# RADIOWAVE PROPAGATION IN THE PRESENCE OF KNIFE-EDGES

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## 1. Introduction

The formulation of radiowave propagation over inhomogeneous irregular ground usually assumes that the terrain varies smoothly [1,2,3]. The situation of vertical terrain discontinuities has recently been considered using the technique of the vector form of Compensation Theorem [4]. However, less attention has been paid to the case of obstacles on the ground although various cases have been considered by Furutsu [5] and Wait [6]. A knife-edge is the simplest geometrical shape with which to model an obstacle on the ground. Previous papers [7,4] have considered the diffraction of a single knife-edge on a flat ground, where the knife-edge is infinitely extended in the transverse direction. In this paper, the work is extended to consider generally the case of either a single or cascaded knife-edges on a flat ground in the transmission path. Experimental results are also presented.

## 2. Theoretical formulation

Referring to Fig 1, the transmitter consists of the vertical electric current element $I_1 dl_1$. The receiver is the horizontal magnetic current element $K_2 dl_2$ which couples maximally to the magnetic field produced by $I_1 dl_1$. These are separated by a horizontal distance 'd' at heights $h_A$ and $h_B$ respectively above a flat ground. Between the transmitter and receiver, there exist 'N' knife-edges of different heights and electric properties and these sit on the flat ground at difference separations. The position of the n-th knife-edges is denoted by a distance $d_{1,n}$ from the transmitter and its height is $h_{c,n}$. The electric properties of the n-th knife-edge are represented by its normalised surface impedance $\Delta_s = \Delta_{k,n}$ and that of the supporting flat ground by $\Delta_s = \Delta_c$, which is assumed to be constant.

Regarding this structure as a perturbation on a reference ground which is now chosen to be the flat supporting ground with a normalised surface impedance $\Delta_o = \Delta_c$, the perturbation occurs only at the positions of those 'N' knife-edges. According to the theory developed in Reference (4), the attenuation factor at the receiving point B, defined with respect to the free space attenuation, is given by Eqn (32) in that Reference. With the application of Eqn (32) in Reference (4), to the above mentioned structure, the attenuation factor at the receiving point B in the far field is obtained as

$$W(B) = W_{10}(B) - \frac{1}{2} \sum_{n=1}^{N} \left[ \frac{jd}{\lambda_o d_{1,n} d_{2,n}} \right]^{\frac{1}{2}} \int_{d_{1,n}}^{d_{1,n}+2h_{c,n}} (\Delta_{k,n} - \Delta_e(P)) W(P) W_{20}(P) \, e^{-jk_o[R_{10}(P)+R_{20}(P)-R_o]} dr \qquad (1)$$

where $W_{10}(B)$ is the attenuation factor, with respect to the free space attenuation, at the receiving point B from the transmitter at A, and $W_{20}(P)$

is the corresponding attenuation at a field point P on the surface of the n-th knife-edge, from the horizontal magnetic current element at the point B if it operates as a transmitter. These attenuation factors are both calculated in the absence of the knife-edges, but W(P) in Eqn (1) is the attenuation factor at P in the presence of the knife-edges. Also in Eqn (1), r is the curved distance along the irregular surface, and $\Delta_e(P)=\pm(E_{z2}/\eta_o H_{\phi 2})_P$ for the front and rear faces respectively. $E_{z2}(P), H_{\phi 2}(P)$ are produced by the horizontal magnetic current element $K_2 dl_2$ at B. The integral in Eqn (1) is evaluated over the front and rear faces of the knife-edges. Since the rear faces are not directly illuminated, it is assumed that W(P)=0 on these rear faces, so that Eqn (1) reduces to

$$W(B) = W_{10}(B) - \frac{1}{2}\sum_{n=1}^{N}\left[\frac{jd}{\lambda_o d_{1,n} d_{2,n}}\right]^{\frac{1}{2}} \int_{d_{1,n}}^{d_{1,n}+h_{c,n}} (\Delta_{k,n}-\Delta_e(P))W(P)W_{20}(P)\, e^{jk_o(R_{o1}(P)+R_{o2}(P)-R_o)}dr \quad (2)$$

If the geometrical structure in Fig 1 satisfies

$$|d_{1,n} - d_{1,n-1}| >> \text{Maximum } (h_{c,n}, h_A, h_B) \quad (3)$$

on the front faces of the knife-edges, $\Delta_e(P)\simeq-1$. In the integrand of Eqn (2), W(P) on each of the front faces is unknown. However it can be approximated, using geometric optics, by considering the field on the front face of the n-th knife-edge to be the resultant field of an incident wave and a reflected wave. The incident wave, denoted by $W_{1,n-1}(P)$, is the wave propagating to the front face of the n-th knife-edge through the diffraction of (n-1) knife-edges in front of it as if the n-th and those knife-edges behind were absent. The reflected wave is the wave reflected by the front face of the n-th inife-edge with a local reflection coefficient $R_{vk,n}$ as if the knife-edge were an infinitely extended plane. From this assumption, there will exist higher order reflections between two knife-edges, but these are neglected under the assumption of Eqn (3), so that $W(P) = (1+R_{vk,n})W_{1,n-1}(P)$. Since $R_{vk,n} = (1-\Delta_{k,n})/(1+\Delta_{k,n})$, we have

$$W(P) = \frac{2}{(1+\Delta_{k,n})} W_{1,n-1}(P)$$

Substituting it into Eqn (2) with $\Delta_e(P)\simeq-1$ as evaluated above yields

$$W(B) = W_{10}(B) - \sum_{n=1}^{N}\left[\frac{jd}{\lambda_o d_{1,n} d_{2,n}}\right]^{\frac{1}{2}} \int_{d_{1,n}}^{d_{1,n}+h_{c,n}} W_{1,n-1}(P)W_{20}(P)e^{-jk_o(R_{o1}(P)+R_{o2}(P)-R_o)}dr \quad (4)$$

for each knife-edge. Changing the variable from r to $\xi_n$ using the substitution $r=d_{1,n}+\xi_n$, allows Eqn (4) to be expressed as

$$W(B) = W_{10}(B) - \sum_{n=1}^{N} \left[ \frac{jd}{\lambda_o d_{1,n} d_{2,n}} \right]^{\frac{1}{2}} \int_{o}^{h_{c,n}} W_{1,n-1}(P) W_{20}(P) e^{-jk_o(R_{o1}(P)+R_{o2}(P)-R_o)} d\xi_n^o \quad (5)$$

which is the final integral expression for W(B), and ready for numerical solution.

## 3. Experiments

The validity of Eqn (5) was first tested with N=1 using microwave modelling at $f_o$=9.6GHz. In the experimental set up the flat supporting ground and the knife-edge were both made of aluminium sheet. The knife-edge had a height $h_{c,1}=\lambda_o$=3.125cm. Keeping d=80cm=25.6$\lambda$ unchanged, and changing $d_{1,1}$, i.e. the position of the knife-edge between the $\lambda/4$ monopole transmitter with $h_A$=0 and the receiver consisting of a tapered waveguide probe at $h_B$=0.5cm=0.16$\lambda$, the measured data are plotted as small circles in Fig 2 together with the theoretical result shown in solid line. The theoretical and experimental results are in very good agreement. the same measure of agreement was also obtained when $d_{1,1}$=40cm=12.8$\lambda_o$ was kept unchanged and d was varied, as shown in Fig 3.

Keeping the transmitter and receiver heights unchanged, but placing two identical aluminium knife-edges of height 0.91$\lambda_o$ at $d_{1,1}$=9.6$\lambda_o$ and $d_{12}$=19.2$\lambda_o$ respectively, the measured data when d is varied behind the second knife-edge are plotted as small 'x's in Fig 4, together with the corresponding theoretical results predicted from eqn (5). The experimental result follows the theoretical prediction, with only a slight variation which is believed to be due to the weakness of the signal after diffractions over two knife-edges.

## 4. Conclusions

A general formulation for inhomogeneous irregular ground has been applied to a structure of N knife-edges sitting on a flat supporting ground. The attenuation factor obtained for this case involves a height integration. The experimental results for N=1 and 2 knife-edges agree reasonably well with the theoretical predicted results.

## 5. References


1. Hufford G.A., Q.J. Appl. Math., Vol. 9(4), P 391-404, 1952
2. Ott R.H., Radio Sci., Vol. 6(4), P 429-435, 1971
3. Monteath G.D., Application of electromagnetic reciprocity Principle, Pergamon, New York, 1973
4. Wu Z. et al., Radio Science, Vol. 23(1), P 33-40, 1988
5. Furutsu K., J. of Rad. Res. Labs., Vol. 4, P 135-153, P 349-393, 1957
6. Wait J.R., Radio Science, Vol. 3(10), P 995-1003, 1968
7. Wu Z. et al., Int. J. Electronics, Vol. 65(1), P 85-89, 1988

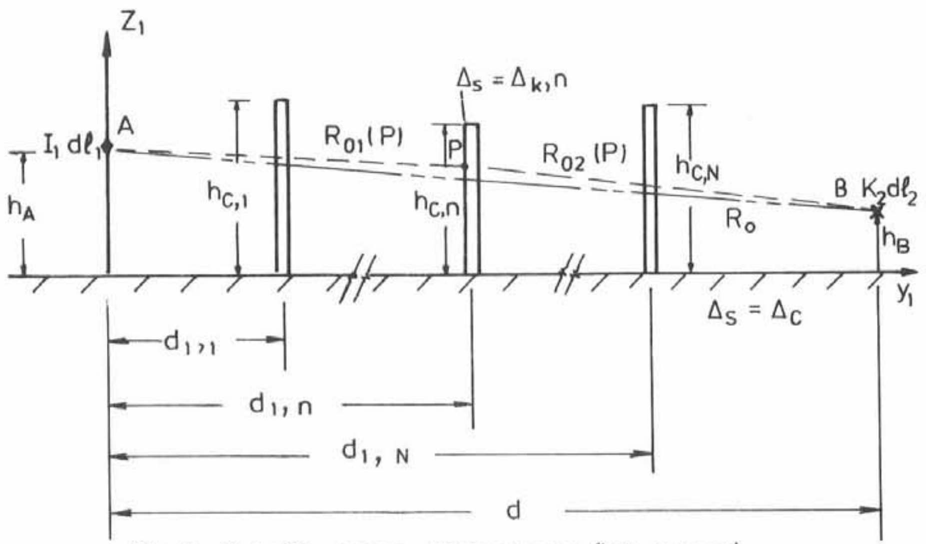


Fig.1 N knife-edges sitting on a flat ground

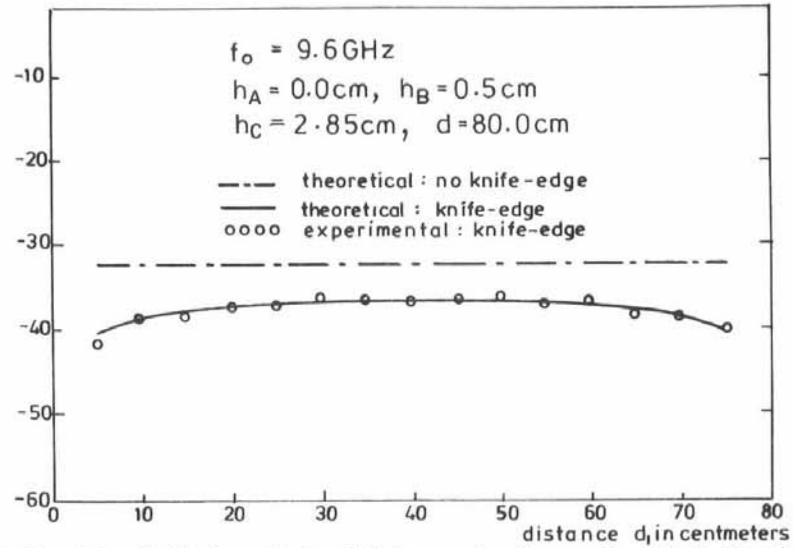


Fig. 2 Receiving field strength Ez at distance d vs the position of a knife edge.

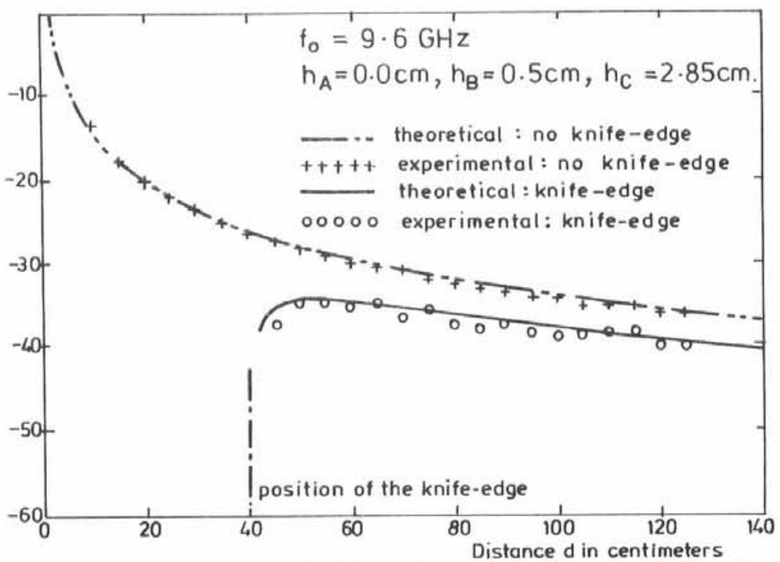


Fig. 3 Field strength Ez along radial direction with/without a knife-edge

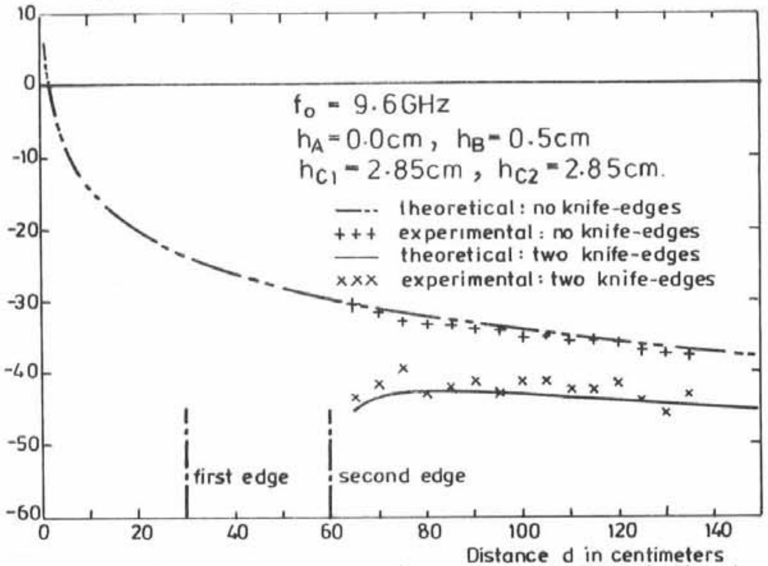


Fig. 4 Field strength Ez with distance d along a path of two knife-edges