TRANSIENT DIPOLE FIELDS SCATTERED BY A PERFECTLY CONDUCTING CYLINDER BURIED IN A LOSSY MEDIUM

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I. INTRODUCTION It is very important for a radar imaging of an invisible target such as underground imaging to investigate the properties of the transient electromagnetic fields transmitted from an antenna and scattered from buried objects. Many authors have investigated the electromagnetic scattering from the cylindrical object buried in a lossy medium for a plane wave incidence or an infinite line current source[1]-[7]. However, most practical underground imaging is carried out with antennas of finite size, e.g., dipole antennas and loaded loop antennas. This paper discusses the transient scattered fields when the dipole antenna is located over the interface separating the two half-spaces and the perfectly conducting cylinder is buried in the lossy medium. The early time scattering fields of the dipole antenna parallel to the buried cylinder are calculated exactly. The geometrical optics(GO) approximation is also derived and is compared with the exact transient fields. It is shown that the GO solution is a very good approximation in the backscattering region above the cylinder, but it fails when the observation point deviates from the right overhead of the buried cylinder.

II. SCATTERING FIELDS The geometry of the problem is illustrated in Fig.1. The space is partitioned into two halves. One of which is filled with air and the other with a homogeneous lossy medium. The perfectly conducting cylinder is buried in the medium whose radius is a. The axis of the cylinder is taken as the z-axis, and is pararell to the interface with depth x_e.

The primary current source $\mathbf{J}^p$ on the transmitting antenna is located in the air. The secondary current $\mathbf{J}^S$ is distributed on the cylinder surface. The electromagnetic fields for the currents $\mathbf{J}^p$ and $\mathbf{J}^S$ can be exactly determined by using the dyadic Green's function for the two half-spaces[8]. The secondary current distribution $\mathbf{J}^S$ for a known $\mathbf{J}^p$ is determined by using the dyadic Green's function and the boundary condition on the cylindrical surface. We define the Fourier transform of $\mathbf{J}^S$ as

$$\mathrm{J}^S_{NR}(\mathbf{k}) = -j\omega\mu_0 \int\int\int_{V'} \mathbf{N}^{R'}(\mathbf{k})\cdot\mathbf{J}^S(\mathbf{R}')dV' \qquad \mathrm{J}^S_{MR}(\mathbf{k}) = -j\omega\mu_0 \int\int\int_{V'} \mathbf{M}^{R'}(\mathbf{k})\cdot\mathbf{J}^S(\mathbf{R}')dV' \tag{1}$$

where $\mathbf{M}^{R'}$ and $\mathbf{N}^{R'}$ are the vector wave functions[8]. Then, the resultant integral equations for $\mathrm{J}^S(\mathbf{k})$ are expressed as follows.

$$\begin{aligned}\mathrm{J}^S_{MR}(-\mathbf{k}_2) &= -\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\epsilon_n}{\eta_2^2}(-1)^n \\ &[k_{2x}k_z\frac{\dot{\mathrm{J}}_n(\eta_2 a)}{\dot{\mathrm{H}}_n^{(2)}(\eta_2 a)}\int_{-\infty}^{\infty}\frac{dk'_y}{k'^2_y+k_z^2}\cos n(\Psi_{2xy}-\Psi'_{2xy})\{-k'_{2x}k_z\mathrm{A}(k'_y,-k_z)+jk_2k'_y\mathrm{B}(k'_y,-k_z)\} \\ &+jk_2k_y\frac{\mathrm{J}_n(\eta_2 a)}{\mathrm{H}_n^{(2)}(\eta_2 a)}\int_{-\infty}^{\infty}\frac{dk'_y}{k'^2_y+k_z^2}\cos n(\Psi_{2xy}-\Psi'_{2xy})\{jk_2k'_y\mathrm{A}(k'_y,-k_z)-k'_{2x}k_z\mathrm{B}(k'_y,-k_z)\}]\end{aligned} \tag{2}$$

$$\mathrm{J}^S_{NR}(-\mathbf{k}_2) = -\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\epsilon_n}{\eta_2^2}(-1)^n$$

$$[k_2 k_y \frac{\dot{\mathrm{J}}_n(\eta_2 a)}{\dot{\mathrm{H}}_n^{(2)}(\eta_2 a)}\int_{-\infty}^{\infty}\frac{dk'_y}{k'^2_y + k_z^2}\cos n(\Psi_{2xy} - \Psi'_{2xy})\{-k'_{2x}k_z \mathrm{A}(k'_y, -k_z) + jk_2 k'_y \mathrm{B}(k'_y, -k_z)\}$$
$$+ jk_{2x}k_z \frac{\mathrm{J}_n(\eta_2 a)}{\mathrm{H}_n^{(2)}(\eta_2 a)}\int_{-\infty}^{\infty}\frac{dk'_y}{k'^2_y + k_z^2}\cos n(\Psi_{2xy} - \Psi'_{2xy})\{jk_2 k'_y \mathrm{A}(k'_y, -k_z) - k'_{2x}k_z \mathrm{B}(k'_y, -k_z)\}] \quad (3)$$

$$\cos\Psi'_{2xy} = -\frac{k'_{2x}}{\eta_2}\ ,\ \sin\Psi'_{2xy} = -\frac{k'_y}{\eta_2}\ ,\ k'_{2x} = \sqrt{k_2^2 - k'^2_y - k_z^2}\ ,\ \eta_2 = \sqrt{k_2^2 - k_z^2}$$

where $\varepsilon_0=1$, $\varepsilon_n=2$ for n>0, and the dots on the Bessel functions J_n and $H_n^{(2)}$ mean the differentiation with respect to a. The coefficients A and B in eqs.(2) and (3) are given by

$$\mathrm{A}(k_y, k_z) = \frac{\mathrm{T}_M^{(+)}}{k_{1x}}\mathrm{J}^P_{MR}(\mathbf{k}_1) + \frac{\mathrm{R}_M^{(-)}}{k_{2x}}\mathrm{J}^S_{MR}(-\mathbf{k}_2)$$
$$\mathrm{B}(k_y, k_z) = \frac{\mathrm{T}_N^{(+)}}{k_{1x}}\mathrm{J}^P_{NR}(\mathbf{k}_1) + \frac{\mathrm{R}_N^{(-)}}{k_{2x}}\mathrm{J}^S_{NR}(-\mathbf{k}_2) \quad (4)$$

where

$$\mathrm{T}_M^{(+)} = \frac{2k_{1x}}{k_{1x}+k_{2x}}e^{j(k_{1x}-k_{2x})x_e} \qquad \mathrm{R}_M^{(-)} = \frac{k_{2x}-k_{1x}}{k_{2x}+k_{1x}}e^{-2jk_{2x}x_e}$$
$$\mathrm{T}_N^{(+)} = \frac{2k_1 k_2 k_{1x}}{k_2^2 k_{1x} + k_1^2 k_2 x}e^{j(k_{1x}-k_{2x})x_e} \qquad \mathrm{R}_N^{(-)} = \frac{k_1^2 k_{2x} - k_2^2 k_{1x}}{k_1^2 k_{2x} + k_2^2 k_{1x}}e^{-2jk_{2x}x_e} \quad (5)$$

The second terms of the coefficients A and B express the multiple reflection between the cylinder and the interface of two media. Therefore, the early time scattering fields are exact even if the second terms of A and B are neglected. We refer this fields to the first order scattering fields in this paper.

For a short(infinitesimal) horizontal electric dipole with the current moment $Il\hat{z}$ located at $(x_s, y_s, 0)$ in the air, the z-component first order scattering electric field at same point is given by

$$\mathrm{E}^{S1}_{\mathrm{T}z}(x_s, y_s, 0) = -\frac{120}{\pi^2}Ilk_1\int_0^{\infty}dh\sum_{n=0}^{\infty}\frac{\epsilon_n}{\eta_2^2}(-1)^n$$
$$\left[\frac{\dot{\mathrm{J}}_n(\eta_2 a)}{\dot{\mathrm{H}}_n^{(2)}(\eta_2 a)}\left\{\int_{-\infty}^{\infty}\frac{dk_y}{k_y^2+h^2}e^{-j(k_{1x}x_h + k_{2x}x_e + k_y y_s)}{}^{\cos}_{\sin}(n\Psi_{2xy})\cdot\left(\frac{k_{2x}k_y h}{k_{1x}+k_{2x}} - \frac{k_2^2 k_{1x}k_y h}{k_1^2 k_{2x} + k_2^2 k_{1x}}\right)\right\}^2\right.$$
$$\left.+\frac{\mathrm{J}_n(\eta_2 a)}{\mathrm{H}_n^{(2)}(\eta_2 a)}\left\{\int_{-\infty}^{\infty}\frac{dk_y}{k_y^2+h^2}e^{-j(k_{1x}x_h + k_{2x}x_e + k_y y_s)}{}^{\cos}_{\sin}(n\Psi_{2xy})\cdot\left(\frac{k_2 k_y^2}{k_{1x}+k_{2x}} - \frac{k_2 k_{1x}k_{2x}h^2}{k_1^2 k_{2x} + k_2^2 k_{1x}}\right)\right\}^2\right] \quad (6)$$

On the other hand, the geometrical optics(GO) approximation of the first order scattering field is given by

$$\mathbf{E}^{S1}_{\mathrm{T}}(x_s, y_s, 0) \sim -j60\left(\frac{n\cos i\cos\theta}{\cos i + n\cos\theta}\right)^2 e^{-2j(k_1 R_1 + k_2 R_2)}\cdot Il$$
$$\cdot\left[\frac{a}{(nR_1\cos^2\theta + R_2\cos^2 i)(nR_1 + R_2)\{(R_2+a)\cos^2 i + nR_1\cos^2\theta\}}\right]^{\frac{1}{2}}\cdot\hat{z} \quad (7)$$

where the distances R_1 and R_2, incident angle i, the refraction angle θ are determined so as to minimize the electric length between the source point and the observation point via the cylinder surface. The transient fields are obtained by transforming eqs.(6) and (7) to the time domain. In this paper, the numerical Fourier transformation is used.

III. NUMERICAL EXAMPLES Fig.2 shows the first order transient scattered fields of the short horizontal dipole at the source point with a parameter of the distance along to the y-axis y_s. The height of the antenna from the interface x_h=5cm and the depth of the cylinder x_e=100cm. The dipole antenna is excited by the following $\cos^6$-type pulse.

$$Il(t) = (Il)_0 \cos^6(\pi \frac{t}{P_d}) , \qquad |t| \le \frac{P_d}{2} . \tag{8}$$

The solid lines are exact scattering field and the broken lines are GO approximation. It is found that the GO solution is a very good approximation for small y_s. However, the approximation error of GO is greatly increased as the antenna goes away from the right overhead of the buried cylinder. This obviously indicates the effect of the wave propagating along the interface.

Fig.3 shows the transient responses of the dipole antenna parallel to the cylinder whose length and radius are 15cm and 1.25mm, respectively. The pulse width P_d of the $\cos^6$-type excitation voltage V_e is 3[nsec]. The period of the ringing of the response corresponds to the period that the current on the antenna goes to the endpoint of the antenna and is reflected back to the driving point with the light speed.

IV. CONCLUSIONS The early time scattering fields have been calculated exactly when the short(infinitesimal) or the finite-length dipole antenna is located over the interface separating the air and the lossy medium, and the perfectly conducting infinite cylinder is buried in the lossy medium. The GO approximation has also been derived and compared with the exact transient field. It has been shown that the GO solution is a good approximation in the backscattering region above the cylinder. Thus, it has been found that the GO approximation is very effective for applying to the underground time domain imaging when the propagation path is nearly normal to the ground surface. It has been also found that the GO approximation is invalid for the slant propagation path.

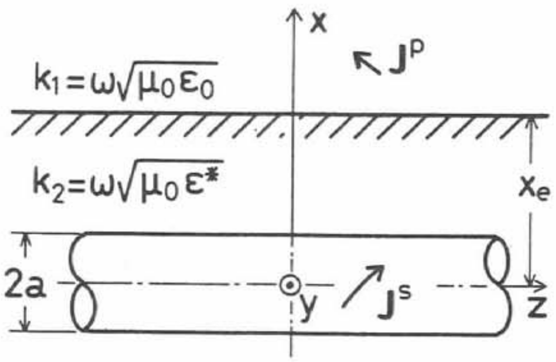

Fig.1 Geometry of the problem. $\varepsilon^*= \varepsilon_0 \varepsilon_r - j\,\sigma/\omega$ is a complex dielectric constant.

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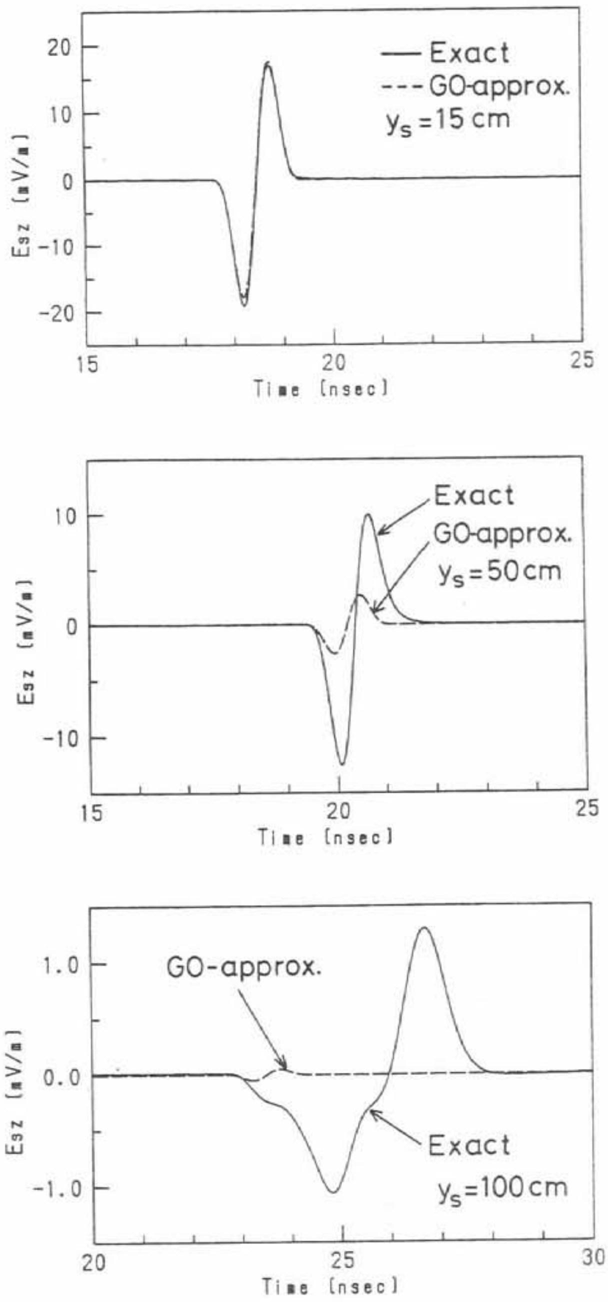

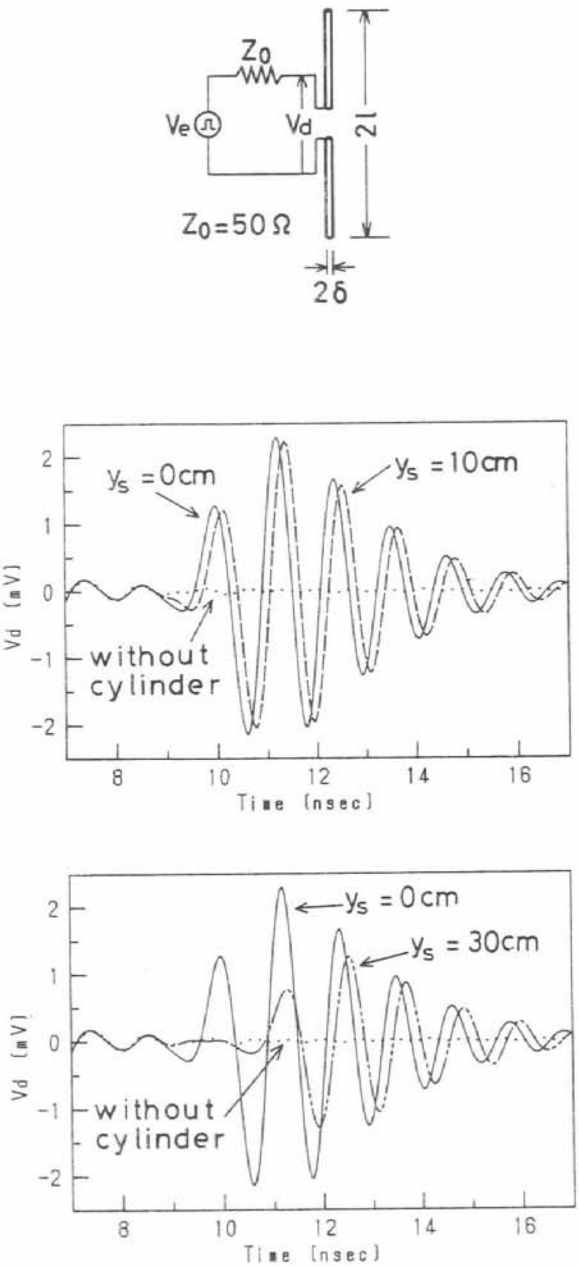

Fig.2 First order transient scattered fields of the short horizontal dipole antenna with a parameter of y_s. —:exact, - -: GO approximation.
(a=15cm, x_e=100cm, x_h=5cm, ε_r=10, σ=0.01S/m, $(Il)_0$=1mAm, P_d=2nsec)

Fig.3 Transient responses of the finite length dipole antenna.
---:y_s=0cm, - -:y_s=10cm,
-.-:y_s=30cm.
(2l=15cm, 2δ=2.5mm, P_d=3nsec, a=5cm, x_e=50cm, x_h=5cm, ε_r=10, σ=0.005S/m)