

CALCULATION OF THE SCATTERING AND RECEIVING PROPERTIES  
OF AN ANTENNA COVERED BY A LOSSY LAYER

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1. Introduction

Conformal aircraft antennas are often covered by lossy materials in an effort to reduce their radar cross-sections. It is therefore important to understand the relationship between the power received and the backscattered field as a function of the parameters of the cover material. While accurate models of aircraft antennas, such as cavity backed dipoles, are currently being analyzed by the authors using the finite difference time domain method, a simpler model can give much insight into the behavior of covered antennas.

2. Geometry

A simple but informative model of a covered aircraft antenna is shown in figure 1. Here a monopole above a conducting ground screen is covered by a lossy dielectric sheet of infinite extent and is illuminated by an incident plane wave. The monopole is terminated in a load impedance, and the power delivered to the load and the field scattered by the antenna are to be determined. This is done by solving the transmitting and scattering cases and using the concept of superposition.

3. Integral Equation for the Monopole Current

An integral equation for the current on the monopole is established by using the boundary condition on the antenna surface

$$E_z^s(z) = -E_z^i(z) \tag{1}$$

where  $E_z^s(z)$  is the scattered field maintained on the monopole, and  $E_z^i(z)$  is the impressed field due to the incident wave in the scattering case or the load voltage in the transmitting case. Assuming the monopole to be a thin wire with its current concentrated on axis, the scattered and impressed fields can be written as

$$E_z^s(z) = k^2 \Pi_z^s(z) + \frac{\partial^2 \Pi_z^s(z)}{\partial z^2} \tag{2}$$

$$E_z^i(z) = \begin{cases} V_0 \delta(z+d) & \text{transmitting case} \\ W \cos(kz \sin \theta) & \text{scattering case} \end{cases} \tag{3}$$

where  $W$  is a constant depending on the lossy parameters of the dielectric sheet[1], and  $\Pi_z^s(z)$  is the scattered Hertzian potential. Due to the angular symmetry of the monopole, the Hertzian

potential is most conveniently expressed in terms of a Sommerfield integral

$$\Pi_z^s(z) = \frac{1}{2\pi} \int_0^\infty \left[ \int_{-d}^z \frac{I_z(z')}{j\omega\epsilon_3} \tilde{G}_{zz}^> dz' + \int_z^0 \frac{I_z(z')}{j\omega\epsilon_3} \tilde{G}_{zz}^< dz' \right] J_0(\lambda\rho) \lambda d\lambda \quad (4)$$

with

$$\tilde{G}_{zz}^> = \frac{e^{-j\vec{k}\cdot\vec{r}}}{p_3} \cosh[p_3(z'+d)] F(z)$$

$$\tilde{G}_{zz}^< = \frac{e^{-j\vec{k}\cdot\vec{r}}}{p_3} \cosh[p_3(z+d)] F(z')$$

$$F(x) = \frac{Q \cosh(p_3 x) - Z \sinh(p_3 x)}{Q \sinh(p_3 d) + Z \cosh(p_3 d)}$$

$$Q = p_3 \epsilon_2 [\epsilon_1 p_2 \cosh(p_2 t) + \epsilon_2 p_1 \sinh(p_2 t)]$$

$$Z = p_2 \epsilon_3 [\epsilon_1 p_2 \sinh(p_2 t) + \epsilon_2 p_1 \cosh(p_2 t)]$$

$$p_j^2 = \lambda^2 - k_j^2 ; k_j^2 = \omega^2 \epsilon_j \mu_j ; j = 1, 2, 3$$

Here  $I_z(z)$  is the unknown axial current. Substituting (2) into the boundary condition (1) yields an integro-differential equation for  $I_z(z)$ . Solving the differential equation gives the Hallen-type integral equations

$$\Pi_z^s(z) = C \cos[k_3(z+d)] - \frac{V_0}{2k_3} \sin[k_3 |z+d|] \quad \text{transmitting case} \quad (5a)$$

$$\Pi_z^s(z) = C \cos(k_3 z) - \frac{W}{k_3^2} \left[ \frac{\cos(k_3 z \sin\theta) - \cos(k_3 z)}{\cos^2\theta} \right] \quad \text{scattering case} \quad (5b)$$

These equations are easily solved using the moment method with pulse function expansion and point matching.

#### 4. Calculation of Scattered Field

The scattered field in the region above the lossy layer can be calculated using the superposition of scattering and transmitting currents via

$$E_z = \frac{\eta_1}{j\pi k_1} \sum_{n=1}^N a_n \int_0^\infty \frac{\cosh(p_3 \delta_n) \sinh[p_3(\frac{\Delta}{2})]}{\chi(\lambda)} e^{-p_1(z-t)} J_0(\lambda\rho) \frac{\lambda^3}{p_3^2} d\lambda \quad (6)$$

$$E_\rho = \frac{\eta_1}{j\pi k_1} \sum_{n=1}^N a_n \int_0^\infty \frac{\cosh(p_3 \delta_n) \sinh[p_3(\frac{\Delta}{2})]}{\chi(\lambda)} e^{-p_1(z-t)} J_1(\lambda\rho) \frac{p_1 \lambda^2}{p_3^2} d\lambda \quad (7)$$

where  $a_n$  is the coefficient of the  $n$ 'th pulse expansion function,  $\Delta$  is the pulse width,  $\delta_n = (n - \frac{1}{2})\Delta$ , and

$$\chi(\lambda) = \frac{\epsilon_1}{\epsilon_3} \sinh(p_3 d) \cosh(p_2 t) + \frac{\epsilon_1 p_2}{\epsilon_2 p_3} \cosh(p_3 d) \sinh(p_2 t) \\ + \frac{\epsilon_2 p_1}{\epsilon_3 p_2} \sinh(p_3 d) \sinh(p_2 t) + \frac{p_1}{p_3} \cosh(p_3 d) \cosh(p_2 t)$$

### 5. Numerical result

Figure 2 shows the relative power received by a short monopole plotted against the thickness of the dielectric layer. Also shown is the relative power density of the backscattered wave. While the load power is reduced by the presence of the lossy layer, the backscattered field is reduced by a greater amount. Similar results are obtained using lossy magnetic materials as shown in figure 3.

### REFERENCE

[1] M.A. Blischke, E.J. Rothwell, K.M. Chen, and J.L. Lin, "Receiving and Scattering characteristics of a circular patch antenna array", *Journal of Electromagnetic Waves and Applications*, vol. 2, No. 3/4, 1988, pp 353-378.

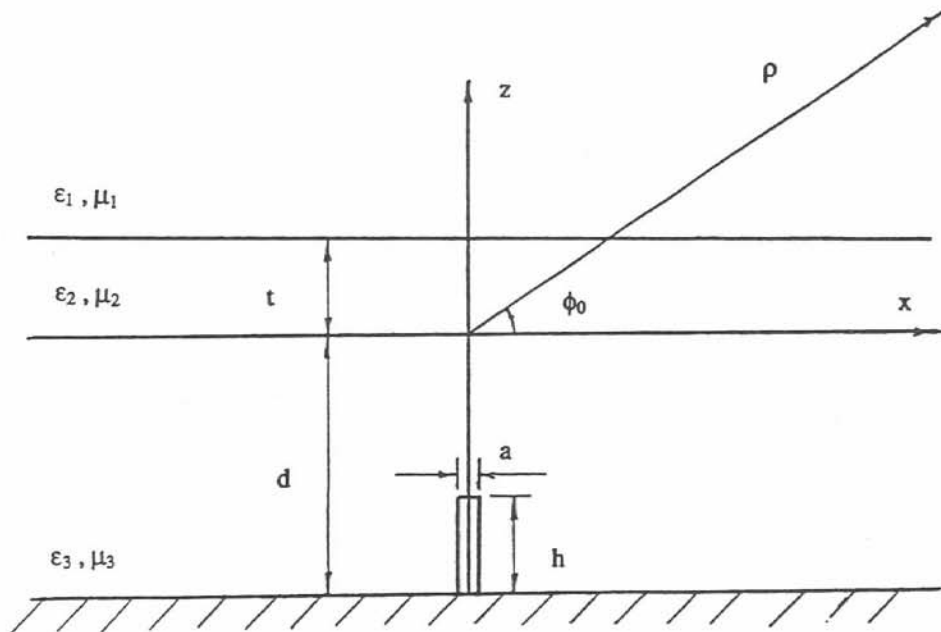


Fig. 1. geometry of antenna covered by lossy layer

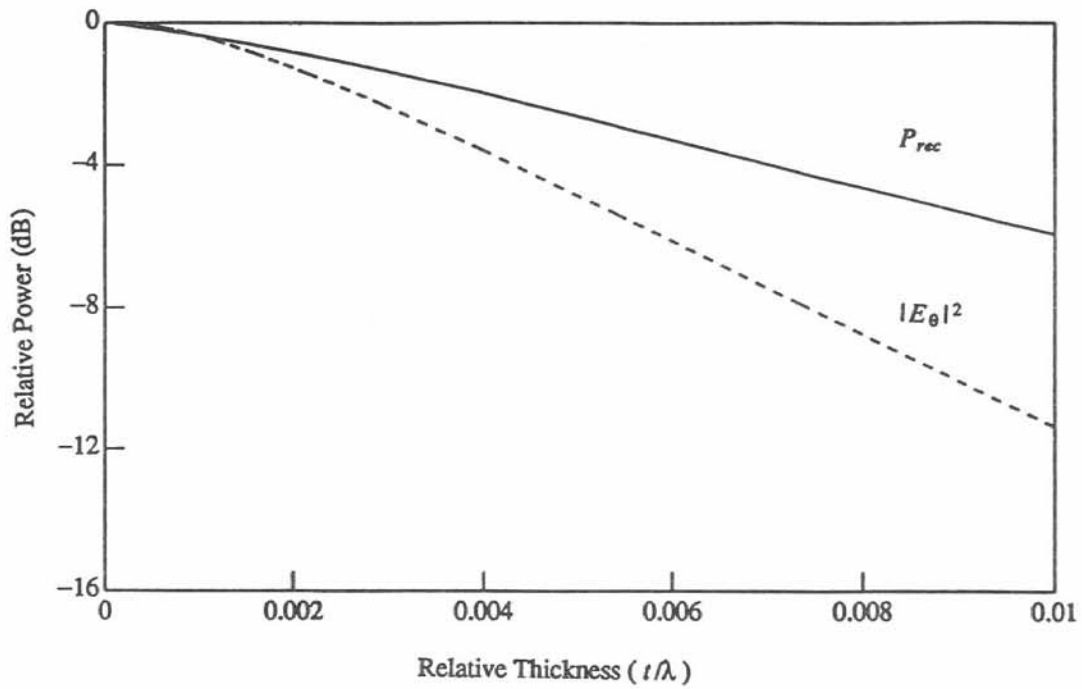


Fig. 2. Received power (solid line) and back scattered power density (dashed line) normalized to the case  $t=0$  for  $\frac{d}{\lambda} = 0.2$   $\frac{a}{\lambda} = 0.001$   $\frac{p}{\lambda} = 8.0$   $\frac{h}{\lambda} = 0.1$   $\phi_0 = 15^\circ$   $Z_l = 50 \Omega$   $\epsilon_2 = 10 - j200$   $\mu_2 = 1$

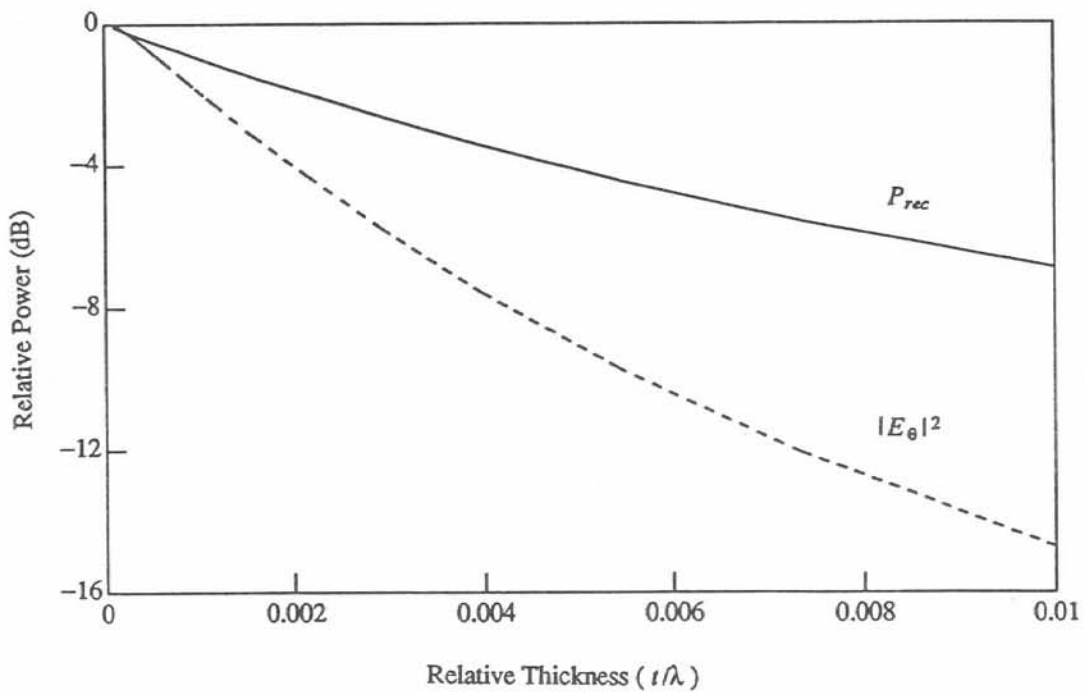


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