SCATTERING FROM A CAVITY-BACKED SLIT ON A GROUND PLANE -- TM CASE

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** Supported by the National Science Council, Republic of China, under Grant NSC 77-0404-E002-27.

INTRODUCTION - Recently, the finite element method is proposed to compute the cavity admittance matrix for a 2D arbitrary cavity in the problem of TE scattering from a cavity-backed slit [1]. With the finite element method the medium in the cavity can even be inhomogeneous. The computed admittance matrix of the cavity, when combined with an admittance matrix of the equivalent magnetic current on the exterior side of the closed aperture, can solve a large class of 2D aperture coupling problems. In this paper we will apply the same method to TM scattering problems. The equivalent magnetic current on the slit will be solved.

PROBLEM AND FORMULATION - Consider a 2D cavity-backed slit which is illuminated by a plane wave (Fig. 1). By a TM assumption, the incident as well as the excited electric field is in the y-direction, and the magnetic field is on the xz plane. The whole problem can be formulated by the generalized network formulation [2].

In the generalized network formulation, an equivalent magnetic surface current

$$M = \hat{z} \times E = \hat{x} \; M(x) \tag{1}$$

and its negative are attached on the exterior and the interior side of the closed aperture, respectively. Assume that M(x) is expanded into

$$M(x) = \sum V_q \; b_q(x) \tag{2}$$

where $b_q(x)$ are the triangular basis functions, and V_q are the unknown coefficients to be solved. Then by the continuitity of the tangential magnetic field at the aperture, and by taking inner product to the equation, we achieve

$$([Y^R] + [Y^C])[V] = [I] \qquad (3)$$

The source term [I] is related to the short-circuit magnetic field on the closed aperture excited by the obliquely incident plane wave. Its value can be easily evaluated by elementary electromagnetics.

The matrix element of the radiation admittance matrix [Y^R] relates to the aperture magnetic field due to a magnetic current segment radiating in a half space. By the image principle, it may be expressed as an integral involving Hankel functions of the second kind, and can be computed by 10-point Gaussian quadrature and special treatment of the singularity.

The matrix element of the cavity admittance matrix [Y^C] relates to the cavity magnetic field due to a magnetic current segment b_q attached on the interior side of the closed aperture. It is to be computed by the finite element method. To compute it, we must consider a 2D cavity excited by a magnetic surface current $b_q(x)$. For simplicity, we use (E, H) to stand for the field in the cavity problem. By a procedure similar to those given in [1], we obtain a variational equation for the magnetic field H(x, z) in the cavity

$$\delta L = 0$$

$$L = \iint [\frac{1}{\varepsilon_r} \nabla \times H \bullet \nabla \times H - k^2 \mu_r H \bullet H] dxdz + 2j\frac{k}{\eta}\int b_q \hat{x} \bullet H \, dx \qquad (4)$$

Here, ε_r and μ_r are the relative permittivity and permeability of the medium in the cavity, respectively; η is the intrinsic impedance of free space, and k is the wave number in free space.

The variational equation (4) is solved for every q via the finite element method. In applying the finite element method, the cavity domain is first subdivided into several subdomains (elements). The magnetic field in each element is then expanded by parabolic basis functions [3]. The Ritz procedure [3] is applied to get a sparse matrix equation. This matrix equation is solved by the modified frontal solution algorithm [1] , which saves memory and avoids unnecessary operations on zero entries. The solved cavity magnetic field is finally manipulated to get the cavity admittance matrix element.

NUMERICAL RESULTS - Combining [Y^R] and [Y^C], from (3) we can solve the equivalent magnetic current on the slit. Figures 2 and 3 show some typical numerical results. In Fig. 2, we check our results with the one obtained by mode matching. In this figure, the solid (normal incidence) and the dashed (oblique incidence, $\theta = 60^o$) curves represent the equivalent magnetic current on the slit computed by the mode matching, while the black squares are those evaluated by the finite element techniques. For the mode-matching curves, 100 modes were added for each cavity admittance matrix element. For the finite element points, 20 X 20 elements (1023 nodes) were applied. Both results were obtained on a VAX-11/780 minicomputer. The finite element results match well with the mode-matching results. Although the mode matching is much faster, it can not handle cavities of arbitrary shape.

In Fig. 3 we show the power of the finite element technique in dealing with arbitrary cavitiy and inhomogeneity. In this figure, we present the equivalent magnetic current on the opening of a staggered inlet coated by Crowloy BX113 ($\varepsilon_r = 12 - j0.144$, $\mu_r = 1.74 - j3.306$) under normal incidence, which can not be solved by mode-matching. The axis of the inlet is described by a polynomial

$$\frac{x}{0.6} = -3\left(\frac{z}{0.8}\right)^2 + 2\left(\frac{z}{0.8}\right)^3$$

The figure shows the magnetic currents under various coating thicknesses.

REFERENCES

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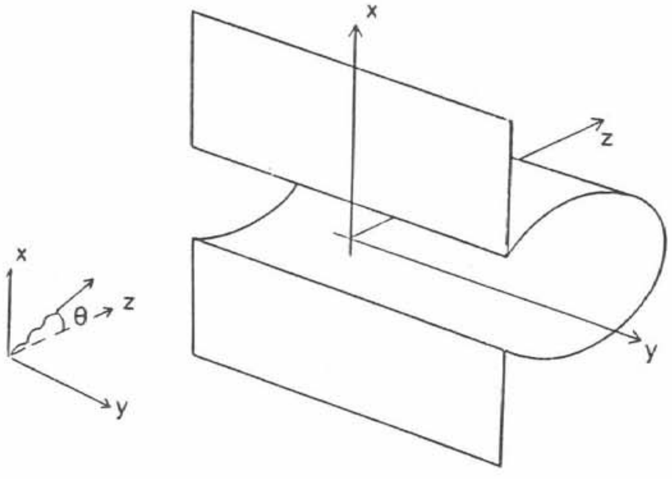


Fig. 1

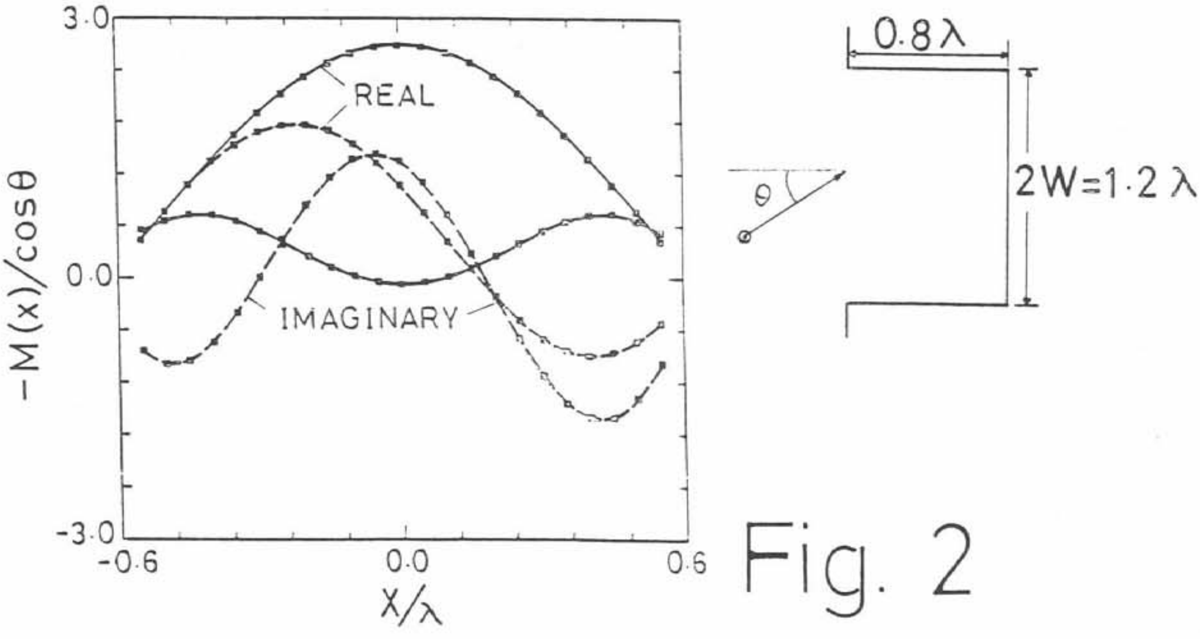
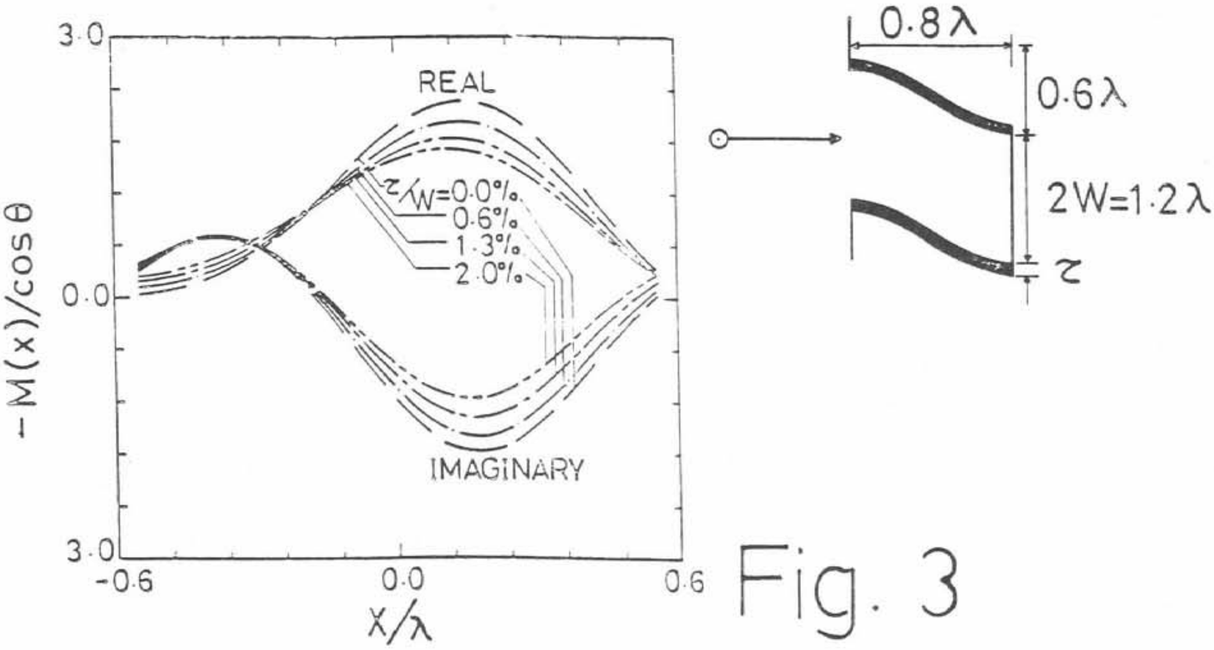

3.0
REAL
IMAGINARY
0.0
-3.0
-0.6
0.0
0.6
X/λ
-M(x)/cosθ
θ
0.8λ
2W=1.2λ

Fig. 2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REAL
τ/W=0.0%
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0.0
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0.6
X/λ
-M(x)/cosθ
0.8λ
0.6λ
2W=1.2λ
τ

Fig. 3