PERFORMANCE ANALYSIS OF SUBBAND ARRAYS

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1. Introduction

The use of the space-time adaptive processing (STAP) provides excellent performance of combating both the co-channel interference (CCI) and inter-symbol interference (ISI) problems in mobile communications. Usually a large number of weights are involved in a STAP system. Therefore, it either requires the inversion of a large size matrix, or makes the convergence slow.

Subband array processing has been proposed [1, 2, 3]. The localized feedback subband array enables one to greatly reduce the circuit size within each single feedback loop, and as such to improve the convergence performance. However, its performance has not been theoretically analyzed. In this paper, we propose the partial feedback scheme, and the subband array performance with these difference feedback schemes is analyzed.

2. Space-Time Adaptive Processing

We consider a base station using an antenna array of N sensors with P users where $P < N$. The user signal of interest is denoted as $s_1(n)$, whereas the signals from other users as $s_p(n)$, $p = 2, ..., P$. The received signal vector $\underline{\mathrm{x}}(n)$ at the array is expressed in discrete form as

$$\underline{\mathrm{x}}(n) = \sum_{p=1}^{P} \sum_{i=-\infty}^{\infty} s_p(i)\underline{\mathrm{h}}_p(n-i) + \underline{\mathrm{b}}(n) \tag{1}$$

where $s_p(n)$ and $\underline{\mathrm{h}}_p(n)$ are the information symbol and the channel response vector of the pth user, respectively, and $\underline{\mathrm{b}}(n)$ is the additive noise vector.

In this paper, we restrict the discussion to T-spaced equalization (i.e., sampled at the symbol rate) in order to simplify the analysis. Fractionally-spaced array processing is analogous by using extended channel model in stead of the array channel model. We make the following assumptions. A1) The user signals $s_p(n), p = 1, 2, ..., P$, are wide-sense stationary and i. i. d. (independent and identically distributed). A2) All channels $\underline{\mathrm{h}}_p, p = 1, 2, ..., P$, are linear time-invariant, and of a finite duration within $[0, D]$. A3) The noise vector $\underline{\mathrm{b}}(n)$ is zero-mean, temporally and spatially white with variance σ at each array sensor.

When an M tap FIR filter is used at the output of each array sensor, we get a vector that contains all the input values at the STAP system,

$$\mathbf{x}(n) = [\, \underline{\mathrm{x}}^T(n) \quad \underline{\mathrm{x}}^T(n-1) \quad \cdots \quad \underline{\mathrm{x}}^T(n-M+1) \,]^T . \tag{2}$$

where the superscripts ^{T} denotes transpose, and ^{H} will be used to denote conjugate transpose. The optimum weight vector under the minimum mean square error (MMSE) criterion is given by the Wiener-Hopf solution

$$\mathbf{w}_{opt} = \mathbf{R}^{-1}\mathbf{r} \tag{3}$$

with

$$\mathbf{R} = E[\mathbf{x}^*(n)\mathbf{x}^T(n)], \quad \mathbf{r} = E[\mathbf{x}^*(n)s_1(n-v)] \tag{4}$$

where the training signal is assumed to be an ideal replica of $s_1(n)$. The superscript * denotes complex conjugate. v in (4) is the delay that minimize the following MMSE.

$$\mathrm{MMSE} = E\left|\mathbf{w}_{opt}^T\mathbf{x}(n) - s_1(n-v)\right|^2 = 1 - \mathbf{r}^H\mathbf{R}^{-1}\mathbf{r}. \tag{5}$$

3. Subband Arrays with Different Feedback Schemes

A. Centralized Feedback Scheme

Performing a transform of $\mathbf{x}(n)$ by using an orthogonal matrix $\mathbf{T}$, we obtain the received signal vector at the transformed domain as

$$\mathbf{x}_T(n) = \mathbf{T}\mathbf{x}(n) \tag{6}$$

where $\mathbf{x}_T(n) = \left[(\mathbf{x}_T^{(1)}(n))^T \quad (\mathbf{x}_T^{(2)}(n))^T \quad \cdots \quad (\mathbf{x}_T^{(M)}(n))^T\right]^T$, and $\mathbf{x}_T^{(m)}$ is the signal vector at the mth transformed domain bin. Denote $\mathbf{w}_T$ as the weight vector, the optimum weight vector under the MMSE criterion is

$$\mathbf{w}_{T,opt} = \mathbf{R}_T^{-1}\mathbf{r}_T = (\mathbf{T}^T)^{-1}\mathbf{w}_{opt} \tag{7}$$

where

$$\mathbf{R}_T = E[\mathbf{x}_T^*(n)\mathbf{x}_T^T(n)] = \mathbf{T}^*\mathbf{R}\mathbf{T}^T, \qquad \mathbf{r}_T = E[\mathbf{x}_T^*(n)s_1(n-v)] = \mathbf{T}^*\mathbf{r}. \tag{8}$$

It is easy to confirm that the transformed domain array with centralized feedback scheme provides the same steady-state MMSE performance [4].

B. Localized Feedback Scheme

In this paper we consider discrete Fourier transform (DFT) as a specific example of the orthogonal transform. Denote $\mathbf{T}_o$ as the $M \times M$ DFT matrix at the output of each array sensor, the transform matrix $\mathbf{T}$ becomes

$$\mathbf{T} = \mathbf{P}_2(\mathbf{I}_N \otimes \mathbf{T}_o)\mathbf{P}_1 \tag{9}$$

where $\mathbf{I}_N$ denotes the $N \times N$ identity matrix, and $\otimes$ the Kronecker product. In (9), $\mathbf{P}_1$ and $\mathbf{P}_2$ are permutation matrices to change the order of the vector $\mathbf{x}(n)$ such that the M samples at each array sensor is transformed by the DFT matrix $\mathbf{T}_o$.

Since the signal correlation between different subbands is small, we can approximate $\mathbf{R}_T$ by ignoring its off-block-diagonal elements, yielding a block-diagonal matrix

$$\mathbf{R}_T' = \begin{bmatrix} \mathbf{R}_T^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_T^{(2)} & \cdots & \mathbf{0} \\ \vdots & & & \vdots \\ \mathbf{0} & \mathbf{0} & \vdots & \mathbf{R}_T^{(M)} \end{bmatrix} \tag{10}$$

where $\mathbf{R}_T^{(m)} = E[\mathbf{x}_T^{(m)}(n)(\mathbf{x}_T^{(m)}(n))^H]$. The inversion of (10) can be obtained by simply inverting each submatrix $\mathbf{R}_T^{(m)}$, $m = 1, ..., M$. Therefore, the inversion computation of dimension $NM \times NM$ becomes M parallel group of matrix inversion of dimension $N \times N$, as such the computation burden is greatly reduced. When recursive methods are used, M parallel control loops are used with N weights in each loop.

We also use the subband version of the reference signal

$$s_1^{(m)}(n-v) = \mathbf{T}_o^{(m)}[s_1(n-v) \; s_1(n-v-1) \; \cdots \; s_1(n-v-M+1)]^T \tag{11}$$

at each subband, where $\mathbf{T}_o^{(m)}$ is the mth row of the matrix $\mathbf{T}_o$. The cross-correlation vector between the received signal vector and the reference signal at the mth subband becomes

$$\mathbf{r}_T^{(m)} = E\left[\left(\mathbf{x}_T^{(m)}(n)\right)^* s_1^{(m)}(n-v)\right] = \left[\mathbf{T}^{(m)}\mathbf{H}_1\right]^* \mathbf{J}_v \left[\mathbf{T}_o^{(m)}\right]^T \tag{12}$$

where $\mathbf{T}^{(m)}$ is the $N \times NM$ submatrix of the matrix $\mathbf{T}$ corresponding to the mth bin, $\mathbf{J}_v$ is an $(M+D-1) \times M$ matrix expressed as $[\mathbf{0}_v^T \;\; \mathbf{I}_M \;\; \mathbf{0}_{D-1-v}^T]^T$ provided that we choose $v < D$, and $\mathbf{0}_v$ denotes the zero matrix of size $M \times v$. Therefore, the weight vector at each subband becomes

$$\mathbf{w}_T^{(m)} = (\mathbf{R}_T^{(m)})^{-1}\mathbf{r}_T^{(m)}. \tag{13}$$

The MSE of the localized feedback transformed domain array is given by

$$\text{MSE}_{LF} = 1 + \mathbf{r}'^H_T(\mathbf{R}'_T)^{-1}\mathbf{R}_T(\mathbf{R}'_T)^{-1}\mathbf{r}'_T - 2\text{Re}\left[\mathbf{r}'^H_T(\mathbf{R}'_T)^{-1}\mathbf{r}_T\right] \tag{14}$$

where

$$\mathbf{r}'_T = \left[\left(\mathbf{r}_T^{(1)}\right)^T \;\; \left(\mathbf{r}_T^{(2)}\right)^T \;\; \cdots \;\; \left(\mathbf{r}_T^{(M)}\right)^T\right]^T. \tag{15}$$

(14) implies that such localized feedback transformed domain array approach is suboptimal, and, its performance depends on the significance of the cross-correlation between signals at different subbands, and thus on the channels $\mathbf{H}_p, p = 1, 2, ..., P$.

C. Partial Feedback Scheme

Subband arrays with partial feedback is a generalization of the aforementioned two feedback schemes that provide us more flexibility in balancing the system complexity and the steady-state MSE performance. The M subbands are divided into K groups and $M_0 = M/K$ subbands are used in each feedback loop. In this case, the signal covariance matrix $\mathbf{R}_T$ is approximated by a new block-diagonal matrix $\mathbf{R}''_T$ with *larger* block size M_0N, expressed as

$$\mathbf{R}''_T = \begin{bmatrix} \mathbf{R}_T^{(G_1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_T^{(G_2)} & \cdots & \mathbf{0} \\ \vdots & & & \vdots \\ \mathbf{0} & \mathbf{0} & \vdots & \mathbf{R}_T^{(G_K)} \end{bmatrix} \tag{16}$$

where $\mathbf{R}_T^{(G_i)}$ is of dimension $M_0N \times M_0N$. Analogous to the localized feedback case, the MSE of the partial feedback transformed domain array is

$$\text{MSE}_{PF} = 1 + \mathbf{r}''^H_T(\mathbf{R}''_T)^{-1}\mathbf{R}_T(\mathbf{R}''_T)^{-1}\mathbf{r}''_T - 2\text{Re}\left[\mathbf{r}''^H_T(\mathbf{R}''_T)^{-1}\mathbf{r}_T\right]. \tag{17}$$

where

$$\mathbf{r}''_T = \left[\left(\mathbf{r}_T^{(G_1)}\right)^T \;\; \left(\mathbf{r}_T^{(G_2)}\right)^T \;\; \cdots \;\; \left(\mathbf{r}_T^{(G_K)}\right)^T\right]^T \tag{18}$$

that contains cross-correlation vectors between the subband signal vectors and their respective the subband reference signals at all the K groups.

4. Simulation Results

A three-element linear array with half wavelength inter-element spacing is considered. Two user signals are illuminating the array (P=2), and each of them has a maximum delay spread D of 5 symbols. Six multipaths are randomly generated at the delay interval, and the amplitude of each path is also random and is normalized such that the total power of the paths becomes the input power level. The angles of arrival (AOAs) of the desired signal is uniformly distributed between [-20, 20] degrees, and the AOAs of the interference signal is uniformly distributed between [10, 50] degrees. The input signal-to-noise ratio (SNR) is 20 dB for both signals. The number of subbands M changes from 4 to 32. The MSE performance at different values of M_0 are evaluated.

It is seen from Fig. 1 that the difference between different feedback schemes is large when M is relative small (M is 4 or 8 in this figure). When M is large (M is 16 or 32), the difference between them is small.

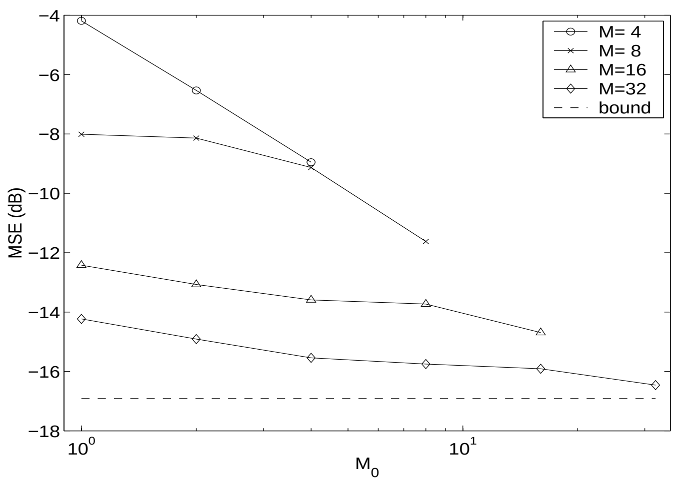

Fig. 1 Mean square error (MSE) performance versus M and M_0.

5. Conclusion

We have analyzed the mean square error (MSE) performance of subband arrays with the centralized, localized, and partial feedback schemes. It has been shown that subband arrays with localized and partial feedback schemes are generally suboptimal, and their MSE performance depends on the channel characteristics. The partial feedback scheme generalizes the subband arrays with centralized and localized feedback schemes, gives us more flexibility to balance the system complexity and the steady-state with the convergence performance.

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