time varying modal forward and backward power are calculated analytically by (10) and (11). Fig.3 shows the procedure for modal scattering power.

$$P_p^n(N)^+ = \frac{|E_p^n(N)^+|^2 ab}{2Z_{TE}} \quad (10)$$

$$P_p^n(N)^- = \frac{|E_p^n(N)^-|^2 ab}{2Z_{TE}} \quad (11)$$

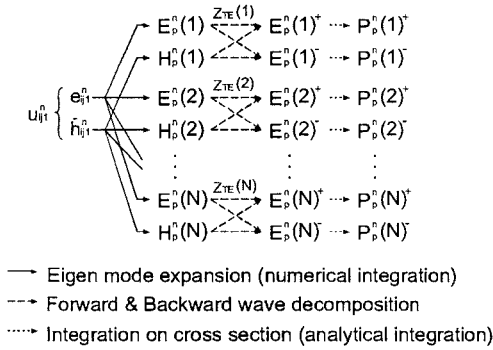


Fig.3 Calculation procedure of a forward and backward modal transmitting power applied at $k = p$ plane based on the eigen mode expansion.

3. Error Analysis

3.1 Time-Domain Characteristics

Fig.4 shows the rectangular waveguide model inserted with a metal post generating infinite TE_{n0} modes. This model is one of the basic H plane discontinuity circuit frequently used in the microwave solid circuit. The waveguide width in region I, II are 40.0mm, 27.0mm each and 5.0mm length of the intermediate region II. The height is 2.0mm constant assuming a narrow gap. The current sheet source J_y [A/m²] equivalent to TE_{20} mode transmitting max 1W power is pulsed with sinusoidal frequency 11.0GHz at two cells inside the absorbing boundary and the modal scattering power is observed in waveguide I at reference planes 5.0mm apart from each discontinuity plane. The cut-off frequencies of TE_{10} , TE_{20} , TE_{30} , and TE_{40} in waveguide I are 3.747GHz, 7.495GHz, 11.242GHz, and 14.990GHz. So two propagating modes and infinite evanescent modes can exit at reference planes and 10 higher modes are taken into account at ABC. The cubic cell $\Delta l = 0.5$ mm is used and the time step somewhat under the courant condition $\Delta t = 0.950$ ps is used. Mode matching method (MM) taking into account 100 higher modes in each region is referred to confirm the operation of the proposed ABC.

Fig.5 shows the envelope of the reflected real modal power at the reference plane 1 in time domain. Connection waveguide I having length L of two type, 10mm and 70mm, are examined compared with the reference value. $L = 70$ mm is selected taking into account the distance which the lowest evanescent mode TE_{30} is attenuated under -30.0 dB at ABC plane in this configuration. On the other hand $L = 10$ mm is selected as one example where reactive power is relative strong at ABC plane. Significant difference in power level is not observed though $L = 70$ mm case needs much time to converge and large memory. The deviation from the reference value is 1.23% in TE_{10} and 0.95% in TE_{20} .

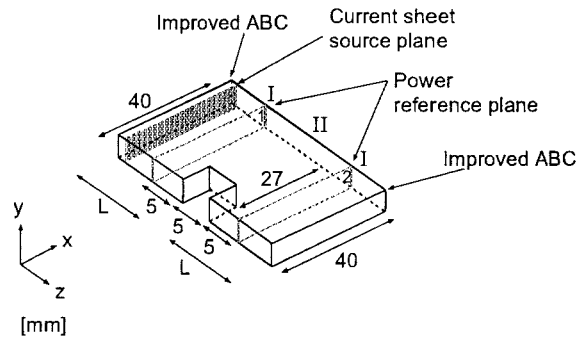


Fig.4 TE_{n0} multi-mode rectangular waveguide having H plane discontinuities. Current sheet source equivalent to TE_{20} mode is incident from the left.

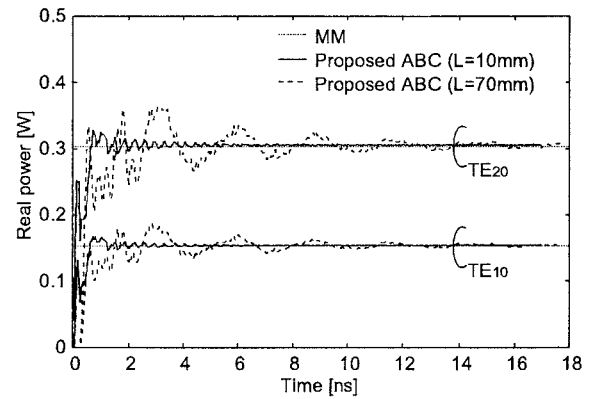


Fig.5 Time domain reflected modal real power at reference plane 1 when TE_{20} (max 1W power) is incident at $f=11.0$ GHz. Connection waveguide length L of two type, 10mm and 70mm, are examined compared with the mode matching method (MM).

The deviation is defined as (12). It is confirmed the evanescent mode absorption can be done successfully even though L is near the discontinuity.

$$Deviation = \frac{|FDTD - MM|}{MM} \times 100[\%] \quad (12)$$

3.2 Frequency-Domain Characteristics

Fig.6 shows the frequency characteristics of the reflected real modal power deviation from the reference value at reference plane 1 when TE_{20} mode is incident with the sinusoidal frequency from 7.5GHz to 14.8GHz. Start frequency 7.5GHz is somewhat above the cut-off frequency of dominant TE_{20} mode and stop 14.8GHz is slightly under the cut-off frequency of TE_{40} mode. Proposed ABC is at 10mm apart from the discontinuity. Small deviation peak under 2.6% was detected near $f=11.3$ GHz where TE_{30} mode starts to propagate and its incident angle is relative high. It is considered the error of the propagating mode near the cut-off frequency range is caused by Mur's differential approximation [10].

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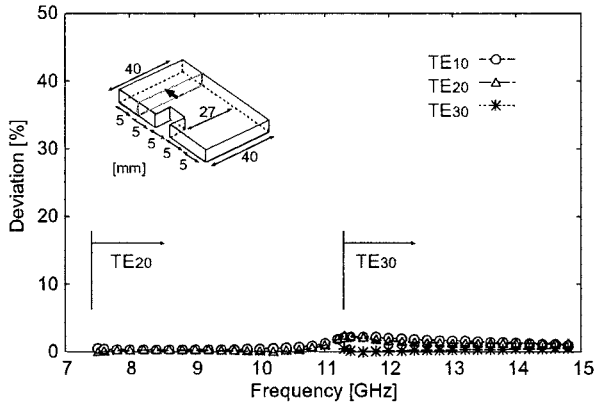


Fig.6 Frequency characteristics of the modal reflected power deviation from the reference value at power reference plane 1 when TE_{20} is incident. Proposed ABC is at 10mm apart from the discontinuity.

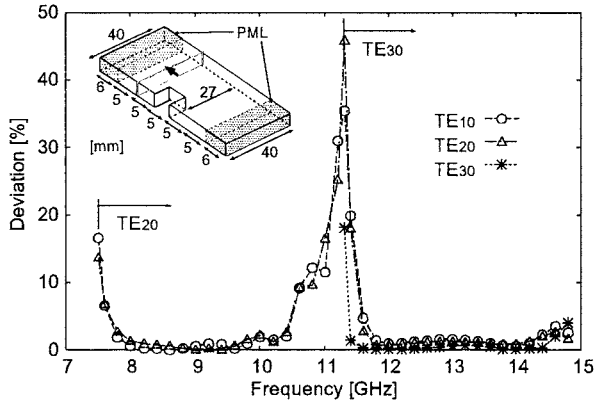


Fig.7 Frequency characteristics of the modal reflected real power deviation from the reference value at power reference plane 1 when TE_{20} is incident. Basic PML 12 layers is at 10mm apart from the discontinuity.

Then the error of TE_{30} spread to another modes because the energy of TE_{30} is large in this configuration. Fig.7 shows the frequency characteristics in the same configuration except for the ABC. The basic PML having 12 layers is used for comparison. Large deviation peaks are detected near $f=7.5\text{GHz}$, 11.2GHz , and 14.9GHz which correspond to the cut-off frequencies. The propagating modes and evanescent modes near the cut-off frequency cannot be absorbed because the basic PML has conductivities only in propagating direction as known. It is confirmed the proposed ABC shows good performance having no significant error in frequency domain.

4. Conclusion

The FDTD method is applied to the modal analysis of a multi-mode waveguide. To absorb the unknown tangential electromagnetic field at the waveguide terminal, the field is expanded into a finite set of eigen modes

then Mur's ABC is applied to each propagating mode and the analytical method is applied to each evanescent mode. Finally, all modes are linearly recombined at the terminal. The proposed method is compared with the mode matching method on the scattering power in a basic rectangular broad waveguide having an H plane discontinuity. It is found that multi-propagating modes are absorbed having no serious error near the cut-off frequency and also the evanescent mode absorption can be done even though ABC is near the discontinuity where reactive energy is strong. The frequency characteristic shows good performance compared with the basic PML. A simple ABC with low calculation cost for the modal analysis of a waveguide is presented though this method is effective only in single frequency analysis.

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