# PARAMETRIC STUDY OF A RECTANGULAR APERTURE ANTENNA EXCITED BY A PROBE INSIDE CAVITY

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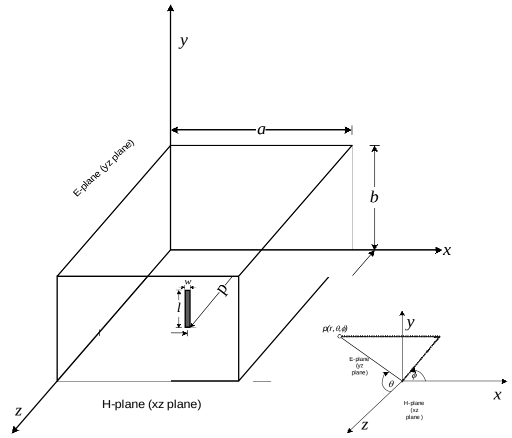
## 1. Introduction

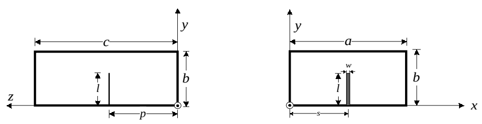
In the wireless communication applications, the antenna is an important role as the key device in transmitting and receiving the signal while the growth of these applications becomes drastically. The antenna in this work is made up of the rectangular aperture mounted on one side of the cavity. In order to design the antenna to meet the application requirements, the parametric study is performed to obtain the optimal parameters. In the previous works, the vector potential is applied in the analysis of the electromagnetic field in the rectangular cavity [1]. The discontinuity of the current at the probe is not included in this approach. Therefore, the dyadic Green function is used to solve this problem [2]-[4]. In this paper, the investigation of rectangular aperture antenna excited by a probe inside cavity is proposed. The analysis starts with the calculation of the electromagnetic field inside the rectangular cavity excited by the probe using the dyadic Green function. Then, the equivalent currents at the aperture are evaluated to determine the radiation pattern from the aperture. The significant parameters that have strong influence to the radiation characteristics are the dimension of the aperture cross section, the length of the cavity, the location and the length of the probe. From the results of this work, it will be utilized as a design parameter of this structure in the microwave and millimeter wave devices.

## 2. Geometry of the Problem

The geometry of the problem is made up of the rectangular aperture mounted on one side of the cavity. The other walls of the cavity are perfect electric conductor. The linear electric probe is located in parallel with the aperture to couple the electromagnetic field from the generator to the cavity.




*(a) perspective view*




*(b) side view* *(c) front view*

**Fig.1** Geometry of the problem

Fig.1 illustrates the configuration of the cavity used for this work, where *a* and *b* parameters are the dimension of the aperture cross section with the length of the cavity of *c*, the length of the probe of *l* located at the location of (*s*, 0, *p*) with the width of the probe of *w*.

## 3. Formulations

In this section, the dyadic Green functions of the rectangular cavity will be derived. In order to derive these functions, it starts with the functions for a rectangular waveguide with the same cross-sectional dimension and applies the method of scattering superposition to find the reflection from the other shorted wall. The functions for semi-infinite waveguide defined in region $\infty > z \geq 0$ are considered first. The electric dyadic Green function for the semi-infinite denoted by $\overline{\overline{G}}_{E1}$ can be written in the form

$$\overline{\overline{G}}_{E1}(\overline{R},\overline{R}')=-\frac{1}{k^2}\hat{z}\hat{z}\delta(\overline{R}-\overline{R}')+\frac{2}{ab}\sum_{m,n}\frac{2-\delta_0}{k_c^2 k_g}\begin{cases}\overline{M}_e(k_g)\overline{M}'_{eo}(z')+i\overline{N}_o(k_g)\overline{N}'_{oe}(z'), z>z' \\ \overline{M}_{eo}(z)\overline{M}'_e(k_g)+i\overline{N}_{oe}(z)\overline{N}'_o(k_g), z<z'\end{cases} \tag{1}$$

where the notation is usual as in [2]. The electric field in the waveguide can be calculated by using the formula

$$\overline{E}(\overline{R}) = i\omega\mu_0 \iiint \overline{\overline{G}}_{E1}(\overline{R},\overline{R}') \cdot \overline{J}(\overline{R}')dV' \tag{2}$$

where $\overline{J}(\overline{R}')$ is the current distribution of the probe. In this work, it can be expressed as

$$\overline{J}(\overline{R}') = \frac{I_m}{w}\sin(k(l-y'))\hat{y} \tag{3}$$

Therefore, the electric field in the rectangular waveguide can be written as

$$E_x = \frac{-4i\omega\mu_0 I_m}{abw}\sum_{m,n}\frac{(2-\delta_0)(k_g^2-k^2)k_y}{k_c^2 k_g k(k_y^2-k^2)}e^{ik_g z}C_x S_y \sin(k_g p)\sin(k_x s)\sin(k_x \frac{w}{2})\left(\cos(k_y l)-\cos(kl)\right) \tag{4}$$

$$E_y = -\frac{4i\omega\mu_0 I_m}{abw}\sum_{m,n}\frac{(2-\delta_0)(k_x^2+k_g^2)k}{k_g k^2 k_x (k_y^2-k^2)}e^{ik_g z}S_x C_y \sin(k_g p)\sin(k_x s)\sin(k_x \frac{w}{2})\left(\cos(k_y l)-\cos(kl)\right) \tag{5}$$

$$E_z = -\frac{4\omega\mu_0 I_m}{abw}\sum_{m,n}\frac{(2-\delta_0)k_y}{k_x k(k_y^2-k^2)}e^{ik_g z}S_x S_y \sin(k_g p)\sin(k_x s)\sin(k_x \frac{w}{2})\left(\cos(k_y l)-\cos(kl)\right) \tag{6}$$

where

$S_x = \sin k_x x,\ C_x = \cos k_x x,\ S_y = \sin k_y y,\ C_y = \cos k_y y$

$k_x^2 + k_y^2 + k_g^2 = k^2 \qquad k_c^2 = k_x^2 + k_y^2$

$k_x = \frac{m\pi}{a}, \quad m = 0,1,\ldots. \ k_y = \frac{n\pi}{b}, \quad n = 0,1,\ldots.$

It is noted that *m* and *n* must not be zero simultaneously, *k* denotes the wave number of the free space at the operating frequency, $I_m$ is the maximum current distribution. Next, the magnetic field in the waveguide is calculated by using the formula of

$$\overline{H}(\overline{R}) = \iiint_v \left[\nabla \times \overline{\overline{G}}_{E1}(\overline{R},\overline{R}')\right] \cdot \overline{J}(\overline{R}')dv' \tag{7}$$

In the same way, the magnetic field in the rectangular waveguide can be expressed as

$$H_x = \frac{4iI_m}{abw}\sum_{m,n}\frac{(2-\delta_0)k}{(k_y^2-k^2)k_x}e^{ik_g z}S_x C_y \sin(k_g p)\sin(k_x s)\sin(k_x \frac{w}{2})\left(\cos(k_y l)-\cos(kl)\right) \tag{8}$$

$$H_y = 0 \tag{9}$$

$$H_z = -\frac{4I_m}{abw}\sum_{m,n}\frac{(2-\delta_0)k}{k_g(k_y^2-k^2)}e^{ik_g z}C_x C_y \sin(k_g p)\sin(k_x s)\sin(k_x \frac{w}{2})\left(\cos(k_y l)-\cos(kl)\right) \tag{10}$$

Once the electromagnetic field is known, the equivalent current at the aperture is determined by using

$$\overline{J}(\overline{R}) = \hat{z}\times\overline{H}(\overline{R})\big|_{z=c} \text{ and } \overline{M}(\overline{R}) = \overline{E}(\overline{R})\big|_{z=c}\times\hat{z} \tag{11}$$

where $\overline{J}(\overline{R})$ and $\overline{M}(\overline{R})$ are the electric and magnetic current sources at *z*=*c*, respectively. The far field radiation of the rectangular aperture is determined by [5]

$$E_\theta(r,\theta,\phi) \simeq \frac{ike^{ikr}}{4\pi r}\left(L_\phi + \eta N_\theta\right) \text{ and } E_\phi(r,\theta,\phi) \simeq -\frac{ike^{ikr}}{4\pi r}\left(L_\theta - \eta N_\phi\right)$$

where $N_\theta = \iint_s \left[J_x\cos\theta\cos\phi + J_y\cos\theta\sin\phi - J_z\sin\theta\right]e^{-ikr'\cos\psi}ds'$

$N_\phi = \iint_s \left[-J_x\sin\phi + J_y\cos\phi\right]e^{-ikr'\cos\psi}ds'$

$L_\theta = \iint_s \left[M_x\cos\theta\cos\phi + M_y\cos\theta\sin\phi - M_z\sin\theta\right]e^{-ikr'\cos\psi}ds'$

$L_\phi = \iint_s \left[-M_x\sin\phi + M_y\cos\phi\right]e^{-ikr'\cos\psi}ds'$

$ds' = dx'dy'$ and $r'\cos\psi = x'\sin\theta\cos\phi + y'\sin\theta\sin\phi$

## 4. Results

In this section, the parametric study of radiation characteristics of the rectangular aperture will be investigated for various parameters of the rectangular cavity and the probe. The operating frequency of 2.45 GHz is used for this work. By following the formulation of the rectangular aperture, the radiation pattern is obtained in Fig.2. It is obvious that the unidirectional pattern is achieved. The beamwidth in E-plane is wider than that of the H-plane because the dimension in *x* direction is larger than the dimension in *y* direction.

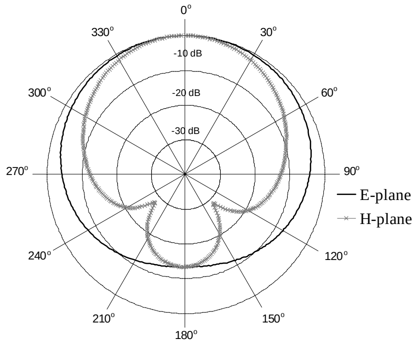
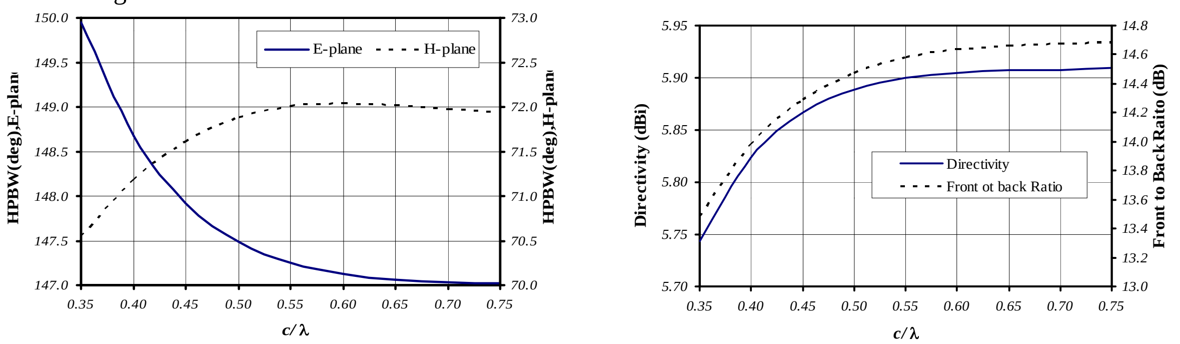



**Fig.2** Radiation pattern ($a= 0.7\lambda$, $b=0.35\lambda$, $c= 0.35\lambda$, $l= 0.25\lambda$, $p=0.25\lambda$, $s= 0.35\lambda$ and $w=0.015\lambda$)

### 4.1 Length of the cavity (*c*)

In this section, the radiation characteristics from rectangular aperture for various the length of the cavity is presented. The half power beamwith, the directivity and the front to back ratio for various *c*-parameters are shown in Fig 3.




(a) Half power beamwith

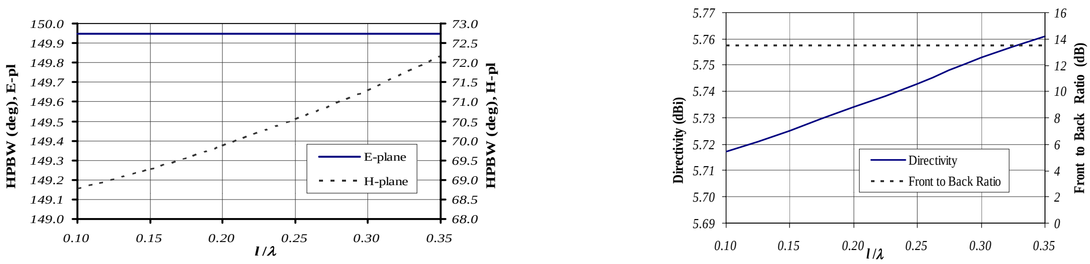
(b) Directivity and front to back ratio

**Fig. 3:** Radiation characteristics for various *c* ($a= 0.7\lambda$, $b=0.35\lambda$, $l=0.25\lambda$, $p=0.25\lambda$, $s=0.35\lambda$ and $w=0.015\lambda$)

It can be found from Fig.3 that when we increase the length of the cavity the beamwidth in E-plane is narrower but the beamwidth in H-plane is wider. However, the directivity and F/B ratio is increased when the length of the cavity is longer.

### 4.2 Length of the probe (*l*)

The analysis of the effect of the length of the probe (*l*) will be reported in this section. Fig. 4 illustrates the radiation characteristics for various *l*-parameters. From the Fig 4, the length of the probe has affected only the H-plane pattern. The beam in E-plane is not change with the variation of the probe length. The longer the probe length, the higher the directivity. The probe length has no influence with F/B ratio as well.



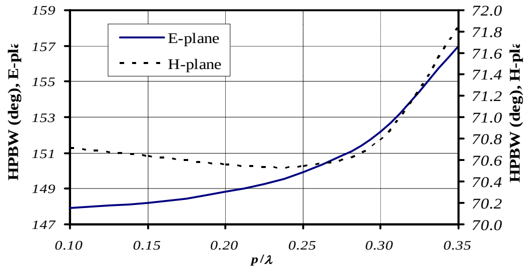

(a) Half power beamwith

(b) Directivity and front to back ratio

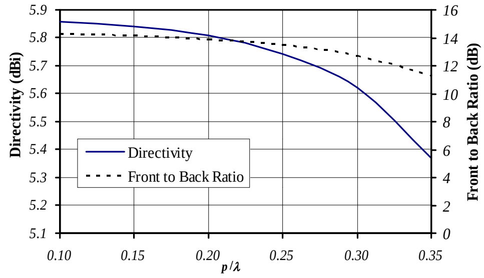
**Fig. 4:** Radiation characteristics for various *l* ($a= 0.7\lambda$, $b=0.35\lambda$, $c=0.35\lambda$, $p=0.25\lambda$, $s=0.35\lambda$ and $w=0.015\lambda$)

### 4.3 Location of the probe on *z*-axis (*p*)

In this section, the location of the probe on *z*-axis (*p*) is changed as illustrated in Fig.5. From the result in Fig.5, the half power beamwidth is increased, the directivity and the front to back ratio are decreased when the position of the probe is closed to the aperture.




(a) Half power beamwith

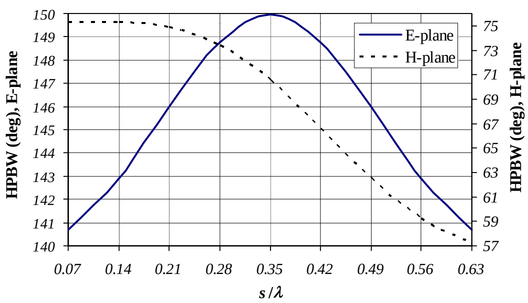


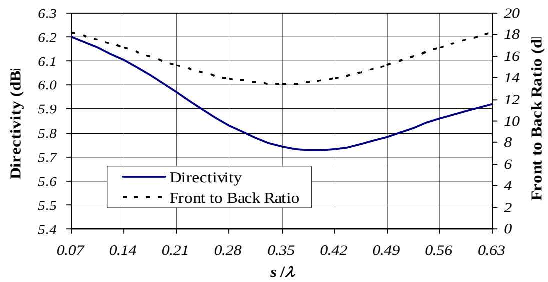

(b) Directivity and front to back ratio

**Fig. 5:** Radiation characteristics for various *p* ($a= 0.7\lambda$, $b=0.35\lambda$, $c=0.35\lambda$, $l=0.25\lambda$, $s=0.35\lambda$ and $w=0.015\lambda$)

### 4.4 Location of the probe on *x*-axis (*s*)

The final parameter is the location of the probe in *x*-axis (*s*). The radiation characteristics for various *s*-parameters are shown as the Fig.6. From Fig.6, the half power beamwidth in E-plane and F/B ratio is symmetry along the center of the aperture in *x* direction. The half power beamwidth in H-plane is decreased when the distance of the position of the probe on *x*-axis is further. The directivity is maximum when the distance from the probe is near $x=0$.




(b) Directivity and front to back ratio

(a) Half power beamwith

**Fig. 6:** Radiation characteristics for various *s* ($a= 0.7\lambda$, $b=0.35\lambda$, $l=0.35\lambda$, $l=0.25\lambda$, $p=0.25\lambda$ and $w=0.015\lambda$)

## 5. Conclusions

This paper presents the characteristic analysis of the rectangular aperture antenna excited by a probe inside cavity. The study starts with the calculation of the electric and magnetic field inside the rectangular cavity excited by the probe using the dyadic Green function. Then, the equivalent currents at the aperture are calculated to determine the radiation pattern from the aperture and the characteristics such as the half power beamwidthin E- and H-plane, the directivity and the front to back ratio for various parameters of the rectangular cavity and the probe. The results from this work are significant to optimal design of this structure to meet the application requirements in the microwave and millimeter wave devices.

## References

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