# TRANSIENT RESPONSES OF ELECTROMAGNETIC WAVES IN MAGNETIZED PLASMA BY PLANE WAVE EXCITATION

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## 1. INTRODUCTION

The transient propagation of electromagnetic waves in a plasma has been of interest in connection with the studies of propagation characteristics and plasma diagnostics [Felsen(ed.), 1976 ; Hosono, 1981 ; McIntosh, 1980] .

In the presence of an external static magnetic field in the direction of propagation(z-axis:see Fig.1), the plasma becomes anisotropic. The transient electromagnetic waves in anisotropic plasmas have been studied by several authors [Case and Haskell, 1969 ; Felsen, 1969 ; Schmitt, 1965] . Their solutions for the waveforms are given by asymptotic expressions and are not generally valid in the region of initial arrival of the pulse. Hill[1972] has investigated the transient responses for an impulse plane wave incidence in a longitudinally magnetized semi-infinite plasma and obtained the waveforms by numerical integration of inverse Fourier transform. However, his method is only valid for the propagation along the static magnetic field.

It is the purpose of this paper to report detailed numerical results in a magnetized plasma as follows:

(1) We study the responses for an unit step modulated carrier incident to a semi-infinite magnetized plasma. The static magnetic field is assumed to have arbitrarily direction in y-z plane(see Fig.1). The plasma is incompressible and homogeneous.

(2) The time responses and the spatial responses of the transmitted waves are given for typical parameters of plasma.

(3) For our analysis, we used a numerical inversion method of the Laplace transform which is proposed by Hosono[1981].

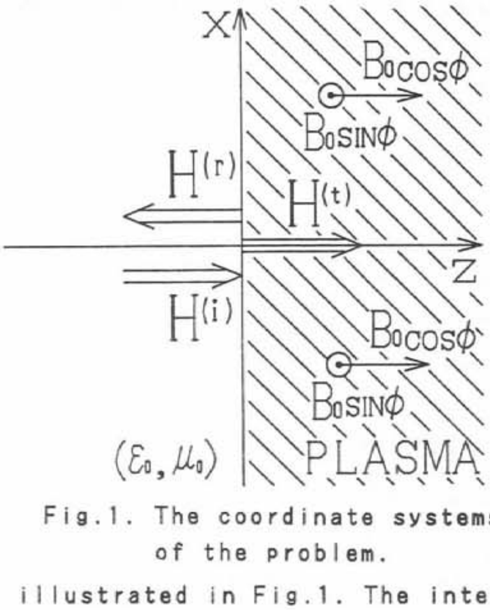


Fig.1. The coordinate systems of the problem.

## 2. FORMULATION

The coordinate system of our problem is illustrated in Fig.1. The interface separating the plasma region(z>0) and free space(z<0) is the plane of z=0. A plane wave is normally incident from free space. A static magnetic field is given by $\mathbf{B}_0 = B_0(\mathbf{a}_y\cos\phi + \mathbf{a}_z\sin\phi)$, where $\mathbf{a}_y$ and $\mathbf{a}_z$ are unit vectors along y and z axes, respectively. It is assumed that the motion of heavy ions can be neglected, but the collision between electrons and heavy particles are accounted. The time-dependent electromagnetic fields are denoted by $\mathbf{E}$ and $\mathbf{H}$, respectively. The linearized equation of motion is

$$m\{\partial/\partial t + \nu\}\mathbf{V} = e\{\mathbf{E} + \mathbf{V}\times\mathbf{B}_0\} \quad (1)$$

$\mathbf{V}$: the mean velocity of electrons, $\nu$ : the collision frequency
$e$ : the charge of electron, $m$ : the mass of electron

Taking the Laplace transform of (1) and Maxwell equations using zero initial condition, we find that the y-components of electromagnetic waves, $\hat{H}_y$, and $\hat{E}_y$, satisfy the following wave equations. $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$ are the Laplace transform of $\mathbf{E}$ and $\mathbf{H}$, respectively.

$$(1+K)\frac{\partial^2 \hat{H}_y}{\partial X^2} + \{(1+K)+\frac{K_z^2}{K}\}\frac{\partial^2 \hat{H}_y}{\partial Z^2} - S\frac{K_z}{Z_o}\{\frac{K_y}{K}\frac{\partial \hat{E}_y}{\partial X}$$
$$+(1+K_P)\frac{\partial \hat{E}_y}{\partial Z}\} - S^2\{(1+K)^2+K_y^2+(1+K)\frac{K_z^2}{K}\}\hat{H}_y=0 \qquad (2)$$

$$\{(1+K)^2+K_y^2+(1+K)\frac{K_z^2}{K}\}\{\frac{\partial^2 \hat{E}_y}{\partial X^2}+\frac{\partial^2 \hat{E}_y}{\partial Z^2}\} + SZ_oK_z\{\frac{K_y}{K}\frac{\partial \hat{H}_y}{\partial X}$$
$$+(1+K_P)\frac{\partial \hat{H}_y}{\partial Z}\} - S^2(1+K_P)\{(1+K)^2+K_y^2+K_z^2\}\hat{E}_y=0 \qquad (3)$$

$\omega_P^2 \triangleq N_o e^2/m\varepsilon_o$, $S \triangleq s/\omega_P$, $Z \triangleq \omega_P z/c$, $X \triangleq \omega_P x/c$, $K_P \triangleq 1/(S^2U)$
$D_o \triangleq \nu/\omega_P$, $U \triangleq 1+D_o/S$, $\Omega_C \triangleq -eB_o/(m\omega_P)$, $Y \triangleq \Omega_C/(SU)$, $K \triangleq K_P/(1+Y^2)$
$K_y \triangleq KY\sin\phi$, $K_z \triangleq KY\cos\phi$, $Z_o \triangleq \sqrt{\mu_o/\varepsilon_o}$, c: velocity of light

Eqs.(2) and (3) are coupled differential equations for $\hat{H}_y$ and $\hat{E}_y$. These equations decouple when $K_z=0$ i.e. $\phi=\pi/2$. To obtain the solution of the coupled equations, we assume that the electromagnetic waves propagate according to the factor $\exp\{-S(\Gamma_x X+\Gamma_z Z)\}$, where $\Gamma_x$ and $\Gamma_z$ are the propagation constants along x and z axes.

In this paper, we consider the case in which a plane wave with magnetic vector polarized in the y direction is normally incident($\Gamma_x=0$) on the interface.

Using (2),(3), $\Gamma_z$ is given by

$$\Gamma_{1z} = [\{\alpha_2+(\alpha_2^2-4\alpha_4\alpha_0)^{1/2}\}/(2\alpha_4)]^{1/2}$$
$$\Gamma_{2z} = [\{\alpha_2-(\alpha_2^2-4\alpha_4\alpha_0)^{1/2}\}/(2\alpha_4)]^{1/2} \qquad (4)$$

$\alpha_4=1+K+K_z^2/K$, $\alpha_2=2\{(1+K)^2+K_y^2+(1+K)\cdot K_z^2/K\}+K_y^2/K$
$\alpha_o=(1+K_P)\{(1+K)^2+K_y^2+K_z^2\}$

The y-components of the transmitted electromagnetic fields are expressed as follows:

$$\hat{H}_y^{(t)} = \hat{H}_{1y}\exp\{-S\Gamma_{1z}Z\} + \hat{H}_{2y}\exp\{-S\Gamma_{2z}Z\} \qquad (5)$$
$$\hat{E}_y^{(t)} = \hat{E}_{1y}\exp\{-S\Gamma_{1z}Z\} + \hat{E}_{2y}\exp\{-S\Gamma_{2z}Z\} \qquad (6)$$

$\hat{H}_{1y}$ and $\hat{H}_{2y}$ in (5) and (6) are unknown coefficients which are determined by the boundary conditions at z=0.

The incident magnetic fields have only y-component given by

$$\hat{H}_y^{(i)}=\hat{H}_o\exp(-SZ), \quad \hat{E}_y^{(i)}=0 \qquad (7)$$

$\hat{H}_o=\Omega_o/(S^2+\Omega_o^2)=\mathcal{L}[u(\tau)\sin(\Omega_o\tau)]$, $u(\tau)$: unit step function
$\tau=\omega_P t$, $\Omega_o=\omega_o/\omega_P$, $\omega_o$: signal angular frequency

The reflected waves, $\hat{H}_y^{(r)}$ and $\hat{E}_y^{(r)}$, from the plasma are expressed by

$$\hat{H}_y^{(r)}=\hat{R}_y\exp\{SZ\} \qquad \hat{E}_y^{(r)}=Z_o\hat{R}_x\exp\{SZ\} \qquad (8)$$

, where $\hat{R}_y$ and $\hat{R}_x$ are the reflection coefficients. Using the boundary conditions, the following expressions are obtained for $\hat{H}_{1y}, \hat{H}_{2y}, \hat{E}_{1y}$, and $\hat{E}_{2y}$:

$$\hat{H}_{1y} = 2\hat{H}_o \Lambda_1 \Gamma_{1z}/(1+\Gamma_{1z}), \quad \hat{H}_{2y} = 2\hat{H}_o \Lambda_2 \Gamma_{2z}/(1+\Gamma_{2z}) \tag{9}$$

$$\hat{E}_{1y} = -2\hat{E}_o \Lambda_3/(1+\Gamma_{1z}), \quad \hat{E}_{2y} = 2\hat{E}_o \Lambda_3/(1+\Gamma_{2z}) \tag{10}$$

$$\Lambda_1 = \{(1+K_p-\Gamma_{1z}^2)(\Gamma_{2z}^2-1)\} / \{K_p(\Gamma_{2z}^2-\Gamma_{1z}^2)\}$$

$$\Lambda_2 = \{(1+K_p-\Gamma_{2z}^2)(\Gamma_{1z}^2-1)\} / \{K_p(\Gamma_{1z}^2-\Gamma_{2z}^2)\}$$

$$\Lambda_3 = \{K_z(\Gamma_{1z}^2-1)(\Gamma_{2z}^2-1)\} / \{K \cdot K_p(\Gamma_{2z}^2-\Gamma_{1z}^2)\}$$

The time-dependent fields can be evaluated by using the formula of numerical inversion [Hosono, 1981] :

$$f(t) \triangleq \mathcal{L}^{-1}[F(s)] = \{\exp(a)/t\} \cdot \{\sum_{n=1}^{k-1} F_n + 2^{-p-1} \sum_{q=0}^{p} A_{p,q} F_{k+q}\} \tag{11}$$

$F_n \triangleq (-1)^n \,\mathrm{Im}[\{a+j(n-0.5)\pi\}/t]$, $A_{p,p} \triangleq 1$, $A_{p,q-1} = A_{p,q} + {}_pC_{q+1}$

a: approximation parameter [the relative error is small than exp(-2a)]

k, p: the numbers of truncation and Euler transformation terms, respectively

## 3. RESULTS

The subsequent results are given for lossless plasmas ($D_o$=0). Fig.2 shows $H_y^{(t)}$ at $\tau$=6 and Z=7 as a function of (1/N), where N($\triangleq$k+p) is the total number of terms in the summation. The relative errors of $H_y^{(t)}$ are less than $10^{-5}$ for N $\geqq$ 800 (a=5, p=a+3). In this paper, the numerical computation is carried out using the parameters a, p, and N which make the relative error less than $10^{-5}$ ($\fallingdotseq 10^{-a}$).

Fig.3 is a comparison of our results with those of Hill[1972] for the transmitted waves at Z=2 when excited by delta function ($E_o$=1). There is an excellent agreement between our result and Hill's[1971] result. The interesting transient responses for $\tau$ >20 are not investigated in Hill's[1972] paper. Our numerical inversion, however generally speaking the numerical analysis for long values of time is difficult, gives accurate responses for a wide range of time. It is seen from Fig.3 that the exited plasma frequency oscillation decreases for $\tau$ >50, leaving small beatwave.

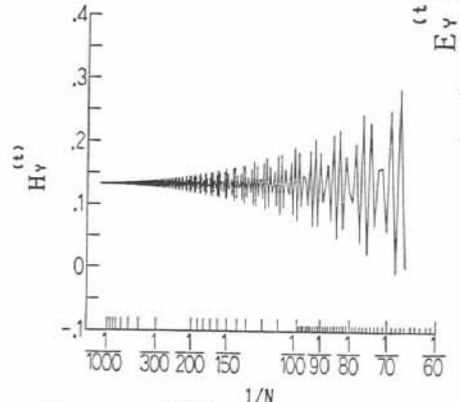


Fig.2. $H_y^{(t)}$ versus (1/N) for $\tau$=6, Z=7, $\phi$=0°, $\Omega_o$=2.0, and $\Omega_c$=1.0.

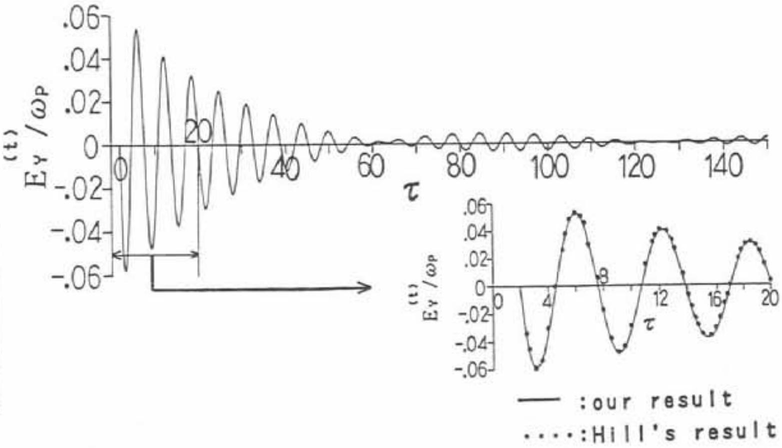


Fig.3. Transient response of $E_y^{(t)}/\omega_P$ for $\phi$=0°, Z=2, $\Omega_c$=0.1, and a comparison of our results with those of Hill[1972].

The transient spatial variations of $H_y^{(t)}$ and $H_x^{(t)}$ for sinusoidal excitation are plotted in Figs.4 and 5 as a function of Z for $\tau=60$ and $\phi=70°$. Fig.6 illustrates the dispersive properties of group velocity $V_{1g}/c$ and $V_{2g}/c$ which are defined by

$$V_{Qg}/c=[\partial \beta_Q(\Omega)/\partial \Omega]^{-1},\quad \beta_Q(\Omega) \triangleq \mathrm{Im}\,[S\Gamma_{Qz}]_{S\rightarrow j\Omega},\quad Q=1,2 \qquad (12)$$

The results of Figs.4-6 tell us the followings:

(1) The main signal with $\Omega_o=2$ starts at $Z_2 \triangleq \tau(V_{2g}/c)=60\times 0.88 \fallingdotseq 53$ because $V_{1g}/c=0.62$ and $V_{2g}/c=0.88$ at $\Omega_o=2$ (Fig.6).

(2) In the region of $Z_1=37 < Z < Z_2=53$ [$Z_1 \triangleq \tau(V_{1g}/c)$], one wave can propagate and in the region of $Z<Z_1$, two waves can propagate.

(3) The waveform in $Z < Z_1$ becomes beatwave because two propagating waves have different wavelength, $\lambda_1 \triangleq 2\pi/\beta_1(\Omega_o)=4.1$, $\lambda_2 \triangleq 2\pi/\beta_2(\Omega_o)=3.6$, respectively.

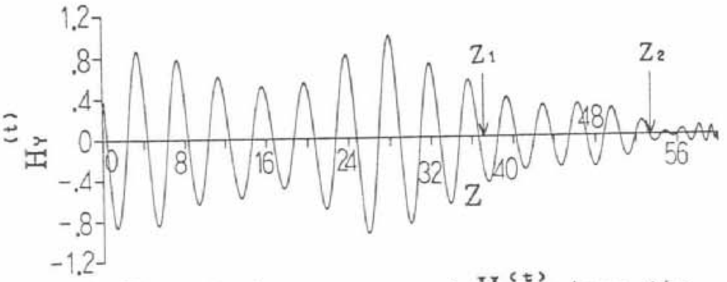


Fig.4. Transient response of $H_y^{(t)}$ along the Z axis for $\tau=60$, $\phi=70°$, $\Omega_o=2.0$, and $\Omega_c=1.0$.

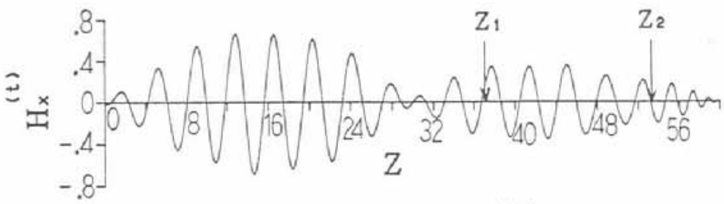


Fig.5. Transient response of $H_x^{(t)}$ along the Z axis for $\tau=60$, $\phi=70°$, $\Omega_o=2.0$, and $\Omega_c=1.0$.

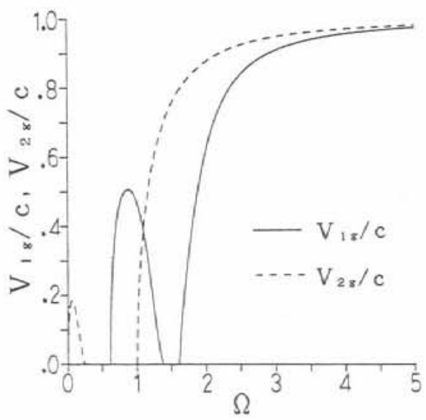


Fig.6. $V_{1g}/c$ and $V_{2g}/c$ versus $\Omega$ for $\phi=70°$ and $\Omega_c=1.0$.

## 4. CONCLUSIONS

We investigated the transient responses for a plane wave normally incident on a magnetized plasma half space. The superimposed static magnetic field is assumed to have arbitrarily direction in y-z plane. The time and the spatial responses are graphically shown, including the cases which have been difficult to analyze by other methods. One possible extension is to analyze the responses in a compressible magnetoplasma [Hinata et al.,1982].

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