CONVERGENCE PERFORMANCE OF THE CMA ADAPTIVE ARRAY UNDER WEIGHT NORM CONSTRAINT

Kazuaki TAKAO and Hiromichi MATSUDA
Faculty of Engineering, Kyoto University
Yoshidahonmachi, Sakyo-ku, Kyoto, 606-01 Japan

1 Introduction

Among various guiding principles of adaptive arrays, the CMA(Constant Modulus Algorithm) is one of the efficient ways to cope with the multipath environment although its application is limited to the cases where the desired signal has the property of constant modulus such as phase or frequency modulated signals.

One of the serious problems of the CMA results from the definition of its cost function to be minimized. There are two or more local minimum points of the cost function in the weight space. Hence, there exists two kinds of states of the convergence. One is the case where the strongest wave among all the arriving waves survives and other waves are suppressed and the other is the case where one of the weaker waves survives and the strongest is suppressed. We prefer the formar case since it results in higher output SINR(Signal to Interference plus Noise Ratio) and call it "lock" condition and call the latter case "capture" condition. It is desirable to discover how "capture" can be prevented and there have been efforts to solve the problem[1–3].

Unlike the CMP(Constrained Minimization of Power) adaptive arrays, the CMA does not perform power minimization and hence it has no interest in the output power of internal noise. Since the output power of internal noise in the "capture" condition is higher than that in the "lock" condition, the two conditions may be discriminated by the amplification factor, that is the value of weight norm.

In this paper, we investigate the CMA adaptive array under weight norm constraint and evaluate its effect on the guarantee of the "lock" condition.

2 CMA under weight norm constraint

We treat the narrow-band signals and express them by complex notation. Let W and X represent complex adjustable weights and input signals respectively, the array output y is given by $y = W^{H}X$ in which the superscript H denote the complex conjugate transpose. the CMA utilizes the property of constant envelope of the desired signal such as FM or PSK. The method exploits the fact that multipath reception and various interference sources generate incidental amplitude modulation on the received signal. The CMA detects this change of envelope and tries to minimize it by adjusting the weights. As the results, the radiation pattern is formed whose nulls are pointed to the directions of the incoming interferences. The cost function to be minimized is defined as

$$Q(W) = \mathrm{E}[\ (|y|^2 - \sigma^2)^2\ ] \tag{1}$$

where σ is the specified value which the envelope of the array output should accord with and $\mathrm{E}[\cdot]$ denotes statistical expectation.

Next, we add a constaint condition to (1) which is expressed as

$$Q(W) = \mathrm{E}[\ (|y|^2 - \sigma^2)^2\ ] + \beta W^{H}W \tag{2}$$

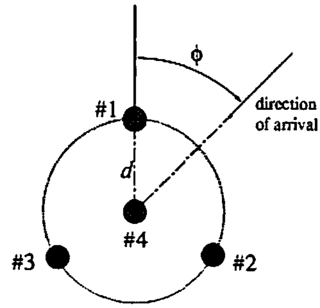

Fig. 1. Four-element Y-shape array

Table 1 Radio environment used in Sect.3

	power	direction of arrival	delay(T)
1st wave	1.0	0°	0
2nd wave	0.5	180°	1.0
internal noise	0.01	——	——

where β is called a constraint coefficient. Due to the constraint term, the second term, it is expected to minimize the output power of internal noise and get higher SINR[4].

3 Effect of the Constraint Term

Here, we analyze the effect of the constraint term and determine the appropriate value of β. We carry out computer simulation under the environment of Table 1. We use the four-element Y shaped half-wavelength equispaced array which consists of isotropic antennas, shown as Fig.1. The direction of the arriving wave ϕ is measured from the reference line connecting #1 and #4. We treat the transmitted signal as QPSK modulated and the internal noise is generated at each antenna which has equal power. Since no *a priori* information about the desired signal is available, the initial radiation pattern should be isotropic. Thus, we set the initial value of the weight vector $W \doteq [0,0,0,1]^{\mathrm{T}}$, where the superscript T denotes the transpose. A quasi-Newton method is adopted for minimizing the cost function (2).

Figure 2 shows the relation of the value of weight norm and the output SINR after convergence to β respectively. By constraining, we obtain the minimum point whose weight norm is smaller (the left figure) which reduces the output power of the internal noise. On the other hand, we cannot get $Q = 0$ because of the constraint factor which results in the reduction of the response to the direction of the desired signal. Hence, the desired output is reduced. Consequently, the output SINR is decided from these two terms as shown in the right figure. In this environment, we find the most suitable value of β being between 0.1 and 1.5.

4 Variable Constraint Coefficient Method

In this section, we introduce the new CMA system with weight norm constraint. To avoid the reduction of the response to the direction of the desired signal, it is desirable to set β to be zero

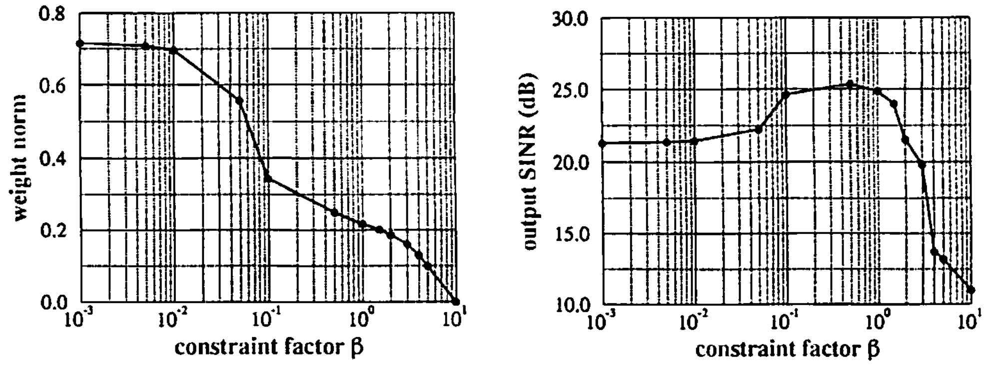

Fig. 2. The relation of the weight norm and the output SINR to the constraint factor β

after system is considered to be stable. It is desirable that the constraint term has its effect only during the transient state. Therefore β should be set in proportion to the input power of incoming waves. Thus we define β as follows:

$$\beta = \begin{cases} \frac{1}{K}\sum_{k=1}^{K}|x_k|^2 & \text{before convergence (transient state)} \\ 0 & \text{after convergence (stable state)} \end{cases} \tag{3}$$

where K is the number of elements and x_k is the input signal at k-th element.

To discriminate between transient and stable states, we use the value of cost function Q. Here we set the criterion n and ε. When the difference of Q between the successive iterations is less than the threshold ε which is a small number for n times continuously, we consider the system to be stable and set β to be zero. Otherwise we consider it to be transient. n and ε should be decided from the environment we assume. We set $n = 4, \varepsilon = 0.05$ in this paper.

5 Computer Simulation

In this section we carry out computer simulation based on the method we expressed as above. Table 2 shows the model of the four incoming waves.

Figure 3 shows the convergence behavior of the conventional CMA and the constrained CMA. The constrained CMA (variable constraint coefficient method) catches the first wave whose input power is the largest and reaches "lock" while the conventional CMA catches the second wave and reaches "capture". The values of weight norm at 40-th iteration are 0.41 and 1.15 respectively, which proves the constrained CMA can reach the minimum point of the weight plane whose norm is smaller.

We also find in Fig.3 that convergent speed is less rapid by the constrained CMA than by the conventional one. The reason is that the constrained CMA spends first 10 iterations for extracting the strongest wave and stays in the transient state.

6 Conclusion

We have introduced the new system of CMA adaptive array. The purpose of this paper is how to attain a high output SINR by preventing the system from falling into the local minimum point, "capture". Here, we modified the cost function of CMA and constrained the system to minimize the weight norm in addition to minimizing the fluctuations of the envelope. We have

Table 2 Radio environment used in Sect.5

	power	direction of arrival	delay(T)
1st wave	1.0	0°	0
2nd wave	0.5	180°	1.0
3rd wave	0.5	90°	2.0
4th wave	0.25	−30°	3.0
internal noise	0.01	——	——

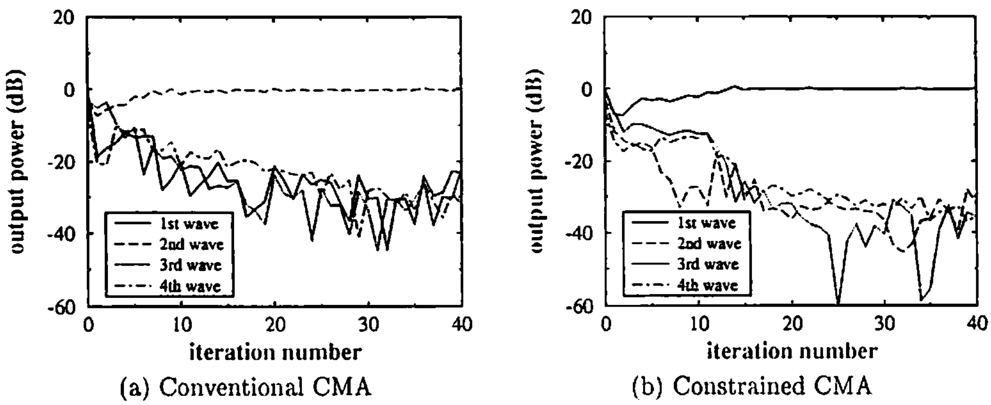

Fig. 3. The transient behavior of the two CMA systems

ascertained the proposed CMA system, the variable constrained coefficient method, can always guarantee "lock" condition and get higher SINR by means of computer simulation.

References

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